Gleason parts and fibers for the spectrum of $\mathcal{A}_u(B_{\ell_p})$

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Talk based on joint works with
Richard Aron, Daniel Carando, Silvia Lassalle,
Manuel Maestre and Tomás Rodríguez

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Richard Aron, V. D., Silvia Lassalle and Manuel Maestre. *Gleason parts for algebras of holomorphic functions in infinite dimensions*. Rev. Matemática Complut. 33, 2020.

V. D., Silvia Lassalle and Manuel Maestre. *Fibers and Gleason parts for the maximal ideal space of $\mathcal{A}_u(B_{\ell_p})$* . Banach J. Math. Anal. 19, 2025.

Daniel Carando, V. D. and J. Tomás Rodríguez. *A new tour to Gleason parts for an algebra of holomorphic functions*. Preprint.

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Equivalent definition

f is continuous and for $x_0 \in U$, $x \in X$, the map $\lambda \mapsto f(x_0 + \lambda x)$ is holomorphic in some neighborhood of 0.

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$\mathcal{A}_u(B_X)$ is a Banach algebra:

$$\|f \cdot g\| \leq \|f\| \|g\|.$$

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In the spectrum $\mathcal{M}(\mathcal{A}_u(B_X))$ we consider the w^*-topology inherited from $\mathcal{A}_u(B_X)'$. With this topology, $\mathcal{M}(\mathcal{A}_u(B_X))$ is a compact set.

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2.2. THEOREM: In any function algebra, if

$$||h_1 - h_2|| < 2 \text{ and } ||h_2 - h_3|| < 2, \text{ then } ||h_1 - h_3|| < 2.$$

In other words, the relation $||h_1 - h_2|| < 2$ is transitive. Since it is trivially symmetric and reflexive it is an equivalence relation. We shall call the equivalence classes of this relation <u>parts</u>

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For $\varphi,\ \psi \in \mathcal{M}(\mathcal{A}_u(B_X))$,

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Gleason parts split $\mathcal{M}(\mathcal{A}_u(B_X))$ into equivalent classes.

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$$\mathcal{GP}(\varphi) = \{\psi \in \mathcal{M}(\mathcal{A}_u(B_X)) \ : \ \rho(\varphi, \psi) < 1\}.$$

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If U is an open convex set in a Banach space Y, we say that $\Phi\colon U \to \mathcal{M}(\mathcal{A}_u(B_X))$ is an analytic inclusion if it is injective and for any $f \in \mathcal{A}_u(B_X)$ the mapping

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As a consequence of Schwarz' lemma we have

The image of an analytic inclusion Φ is contained in a single Gleason part.

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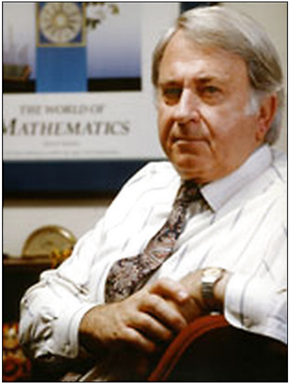

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The fiber over $z \in \overline{B}_{X''}$ is given by

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Through the canonical extension to the bidual $f \rightsquigarrow \widetilde{f}$ we can consider evaluation homomorphisms δ_z in $\mathcal{M}(\mathcal{A}_u(B_X))$ for all $z \in \overline{B}_{X''}$: $\delta_z(f) = \widetilde{f}(z)$.

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If $\varphi = w^* - \lim \delta_{z_\alpha}$ with $\|z_\alpha\| \leq r < 1$ then $\varphi \in \mathcal{GP}(\delta_0)$.

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The goal of this talk is to show how to construct homomorphisms in fibers over interior points belonging to different Gleason parts than $\mathcal{GP}(\delta_0)$.

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Thus, $\rho(\varphi, \delta_0) = 1$ meaning that $\mathcal{GP}(\varphi) \neq \mathcal{GP}(\delta_0)$.

