Bond Risk Premia and the "Return Forecasting Factor"

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April 3, 2017

Abstract

The return forecasting factor is a linear combination of forward rates that predicts future one-year excess bond returns of bond of all maturities better than traditional measures obtained from the yield curve. We analyze if this single factor structure in excess bond returns is robust to including other holding periods in the predictive bond return regressions. We find that it is not. We thus conclude that including the return forecasting factor as an additional risk factor in traditional three-factor term structure models, as has been suggested, may not be necessary.

Keywords: Excess returns, bond risk premia, return forecasting factor, affine term structure models.
1 Introduction

For many years researchers agreed that the first three principal components of yields, often labeled level, slope, and curvature factors, contain all the relevant information about the cross section of yields. As a result, a large class of affine term structure models assumes that only those three factors determine the evolution of all zero coupon bond prices. Recent studies, however, argued that other variables contain information that is useful for predicting bond returns beyond that captured by the first three principal components. This evidence comes from predictive regressions of bond returns on variables other than (and possibly including) the first three principal components. Those additional variables appear to have predictive power to explain bond returns. Bauer and Hamilton (2017) call this result a violation of the spanning hypothesis.\(^1\)

In a very influential paper, Cochrane and Piazzesi (2005) argue that a linear combination of current forward rates, labeled the “return forecasting factor,” predicts one-year excess bond returns of each maturity with R\(^2\)s of about 0.35. To put this number in context, the traditional Fama and Bliss (1987) regressions of excess returns on forwards rates attain R\(^2\)s of about 0.17, while regressions on the first three principal components deliver R\(^2\)s of about 0.27. Furthermore, the return forecasting factor contains information that is unrelated to the level, slope, and curvature factors. One implication of this observation is that a single factor captures all the economically relevant variation in expected returns. As a result, Cochrane and Piazzesi (2005, 2008) propose to augment the traditional three-factor affine term structure model by adding a fourth (return forecasting) factor that drives variation in all excess returns.

Recently, however, Bauer and Hamilton (2017) have questioned the value of variables other than the first three principal components as predictors of bond returns. The

\(^1\)Bauer and Hamilton (2017) define the “spanning hypothesis” as the claim that the first three principal components contain all the relevant information for predicting future interest rates and bond returns. This paper also contains an extensive list of papers that document violations of the spanning hypothesis.
conventional tests that support including those variables in predictive regressions are subject to serious small-sample distortions that lead to oversized tests.

In this paper we evaluate the information contained in the return forecasting factor from a different, yet complementary, point of view. In Cochrane and Piazzesi (2005, 2008) the return forecasting factor is the linear combination of current forwards rates that maximizes the predictive power in one-year excess returns regressions. If it is to be regarded as the main driver of all excess returns, this “one-year-ahead” forecasting factor should also drive variation in excess returns for holding periods other than one year. The purpose of this paper is to evaluate this hypothesis.

First, we find that the return forecasting factor outperforms the first three principal components only for the holding period for which it was created for. In regressions of excess returns for holding periods longer than one year, the three principal components provide a better fit of excess returns. And for sufficiently long holding periods, the principal components have more than three times the explanatory power of the return forecasting factor.

Second, we test whether there is a single factor structure in expected excess returns. Although Cochrane and Piazzesi (2005) statistically reject the single factor structure in one-year excess returns, they argue that a single factor captures all the economically relevant variation in one-year expected excess returns. When we consider different holding periods, we also reject the single factor structure. And, in contrast with the finding in Cochrane and Piazzesi (2005), a single factor does not capture all the economically relevant variations in expected excess returns. Taken together, these results imply that a model in which a single factor drives fluctuations in all excess returns fails to capture all the economically relevant variations in the data.

Finally, we run unrestricted regressions of yields on the return forecasting factor and the first three principal components. If the return forecasting factor is a relevant state variable in an affine term structure model, except for knife-edge restrictions imposed on
the risk-neutral distribution of the state variables, it should be a significant predictor of current yields and not only of excess returns. We use the robust t-tests developed by Ibragimov and Müller (2010), and recommended by Bauer and Hamilton (2017) in the context of returns regressions, to test the statistical significance of the return forecasting factor in yield regressions that also include the first three principal components. We find that the return forecasting factor is not a significant predictor of bond yields while the first three principal components are highly significant.

