Bounds for the constant of analyticity via complexification techniques

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Algebraic complexification

Given a real vector space E, we consider its **complexification**

$$\tilde{E} = \mathbb{C}\text{-span}(E) = E \oplus iE = \{x + iy : x, y \in E\}.$$

Complexifications

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Alternatives:

- $\tilde{E} = \mathbb{R}^2 \otimes E$, identifying $z = x + iy$ with $e_1 \otimes x + e_2 \otimes y$.
- $\tilde{E} = \mathscr{L}(\mathbb{R}^2, E)$, identifying $z = x + iy$ with $S \in \mathscr{L}(\mathbb{R}^2, E)$ defined by $S(e_1) = x$ and $S(e_2) = y$.

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Norms on complexifications

If $\| \bullet \|$ is a norm on E, how can we define a norm $\nu(\bullet)$ on $\tilde{E}$?

The norm $\nu(\bullet)$ is called **reasonable**

$$\nu(x; \tilde{E}) = \|x\|_E \quad \text{and} \quad \nu(x + iy; \tilde{E}) = \nu(x - iy; \tilde{E}) \quad \forall x, y \in E.$$

Some Examples

- If $E = \ell_1(\mathbb{R})$ and we want $\tilde{E} = \ell_1(\mathbb{C})$, we use the **Bochnak norm**:

$$\mathfrak{b}(x + iy) := \inf \left\{ \sum |a_j| \|u_j\| : x + iy = \sum a_j u_j,\ a_j \in \mathbb{C},\ u_j \in E \right\}.$$

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This corresponds to using the *projective norm* on $\mathbb{R}^2 \otimes E$.

- If $E = \ell_\infty(\mathbb{R})$ and we want $\tilde{E} = \ell_\infty(\mathbb{C})$, we use the **Taylor norm**:

$$\mathfrak{t}(x+iy) := \sup_{0\le t\le 2\pi} \|x\cos(t) - y\sin(t)\|.$$

This corresponds to using the *injective norm* on $\mathbb{R}^2 \otimes E$.

Extension of Functions

- Given $T \in \mathscr{L}(E; \mathbb{R})$, there exists a unique extension $\tilde{T} \in \mathscr{L}(\tilde{E}; \mathbb{C}$:

$$\tilde{T}(x + iy) = \tilde{T}(x) + i\tilde{T}(y) = T(x) + iT(y).$$

- Given $L \in \mathscr{L}({}^k E; \mathbb{R})$, by multilinearity, there exists a unique extension $\tilde{L} \in \mathscr{L}({}^k \tilde{E}; \mathbb{C})$.
- Given $P \in \mathscr{P}({}^k E; \mathbb{R})$, if L is the unique symmetric multilinea form with $P(x) = L(x, \ldots, x)$ then we must have:

$$\tilde{P}(x + iy) = \widetilde{L}\left((x + iy)^k\right).$$

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Our Problem

What happens to the norms of polynomial extensions?

Given $P : E \to \mathbb{R}$ and a reasonable norm ν, we always have:

$$\begin{aligned} \|P\| &= \sup\{|P(x)| : x \in E,\ \|x\| = 1\} \\ &\leq \sup\{|\tilde{P}(z)| : z \in \tilde{E},\ \nu(z; \tilde{E}) = 1\} \\ &= \|\tilde{P}\|_\nu. \end{aligned}$$

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We seek a constant $C > 0$ such that: $\|\tilde{P}\|_\nu \leq C\|P\|$.

