

## SELFIMPROVING PHENOMENA: POINCARÉ-SOBOLEV INEQUALITIES AND BMO ESTIMATES

In this talk I will present some recent results on weighted Poincaré and Poincaré-Sobolev type inequalities with an explicit quantitative analysis on the dependence on the involved weights. This is a consequence of a sort of self-improving property related to a discrete summability property of the functional controlling the oscillation of a given Lipschitz function.

As a consequence of our results (and the method of proof) we obtain further extensions to two weights Poincaré inequalities and to the case of higher order derivatives. We also obtain results in the same spirit valid for the geometry of product spaces.

We also study minimal integrability conditions via Luxemburg-type expressions with respect to generalized oscillations that imply the membership of a given function  $f$  to the space BMO. Our method is simple, sharp and flexible enough to be adapted to several different settings, like spaces of homogeneous type, non doubling measures and also BMO spaces defined over more general bases than the basis of cubes.