2 The return forecasting factor and excess bond returns

Using Fama-Bliss data of one- through five-year zero coupon bond prices, Cochrane and Piazzesi (2005) run predictive regressions of one-year excess returns on government bonds on five forward rates at the beginning of the period. Their main result is that a single linear combination of forwards (the return forecasting factor) predicts one-year excess returns of bonds of each maturity with an $R^2$ of about 0.35, and up to 0.44 if one also includes lagged forward rates as regressors. Furthermore, the return forecasting factor outperforms the first three principal components extracted from the yield curve in terms of forecasting power. Cochrane and Piazzesi (2005, 2008) conclude that conventional three-factor models of the yield curve miss an important component of bond returns and, therefore, researchers should include a fourth (return forecasting) factor to the models even if it does not improve the cross-sectional fit of the estimated yield curve.

Most empirical papers that study time variation in expected excess bond returns run predictive regression using only one-year excess returns. Yet, to be regarded as the main driver of risk premia in an affine term structure model, the return forecasting factor should also explain variations in excess returns for holding periods other than one year. In this section we assess whether the return forecasting factor is also able to predict excess returns for different holding. Relatedly, we analyze whether the one-factor
structure of expected excess returns is robust to including other holding periods. Our main findings are that, when we include holding horizons different from one year, the predictive power of the return forecasting factor is dominated by the traditional first three principal components, and that a single factor structure is not enough to capture all the economically relevant variations in expected excess returns.

2.1 The predictive one-year excess return regressions

Let $p_t^{(n)}$ denote the log price of an $n$-year zero coupon bond at time $t$. Time $t$ and maturity $n$ are both expressed in years. The log yield is $y_t^{(n)} = -p_t^{(n)}/n$ and the log forward rate at time $t$ for loans starting at $t+n$ and maturing at $t+n+h$ is $f_t^{(n,n+h)} = p_t^{(n)} - p_t^{(n+h)}$. We denote the annualized $h$-year log holding return from buying an $n$-period zero coupon bond at time $t$ and selling it at time $t+h$ as an $(n-h)$-year zero coupon bond by $r_t^{(n)} = (p_t^{(n-h)} - p_t^{(n)})/h$. The (annualized) excess return from holding an $n$-year zero coupon bond for $h$ years is defined as $r_x^{(n)} = r_t^{(n)} - y_t^{(h)}$. Finally, we denote the average $h$-year excess return across maturities by $\bar{r}_x^{(n)} = \frac{1}{N} \sum_{n>h} r_x^{(n)}$, where $H$ is the set of bond with maturities longer than $h$ periods ahead.

Most of the empirical literature on predicting bond returns considers predictive regressions of excess returns for holding periods of $h=1$ year. Instead, we consider predictive regressions for holding periods ranging from $h=1/12$ (1-month excess holding returns) through $h=108/12$ (9-year excess holding returns).

With the Fama-Bliss dataset one can construct five yields, five forward rates, and four excess returns. Using these data, Cochrane and Piazzesi (2005) argue that a single linear combination of the five forwards rates predicts a substantial portion of one-year excess return of all maturities (from 2 through 5 years).

The single-factor predictive regression is given by

$$r_x^{(n)} = b_n \left( y' f_t \right) + \epsilon_t^{(n)},$$

(1)
where \( n = 2, 3, 4, 5 \) and \( f_t = \left[ 1, y_t^{(1)}, f_t^{(1,2)}, f_t^{(2,3)}, f_t^{(3,4)}, f_t^{(4,5)} \right]' \) is a vector with a constant and the forwards at time \( t \). Equation (1) imposes a single factor structure on all one-year excess returns. The alternative, unrestricted, regression is given by

\[
rx_{t\rightarrow t+1}^{(n)} = \beta_n' f_t + \epsilon_{t+1}^{(n)},
\]

where the slope coefficients \( \beta_n \) depend on the maturity of the bond \( n \) and are independent of the slope coefficients of bonds with a different maturity. Cochrane and Piazzesi (2005) show that estimating the one-factor model (1) leads to almost identical results than estimating the unrestricted regressions (2), which suggest that variations in all one-year excess returns are driven by a single factor.\(^2\)

Among other experiments, Cochrane and Piazzesi (2005) compare the predictive ability of the return forecasting factor with that of the traditional principal components extracted from the Fama-Bliss yields. Table 1 reports the \( R^2 \) s of a series of predictive regressions. The results under the label “Fama-Bliss data” are based on the same data used by Cochrane and Piazzesi (2005). The column labeled “Individual” shows the \( R^2 \) of regressions of the average one-year excess return \( \overline{rx}_{t\rightarrow t+1} \) on the five current forward rates, from which we construct the return forecasting factor, and on the individual principal components of yields. The \( R^2 \) of 0.35 replicates the result reported in Table 1 of Cochrane and Piazzesi (2005). The column labeled Joint PC\(_1\): PC\(_i\) reports the \( R^2 \) of multivariate regressions of the average one-year excess return on the principal components PC\(_1\), PC\(_2\), …, PC\(_i\).