Complexification Constants

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Complexification Constants

$$\begin{aligned} \mathbf{c}_\nu(k, E) &= \inf\left\{M > 0 : \|\widetilde{P}\|_\nu \leq M\|P\|,\ \text{for all } P \in \mathscr{P}(^k E)\right\} \\ &= \sup\left\{\|\widetilde{P}\|_\nu : P \in \mathscr{P}(^k E),\ \|P\| \leq 1\right\}. \end{aligned}$$

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$$\mathbf{c}_\nu(E) = \limsup_{k\to\infty} \left(\mathbf{c}_\nu(k, E)\right)^{1/k} = \lim_{k\to\infty} \left(\mathbf{c}_\nu(k, E)\right)^{1/k}.$$

Proposition (Muñoz, Sarantopoulos, and Tonge)

Given a real Banach space E and a reasonable norm ν

$$\mathbf{c}_\nu(k,E) \leq 2^{k-1}.$$

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Proposition (R., Dimant, Galicer)

$$\mathbf{c}_{\mathfrak{b}}(\ell_1^2(\mathbb{R}) \geq \sqrt[4]{2}$$

Proof

We define $P \in \mathscr{P}({}^{8k}\ell_1^2(\mathbb{R}))$ as

$$\begin{aligned} P(x,y) &= (xy)^{2k}\,\mathrm{Re}(x+iy)^{4k} \\ &= (xy)^{2k}\frac{\langle (x,y),(1+i)\rangle^{4k} + \langle (x,y),(1-i)\rangle^{4k}}{2}. \end{aligned} \tag{1}$$

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Then,

$$|P(x,y)| \leq |xy|^{2k}(x^2+y^2)^{2k} \leq \frac{1}{2^{6k}}.$$

Since $|P(\frac{1}{2},\frac{1}{2})| = \frac{1}{2^{6k}}$, we obtain:

$$\|P\| = \frac{1}{2^{6k}}.$$

With thi norm $\widetilde{\ell_1^2(\mathbb{R})} = \ell_1^2(\mathbb{C})$, and $\widetilde{P} \in \mathscr{P}({}^{8k}\ell_1^2(\mathbb{C}))$ is given by (1):

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Then,

$$\|\widetilde{P}\| \geq \left|\widetilde{P}\left(\tfrac{1}{2}, \tfrac{i}{2}\right)\right| = \frac{1}{2^{4k+1}} = 2^{2k-1} \cdot \frac{1}{2^{6k}} = 2^{2k-1}\|P\|.$$

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Therefore,

$$\mathbf{c}_{\mathfrak{b}}(8k, \ell_1^2(\mathbb{R}) \geq 2^{2k-1}, \quad \text{in particular} \quad \mathbf{c}_{\mathfrak{b}}(\ell_1^2(\mathbb{R}) \geq \lim_{k\to\infty} \sqrt[8k]{2^{2k-1}} = \sqrt[4]{2}.$$

Power Series and Radii of Convergence

In a Banach space X, consider an analytic function f with

$$S(f,0) = \sum_{j=1}^{\infty} P_j(x)$$

its power series centered at 0 and radius of convergence

$$R(f,0) = 1.$$

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Fix $a \in B(0,1)$ and consider the series of f centered at a.

- If X is a complex Banach space $R(f,a) \geq 1 - \|a\|$.
- If X is a Hilbert space (real or complex) $R(f,a) \geq 1 - \|a\|$.

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Question

What happens in other real spaces?

Constant of analyticity

For any $S(f, 0) = 1$ define

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Some Lower Bounds

One of the firsts (A.E. Taylor -1938) via complexification:

$$\frac{1}{e\sqrt{2}}\leq \mathscr{A}(E).$$

With sharper complexification:

$$\frac{1}{2}\leq \mathscr{A}(E).$$

Best known lower bound (P. Hájek and M. Johanis):

1

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Theorem (Boyd, Ryan, Snigireva)

For any real Banach space

$$\mathscr{A}(\ell_1(\mathbb{R})) \leq \mathscr{A}(E).$$

Upper Bounds

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For any real Banach space

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Theorem

$$\frac{1}{\mathbf{c}_{\mathfrak{b}}(\ell_1^2(\mathbb{R}))} \leq \mathscr{A}(\ell_1^2(\mathbb{R})) \leq \frac{1}{2} + \frac{1}{2\mathbf{c}_{\mathfrak{b}}(\ell_1^2(\mathbb{R}))} < 1.$$

Proof

Fix a norm one polynomial $P \in \mathscr{P}({}^k\ell_1^2(\mathbb{R}))$ such that

$$\sqrt[k]{\|\tilde{P}\|} > \mathbf{c}_{\mathfrak{b}}(\ell_1^2(\mathbb{R}))(1-\varepsilon).$$