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Theorem 3.3 *The fiber $\mathcal{M}_0$ in $\mathcal{M}(\mathcal{A}_u(B_{\ell_p}))$ contains a set of cardinal $2^{\mathfrak{c}}$ such that any two elements of this set belong to different Gleason parts.*

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Theorem (DLM)

Theorem 3.3 *The fiber $\mathcal{M}_0$ in $\mathcal{M}(\mathcal{A}_u(B_{\ell_p}))$ contains a set of cardinal $2^{\mathfrak{c}}$ such that any two elements of this set belong to different Gleason parts.*

What about other fibers?

Gleason parts for $\mathcal{M}(\mathcal{A}_u(B_{\ell_p}))$

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In (DLM) we could prove the same result only in the case $p \in \mathbb{N}$. First we use that $T(z) = (c_n z_n)_n$, with $|c_n| = 1$ for all n, is an onto isometry in ℓ_p.

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Hence, $\varphi \in \mathcal{M}_z$.

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A little more involved arguments than before yields:

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A little more involved arguments than before yields:

Proposition (DLM)

Proposition 3.5 *Let $p \in \mathbb{N}$, $p \geq 2$. Then for each $z \in B_{\ell_p}$, the fiber $\mathcal{M}_z$ in $\mathcal{M}(\mathcal{A}_u(B_{\ell_p}))$ contains a set of cardinal $2^{\mathfrak{c}}$ such that each two elements of this set belong to different Gleason parts.*

Gleason parts for $\mathcal{M}(\mathcal{A}_u(B_{\ell_p}))$

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Let $R \in \mathcal{P}({}^l\ell_p)$ and $Q \in \mathcal{P}({}^k\ell_p)$ that *depend on different variables*. Then,

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$$\|RQ\| = \|R\|\|Q\| \left(\frac{l^l k^k}{(l+k)^{l+k}} \right)^{\frac{1}{p}}.$$

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Define $R \in \mathcal{P}({}^l\ell_p)$ and $Q \in \mathcal{P}({}^k\ell_p)$ by

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Then, $\|R\| = \|Q\| = 1$.

Gleason parts for $\mathcal{M}(\mathcal{A}_u(B_{\ell_p}))$

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Now we consider the polynomial $P \in \mathcal{P}({}^{l+k}\ell_p)$ given by

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Note that R and Q depend on different variables, so

$$\|P\| = \|R\|\|Q\|\left(\frac{l^l k^k}{(l+k)^{l+k}}\right)^{\frac{1}{p}}\left(\frac{(l+k)^{l+k}}{l^l k^k}\right)^{\frac{1}{p}} = 1.$$

Gleason parts for $\mathcal{M}(\mathcal{A}_u(B_{\ell_p}))$

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Recall that $\varphi \in w^* - ac\{\delta_{z+se_n} : n > m\}$ and $\varphi \in \mathcal{M}_z$.

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Recall that $\varphi \in w^* - ac\{\delta_{z+se_n} : n > m\}$ and $\varphi \in \mathcal{M}_z$.

Also, for $n > m$,

$$\begin{aligned} P(z+se_n) &= R(z)Q(se_n)\left(\frac{(l+k)^{l+k}}{l^l k^k}\right)^{\frac{1}{p}} = \|z\|^l s^k \left(\frac{(l+k)^{l+k}}{l^l k^k}\right)^{\frac{1}{p}} \\ &= \left(\frac{l}{l+k}\right)^{\frac{l}{p}} \left(\frac{k}{l+k}\right)^{\frac{k}{p}} \left(\frac{(l+k)^{l+k}}{l^l k^k}\right)^{\frac{1}{p}} = 1. \end{aligned}$$

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Then $\varphi(P) = 1$ and $\delta_z(P) = 0$. Hence, $\mathcal{GP}(\varphi) \neq \mathcal{GP}(\delta_z)$.

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Then $\varphi(P) = 1$ and $\delta_z(P) = 0$. Hence, $\mathcal{GP}(\varphi) \neq \mathcal{GP}(\delta_z)$.

Recall that this was for $z \in B_{\ell_p}$ such that $\operatorname{supp}(z) \subseteq \{1, \ldots, m\}$ and $\|z\|^p \in \mathbb{Q}$.