The \( R^2 \) of 0.35 using a single factor was considered a success.\(^3\) The traditional Fama and Bliss (1987) regressions of excess returns on individual forwards attain an \( R^2 \) of about 0.17. The return forecasting factor duplicates the explanatory power of the individual forwards. The return forecasting factor also outperforms the predictive ability of the level,

\(^2\)Cochrane and Piazzesi (2005) construct the return forecasting factor \( \gamma'f_t \) by regressing the average one-year excess return \( \overline{rx}_{t\rightarrow t+1} \) on the five Fama-Bliss forward rates. In a second step they estimate the factor loadings \( b_n \) by using \( \hat{\gamma}'f_t \) as a generated regressor in equation (1).

\(^3\)But see the critique in Bauer and Hamilton (2017).
slope, and curvature factors \((PC_1, PC_2, \text{ and } PC_3)\) which achieve an \(R^2\) of 0.27 with most of the explanatory power coming from the slope factor. Note, in addition, that the fourth principal component seems to be a relevant predictor of one-year excess returns.\(^4\)

<table>
<thead>
<tr>
<th>Regressor</th>
<th>Fama-Bliss data</th>
<th>GSW data</th>
</tr>
</thead>
<tbody>
<tr>
<td>Return forecasting factor</td>
<td>0.351</td>
<td>0.358</td>
</tr>
<tr>
<td>(PC_1)</td>
<td>0.028</td>
<td>0.033</td>
</tr>
<tr>
<td>(PC_2)</td>
<td>0.213</td>
<td>0.169</td>
</tr>
<tr>
<td>(PC_3)</td>
<td>0.024</td>
<td>0.059</td>
</tr>
<tr>
<td>(PC_4)</td>
<td>0.085</td>
<td>0.009</td>
</tr>
<tr>
<td>(PC_5)</td>
<td>0.000</td>
<td>0.021</td>
</tr>
<tr>
<td>(PC_6)</td>
<td></td>
<td>0.026</td>
</tr>
<tr>
<td>(PC_7)</td>
<td></td>
<td>0.013</td>
</tr>
<tr>
<td>(PC_8)</td>
<td></td>
<td>0.014</td>
</tr>
<tr>
<td>(PC_9)</td>
<td></td>
<td>0.010</td>
</tr>
<tr>
<td>(PC_{10})</td>
<td></td>
<td>0.003</td>
</tr>
</tbody>
</table>

The table report the \(R^2\)'s of predictive regressions of the average one-year excess returns \(r_{x,t} \rightarrow t+1\) on the return forecasting factor and the principal components of bond yields (\(PC_1, PC_2,...\)). The columns labeled \(Fama-Bliss\ data\) are based on the five Fama-Bliss discount bond prices. The columns labeled \(GSW\ data\) uses the treasury yields constructed by Gürkaynak, Sack and Wright (2007). The sample period is that used by Cochrane and Piazzesi (2005), Jan-1964 through Jan-2003.

The right panel, labeled “\(GSW\ data,\)” replicates the previous results but using bond prices constructed from the yield curve estimated by Gürkaynak, Sack and Wright (2007). We use one- through ten-year yields on zero-coupon bonds and construct ten forward rates, nine one-year excess returns, and ten principal components. We construct the return forecasting factor by regressing the average one-year excess return \(r_{x,t} \rightarrow t+1\) on the ten forward rates. This regression delivers an \(R^2\) of 0.36, while a regression of the

\(^4\)The results for the principal components are slightly different from those in Table 4 in Cochrane and Piazzesi (2005) because we standardized yields before computing the principal components, while Cochrane and Piazzesi did not.
average one-year excess return on the first three principal components attains an $R^2$ of just 0.26. And, as above, most of the predictability comes from the slope factor ($PC_2$). With these data, however, the fourth principal component is not as important as it is with the Fama-Bliss data but we still find some predictability in the fifth and sixth principal components (and somewhat less in $PC_7$ and $PC_8$). The main conclusion is not altered by this new dataset: the return forecasting factor captures more information than the first principal components to forecast one-year excess returns.\footnote{We also obtained similar results using the dataset in Jungbacker, Koopman and van der Wel (2014).}