Take a norm one vector $(\alpha, \beta) \in \widetilde{\ell_1(\mathbb{R})} = \ell_1(\mathbb{C})$ such that

$$\tilde{P}(\alpha, \beta) = \|\tilde{P}\|.$$

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Some asumption

- $|\alpha| \geq \frac{1}{2} \geq |\beta|$.
- $\alpha \in \mathbb{R}_{>0}$.
- There is m such that $\tilde{P}(\alpha, \beta)^m \in \mathbb{R}_{>0}$.

Let f on $B_{\ell_1^2(\mathbb{R})}$ be given by the series

$$S(f,0) = \sum_{n=1}^{\infty} P^{mn}(x).$$

This series has radius of convergence

$$R(f,0) = \frac{1}{\limsup_{n\to\infty} \sqrt[mnk]{\|P^{mn}\|}} = \frac{1}{\sqrt[k]{\|P\|}} = 1.$$

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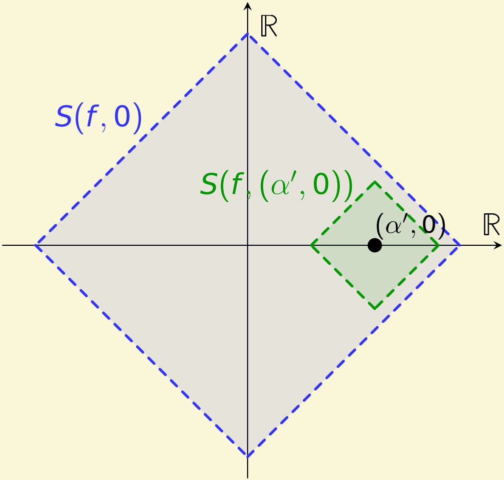
Let

$$\alpha' = \frac{\alpha}{\sqrt[k]{\|\widetilde{P}\|}} < \frac{\alpha}{\mathbf{c}_{\mathfrak{b}}(\ell_1^2(\mathbb{R})))(1-\varepsilon)},$$

and consider

$$S(f,(\alpha',0)) = \sum_{j=1}^{\infty} Q_j(x-(\alpha',0)) \quad \text{and} \quad R(f,(\alpha',0)).$$

We want to see that $R(f, (\alpha', 0)) \leq \frac{|\beta|}{(1-\varepsilon)}$



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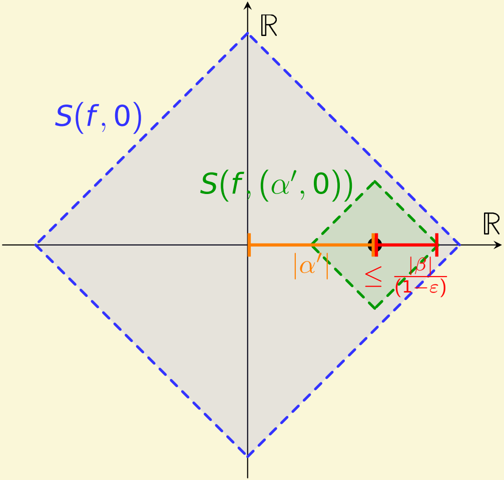

$$|\alpha'| + \frac{|\beta|}{(1-\varepsilon)} < \frac{|\alpha|}{\mathbf{c}_{\mathfrak{b}}(\ell_1^2(\mathbb{R}))(1-\varepsilon)}| + \frac{|\beta|}{(1-\varepsilon)} \leq \frac{1}{1-\varepsilon}\left(\frac{1}{2\mathbf{c}_{\mathfrak{b}}(\ell_1^2(\mathbb{R}))} + \frac{1}{2}\right)$$

Take $S(\tilde{f},0)=\sum_{n=1}^{\infty}\widetilde{P}^{mn}(x)$, then

$$R(\tilde{f},0)=\frac{1}{\limsup_{n\to\infty}\sqrt[mnk]{\|\tilde{P}^{mn}\|_{\mathfrak{b}}}}=\frac{1}{\sqrt[k]{\|\tilde{P}\|_{\mathfrak{b}}}}.$$