Gleason parts for $\mathcal{M}(\mathcal{A}_u(B_{\ell_p}))$

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By Dirichlet's approximation theorem, there exist sequences $(k_j), (l_j) \subset \mathbb{N}$ such that

$$\frac{l_j}{l_j + k_j} \underset{j\to\infty}{\to} \|z\|^p \quad \text{and} \quad (l_j + k_j)\|z\|^p - l_j \underset{j\to\infty}{\to} 0.$$

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Take $\varphi \in w^* - ac\{\delta_{\pi_n(z)+se_{n+1}} : n \in \mathbb{N}\}$.

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Due to $\pi_n(z) + se_{n+1} \overset{w}{\to} z$, we have $\varphi \in \mathcal{M}_z$.

Computations are more intricate here but we could define a sequence of polynomials (which are product of polynomials depending on different variables) to deduce $\mathcal{GP}(\varphi) \neq \mathcal{GP}(\delta_z)$.

Gleason parts for $\mathcal{M}(\mathcal{A}_u(B_{\ell_p}))$

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Again we manage to construct an interpolating sequence and prove:

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Theorem (DLM)

Theorem 3.2. Let $1 < p < \infty$ and $z \in B_{\ell_p}$. Then, there are $2^{\mathfrak{c}}$ different Gleason parts of $\mathcal{M}(\mathcal{A}_u(B_{\ell_p}))$ completely contained in $\mathcal{M}_z$. More precisely, denoting $s = (1 - \|z\|^p)^{1/p}$, the cardinality of

$$w^* - ac\{\delta_{\pi_n(z)+se_{n+1}} : n \in \mathbb{N}\} \subset \mathcal{M}_z$$

is $2^{\mathfrak{c}}$ and each φ in this set belongs to a different Gleason part. Moreover, $\mathcal{GP}(\varphi) \subset \mathcal{M}_z$ for every such φ.

Gleason parts for $\mathcal{M}(\mathcal{A}_u(B_{\ell_p}))$

Again we manage to construct an interpolating sequence and prove:

Theorem (DLM)

Theorem 3.2. Let $1 < p < \infty$ and $z \in B_{\ell_p}$. Then, there are $2^{\mathfrak{c}}$ different Gleason parts of $\mathcal{M}(\mathcal{A}_u(B_{\ell_p}))$ completely contained in $\mathcal{M}_z$. More precisely, denoting $s = (1 - \|z\|^p)^{1/p}$, the cardinality of

$$w^* - ac\{\delta_{\pi_n(z)+se_{n+1}} : n \in \mathbb{N}\} \subset \mathcal{M}_z$$

is $2^{\mathfrak{c}}$ and each φ in this set belongs to a different Gleason part. Moreover, $\mathcal{GP}(\varphi) \subset \mathcal{M}_z$ for every such φ.

Gleason parts for $\mathcal{M}(\mathcal{A}_u(B_{\ell_p}))$

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The set

$$\mathcal{M}(\mathcal{A}_u(B_{\ell_p})) \setminus \overline{\{\delta_z : z \in \overline{B}_{\ell_p}\}}^{w^*}$$

is called the *corona*.

Gleason parts for $\mathcal{M}(\mathcal{A}_u(B_{\ell_p}))$

The set

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The famous *Carleson's corona theorem* states that the corona is empty for the algebra $\mathcal{H}^\infty(\mathbb{D})$.

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We could prove that the Gleason parts constructed in the previous theorem are singleton when restricted to the non-corona part of the spectrum.

Gleason parts for $\mathcal{M}(\mathcal{A}_u(B_{\ell_p}))$

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We could prove that the Gleason parts constructed in the previous theorem are singleton when restricted to the non-corona part of the spectrum.

Proposition (DLM)

Proposition 3.12. Let $1 < p < \infty$, $z \in B_{\ell_p}$ and $s = (1 - \|z\|^p)^{1/p}$. Then, for any $\varphi \in w^* - ac\{\delta_{\pi_n(z)+se_{n+1}} : n \in \mathbb{N}\}$ the only element in $\mathcal{GP}(\varphi)$ outside the corona is φ. In particular, if the corona theorem holds, $\mathcal{GP}(\varphi)$ is the singleton $\{\varphi\}$.

¡Muchas gracias!