Based on these results and on individual regressions of one-year excess returns on the return forecasting factor, as in equation (1), Cochrane and Piazzesi (2005, 2008) conclude that a single factor drives all the economically relevant variations in expected excess returns of bonds of each maturity. As a consequence, they propose to augment the traditional three-factor term structure model with a fourth, return forecasting factor, that drives fluctuations in all excess returns. Otherwise, an important component of return predictability would be lost. Moreover, since expected excess returns do not appear to respond to variations in level, slope, and curvature factors, their market price of risk should be zero (Cochrane, 2011).

For this simplification to be valid, however, the same return forecasting factor should drive variation in expected excess returns for holding periods other than one year. The return forecasting factor should not only predict excess returns better than the first three principal components at the one-year horizon, but for holding periods longer and shorter than one year. With this question in mind, in the next subsection we redo the previous exercise for holding periods ranging from 1 month through 9 years (108 months).

### 2.2 The predictive $h$-year excess return regressions

We construct the return forecasting factor by regressing the average one-year excess return $r_{X_{t \rightarrow t+1}}$ on the forward rates using both the Fama-Bliss and GSW datasets. We call this
the “one-year” return forecasting factor because it maximizes the forecasting performance on a regression of the one-year excess return. Next, we run predictive regressions of the (annualized) h-year excess return averaged over all the available maturities, $\bar{r}_{x_{t \rightarrow t+h}}$, on the one-year-ahead return forecasting factor, $\hat{\gamma}'f_t$, and on the first three principal components, $PC_{1t}$, $PC_{2t}$, and $PC_{3t}$ extracted from the same datasets,

\[
\begin{align*}
\bar{r}_{x_{t \rightarrow t+h}} &= \alpha_0 + \alpha_1 (\hat{\gamma}'f_t) + \epsilon_{t+h} \\
\bar{r}_{x_{t \rightarrow t+h}} &= \delta_0 + \delta_1 PC_{1t} + \delta_2 PC_{2t} + \delta_3 PC_{3t} + \eta_{t+h}.
\end{align*}
\]

The top left panel of Figure 1 shows the $R^2$ of regressions of the h-period average excess return on the one-year-ahead return forecasting factor (dotted-circled line) and on the first three principal components (dotted-squared line) as a function of the holding period. While the return forecasting factor has more predictive power than the principal components at the one-year holding horizon, the relation is reversed at longer holding periods. For example, the return forecasting factor explains less than 10 percent of the variability of the 4-year excess return while the principal components explain about 25 percent of the variance.

The other panels of Figure 1 construct an "h-year-ahead" return forecasting factor by finding the linear combination of forwards that minimize the forecast error using a regression of the average h-period excess holding returns on the current forwards. Again, while the h-year-ahead return forecasting factor outperforms the principal components at the h-year holding horizon, the principal components tend to do better than the return forecasting factor at other holding horizons.

This pattern is reinforced when we use the Gürkaynak, Sack and Wright (2007) zero-coupon bond yields. These data allow us to construct excess returns for holding periods other than one year. Gürkaynak, Sack and Wright (2007) estimate on a daily basis a six parameter smooth function for the yield curve proposed by Svensson (1994). Using
The figure shows the $R^2$'s of a regression of the $h$-period excess holding returns ($h=1,2,3,4$) on the return return forecasting factor and on the first three principal components $PC_{1t}$, $PC_{2t}$, and $PC_{3t}$ together. In the upper left panel the return return forecasting factor is computed using a regression of one-year excess returns on the current forwards; in the upper right panel the return return forecasting factor is computed with a regression of two-year excess returns on the current forwards, and so forth.

The last-day of month estimation of the yield curve, we construct zero coupon yields with monthly maturities ranging from 1 month to 10 years. With these zeros, we extract the first three principal components and compute excess returns for holding periods from 1 month to 9 years (108) months. Afterwards, at any time $t$ and for all holding periods $h$, we compute the cross-sectional average of the $h$-period excess returns across the available maturities. Next, we construct $h$-year-ahead return forecasting factor for $h = 1, 2, 3, 4$ by regressing the average $h$-year holding returns on one- through ten-year
forward rates. Finally, we run regressions of the average excess holding returns $\bar{r}_{t \rightarrow t+h}$ for $h = 1/12, 2/12, \ldots, 108/12$ on the four return forecasting factors and the principal components. The $R^2$'s of the regressions are displayed in Figure 2.