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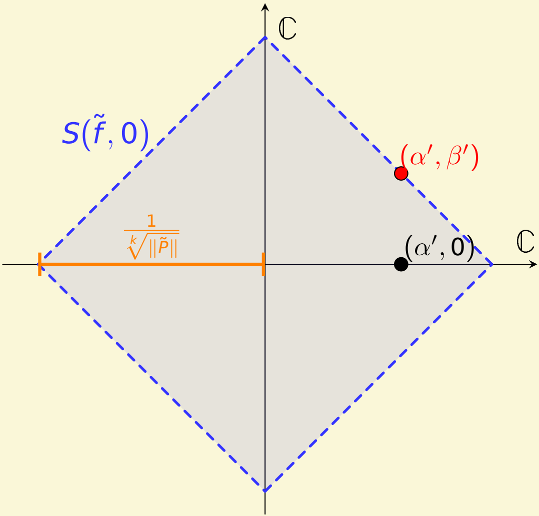
Consider $S(\tilde{f},(\alpha',0))=\sum_{j=1}^{\infty}\widetilde{Q}_j(z-(\alpha',0))$, then

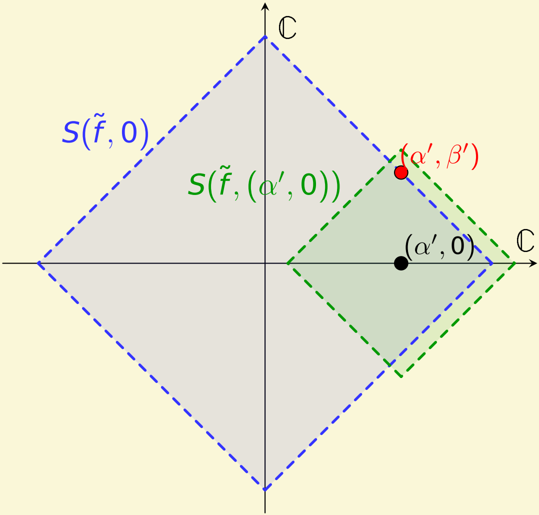
$$\begin{aligned} R(f,(\alpha',0)) &= \frac{1}{\limsup_{j\to\infty}\sqrt[j]{\|Q_j\|}} \\ &\leq \frac{\sqrt[j]{\limsup_{j\to\infty}\mathbf{c}_{\mathfrak{b}}(j,\ell_1^2(\mathbb{R}))}}{\limsup_{j\to\infty}\sqrt[j]{\|\widetilde{Q}_j\|_{\mathfrak{b}}}} \\ &= \mathbf{c}_{\mathfrak{b}}(\ell_1^2(\mathbb{R}))R(\tilde{f},(\alpha',0)). \end{aligned}$$

New goal: bound $R(\tilde{f},(\alpha',0))$.

If $\beta' = \frac{\beta}{\sqrt[k]{\|\tilde{P}\|}}$, $\widetilde{P}^{mn}(\alpha', \beta') = \widetilde{P}^{mn}\left(\frac{(\alpha,\beta)}{\sqrt[k]{\|\tilde{P}\|}}\right) = \frac{\widetilde{P}^{mn}(\alpha,\beta)}{\|\widetilde{P}\|^{mn}} = 1$, then $\tilde{f}(u,v) = \sum_{n=1}^{\infty} \tilde{P}^{mn}(u,v)$ can not be extended to (α', β').



