Homogenization of Mixed Local-Nonlocal PDE Systems

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1. Abstract

Our main goal is to study the homogenization that occurs when one deals with the mix problem that involves the Laplacian and Nonlocal Integral Form operators with two different non-singular kernels that act in different domains A_n, B_n, which is described by the system:

$$\begin{cases} f(x) = \Delta u_n(x) + \displaystyle\int_{B_n} J(x-y)(v_n(y) - u_n(x))dy, \quad x \in A_n \\ \dfrac{\partial u_n}{\partial n}(x) = 0, \quad x \in \partial A_n. \end{cases} \tag{1.1}$$

$$g(x) = \int_{B_n} G(x-y)(v_n(y) - v_n(x))dy + \int_{A_n} J(u-y)(u_n(y) - v_n(x))dy, \qquad x \in B_n, \tag{1.2}$$

where the involved kernels are radial probability densities. Our ambient space Ω is divided into two disjoint A_n and B_n sets. This study introduce two different settings for the domain Ω, that is divided as A_n being the union of finite, periodic and disjoint balls with the same radium $r_n = 1/n$, and the opposite case, where B_n is the union of finite balls described above.

We have a weak convergence of the characteristic functions of A_n as $n \to \infty$ to a bounded function $X : \Omega \to [0, 1]$ and we will prove that, passing to the limit in (1.1)-(1.2), the first case tells us that the homogenized equation (in terms of X) gets the disappearance of local term, while in the second case the local term will survive.

Joint work with Marcone Pereira and Julio Rossi.

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