**Figure 2: $R^2$ of h-year excess holding return regressions using GSW data**

The figure shows the $R^2$'s of a regression of the $h$-year excess holding returns on the return forecasting factor and on the first three principal components. In the upper left panel the return forecasting factor is computed using a regression of one-year excess returns on one- through ten-year forwards; in the upper right panel the return forecasting factor is computed with a regression of two-year excess returns on one- through ten-year forwards, and so forth.

The upper left panel of Figure 2 shows that the highest predictive power of the one-year-ahead return forecasting factor is achieved around the one-year holding period. The return forecasting factor outperforms the principal components for maturities that go from 1 through 28 months, but loses predictive power relative to the principal components for longer holding horizons. When predicting excess returns for holding periods longer
than three years, the predictive power of the regression using the return forecasting factor drops dramatically while it increases substantially when using the principal components. For example, for a 9-year holding period, the predictive power of the principal components is more than three times larger than that of the one-year-ahead return forecasting factor.

The results using the two-year-ahead return forecasting factor (upper right panel) are similar to those using the one-year-ahead return forecasting factor: for holding periods of around two years, the return forecasting factor outperforms the principal components but for maturities longer than three years the principal components dominate. The other two panels show that the three- and four-year ahead return forecasting factor do not add additional information to that contained in the first three principal components.

2.3 The single factor structure in excess returns

Although Cochrane and Piazzesi (2005) statistically reject the single factor model in one-year excess returns, they argue that it captures all the economically relevant variations in expected excess returns. In particular, from the unrestricted regressions (2), Cochrane and Piazzesi (2005) compute an eigenvalue decomposition of expected excess returns \( \hat{\beta}_n' f_t \) for \( n = 2, 3, 4, 5 \) and find that its first principal component, which is almost identical to one-year-ahead return forecasting factor, accounts for 99.5 percent of the total variance of expected excess returns. In this subsection we perform a similar exercise using holding periods other than one year.

Using the average excess returns constructed from the Fama-Bliss data, we estimate
by the method of maximum likelihood the unrestricted system of equations

\[
\begin{bmatrix}
\bar{r}_{x_{t+1}} \\
\bar{r}_{x_{t+2}} \\
\bar{r}_{x_{t+3}} \\
\bar{r}_{x_{t+4}}
\end{bmatrix}
= 
\begin{bmatrix}
\alpha_1 \\
\alpha_2 \\
\alpha_3 \\
\alpha_4
\end{bmatrix}
+ 
\begin{bmatrix}
\beta_{1,1} & \beta_{1,2} & \beta_{1,3} & \beta_{1,4} & \beta_{1,5} \\
\beta_{2,1} & \beta_{2,2} & \beta_{2,3} & \beta_{2,4} & \beta_{2,5} \\
\beta_{3,1} & \beta_{3,2} & \beta_{3,3} & \beta_{3,4} & \beta_{3,5} \\
\beta_{4,1} & \beta_{4,2} & \beta_{4,3} & \beta_{4,4} & \beta_{4,5}
\end{bmatrix}
\begin{bmatrix}
y_{0,1}^{(1)} \\
f_{t}^{(1,2)} \\
f_{t}^{(2,3)} \\
f_{t}^{(3,4)} \\
f_{t}^{(4,5)}
\end{bmatrix}
+ 
\begin{bmatrix}
\epsilon_{t+1}^{(1)} \\
\epsilon_{t+2}^{(2)} \\
\epsilon_{t+3}^{(3)} \\
\epsilon_{t+4}^{(4)}
\end{bmatrix},
\tag{3}
\]