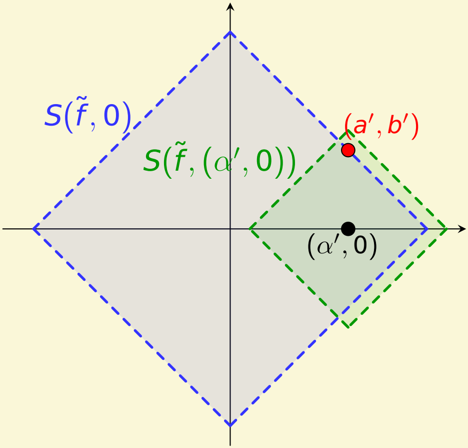
Then the radius of convergence satisfies $R(\tilde{f},(\alpha',0)) \leq |\beta'|$, otherwise we have



We are done:

$$\begin{aligned} R(f,(\alpha',0)) &\leq \mathbf{c}_{\mathfrak{b}}(\ell_1^2(\mathbb{R}))R(\tilde{f},(\alpha',0)), \\ &\leq \mathbf{c}_{\mathfrak{b}}(\ell_1^2(\mathbb{R}))|\beta'|, \\ &= \mathbf{c}_{\mathfrak{b}}(\ell_1^2(\mathbb{R}))\frac{|\beta|}{\sqrt[k]{\|\tilde{P}\|}}, \\ &< \mathbf{c}_{\mathfrak{b}}(\ell_1^2(\mathbb{R}))\frac{|\beta|}{\mathbf{c}_{\mathfrak{b}}(\ell_1^2(\mathbb{R}))(1-\varepsilon)}, \\ &= \frac{|\beta|}{(1-\varepsilon)}. \end{aligned}$$

If m does not exist, we use $(a', b') = e^{i\theta}(\alpha', \beta')$. With this change, we obtain $R(\tilde{S}_2) \leq |b'| + |a' - \alpha'|$, otherwise



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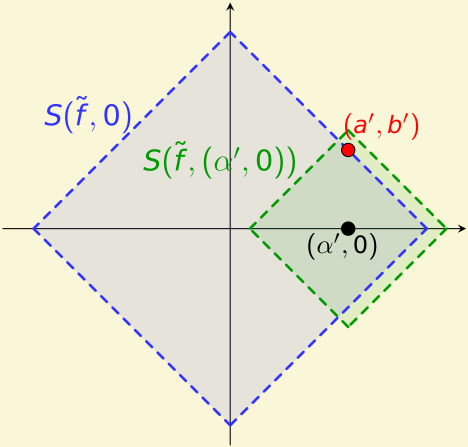

If θ is small $|b'| + |a' - \alpha'| = |\beta'| + |a' - \alpha'| \leq \frac{|\beta|}{\mathbf{c}_{\mathfrak{b}}(\ell_1^2(\mathbb{R}))(1-\varepsilon)}$.

Theorem (Benítez, Sarantopoulos, and Tonge)

Given $P_i \in \mathscr{P}(^{k_i}X)$, for $i = 1, \ldots, n$ and X a complex Banach space, then

$$\prod \|P_i\| \leq \frac{(\sum k_i)^{\sum k_i}}{\prod k_i^{k_i}} \left\| \prod P_i \right\|.$$

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Corollary (Benítez, Sarantopoulos, and Tonge)

Given $P_i \in \mathscr{P}({}^{k_i}E)$, for $i = 1, \ldots, n$ and E a real Banach space, then

$$\prod \|P_i\| \leq 2^{\sum k_i - 1} \frac{(\sum k_i)^{\sum k_i}}{\prod k_i^{k_i}} \left\| \prod P_i \right\|.$$

Proof

Given the polynomials P_i, we have

$$\begin{aligned}\prod \|P_i\| &\leq \prod \|\widetilde{P}_i\|_\nu \\ &\leq \frac{(\sum k_i)^{\sum k_i}}{\prod k_i^{k_i}} \left\|\prod \widetilde{P}_i\right\|_\nu \\ &= \frac{(\sum k_i)^{\sum k_i}}{\prod k_i^{k_i}} \left\|\widetilde{\prod P_i}\right\|_\nu \\ &\leq 2^{\sum k_i - 1} \frac{(\sum k_i)^{\sum k_i}}{\prod k_i^{k_i}} \left\|\prod P_i\right\|.\end{aligned}$$

Other applications: polarization constants

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Corollary (R., Dimant, Galicer)

The polarization constant of $\ell_1^2(\mathbb{R}) = (\mathbb{R}^2, \|\cdot\|_1)$ is greater than one.

Thanks!