and a restricted, one-factor, system of equations given by

\[
\begin{bmatrix}
\bar{r}_{x_{t+1}} \\
\bar{r}_{x_{t+2}} \\
\bar{r}_{x_{t+3}} \\
\bar{r}_{x_{t+4}}
\end{bmatrix}
= 
\begin{bmatrix}
\alpha_1 \\
\alpha_2 \\
\alpha_3 \\
\alpha_4
\end{bmatrix}
+ 
\begin{bmatrix}
\beta_{1,1} & \beta_{1,2} & \beta_{1,3} & \beta_{1,4} & \beta_{1,5} \\
\beta_{2,1} & \beta_{2,2} & \beta_{2,3} & \beta_{2,4} & \beta_{2,5} \\
\beta_{3,1} & \beta_{3,2} & \beta_{3,3} & \beta_{3,4} & \beta_{3,5} \\
\beta_{4,1} & \beta_{4,2} & \beta_{4,3} & \beta_{4,4} & \beta_{4,5}
\end{bmatrix}
\begin{bmatrix}
y_{0,1}^{(1)} \\
f_{t}^{(1,2)} \\
f_{t}^{(2,3)} \\
f_{t}^{(3,4)} \\
f_{t}^{(4,5)}
\end{bmatrix}
+ 
\begin{bmatrix}
\epsilon_{t+1}^{(1)} \\
\epsilon_{t+2}^{(2)} \\
\epsilon_{t+3}^{(3)} \\
\epsilon_{t+4}^{(4)}
\end{bmatrix},
\tag{3}
\]

assuming that the errors are normally distributed. The likelihood ratio statistic, which is
distributed as a chi-square with 12 degrees of freedom, is 133.2. The test strongly rejects
the single factor model.\textsuperscript{6}

Moreover, the rejection is economically relevant. Performing a principal components
decomposition of the unconstrained expected returns obtained from the system (3), we
find that the first principal component explains less than 90 percent of the overall variance
of expected returns, much smaller than the 99.5 percent obtained by Cochrane and
Piazzesi (2005) when considering only one-year excess returns. In other words, a single
factor model fails to capture all the economically relevant variations in expected returns.

\textsuperscript{6}We also performed the test with the individual returns without averaging across maturities and also
strongly reject the single factor model.
3 The return forecasting factor and bond yields

In this section we run regressions of yields on the return forecasting factor and the first three principal components. If the return forecasting factor is a relevant state variable in a term structure model, except for knife-edge restrictions imposed on the risk-neutral distribution of state variables, it should also be a significant predictor of yields.

We use last-day-of-month yields constructed from the Gürkaynak, Sack and Wright (2007) parametric representation of the yield curve and estimate the regressions

\[ y_{t}^{(n)} = \zeta_0^{(n)} + \lambda^{(n)} (\hat{\gamma}' f_t) + \zeta_1^{(n)} PC_{1t} + \zeta_2^{(n)} PC_{2t} + \zeta_3^{(n)} PC_{3t} + \eta_t^{(n)}, \]  

for \( n = \frac{1}{12}, \frac{2}{12}, \ldots, \frac{120}{12} \). We next perform tests of significance of the coefficient \( \lambda^{(n)} \). Since the residuals of this kind of regressions tend to be autocorrelated, we compute standard errors using a conventional Newey-West correction with 18 lags.

The upper left panel of Figure 3 plots the t-statistics of the null hypothesis that \( \lambda^{(n)} = 0 \) (full line) along with the \((0.025, 0.975)\) critical values (dashed-lines). The tests reject the null hypothesis that the coefficient \( \lambda^{(n)} \) is zero for 83 out of the 120 bond maturities. That is, the return forecasting factor seems to be a significant predictor of yields for about 70 percent of the maturities. The other panels of the Figure display the t-statistics and critical values associated with the null hypothesis that the individual coefficients on the principal components, \( \zeta_i^{(n)} \), are zero. In the case of the first and second principal components, we reject a zero coefficient for all maturities while for the third principal components, we reject the null for 115 maturities.

These results suggest that the return forecasting factor contains information on bond yields that is not captured by the first three principal components. Yet, the type of standard error corrections that are typically used in the literature could distort the result and, consequently, we may reject the null hypothesis of a zero coefficient on the return...
Figure 3: Testing if factors do not explain yields using Newey-West correction

The figure shows t-statistics (solid lines) and \((0.025, 0.975)\) critical values (dashed lines) of the null hypothesis that the coefficients of the one-year-ahead return forecasting factor and of the first three principal components are zero in regression (4). Standard errors are corrected using the Newey-West procedure with 18 lags.

Forecasting factor too often when it should not. This problem was noted by Bauer and Hamilton (2017) in the context of regressions of bond returns on the principal components and other forecasting variables.

Conventional procedures for correcting standard errors (such as Newey-West) in a yields regression like (4) are problematic for two reasons. First, the return forecasting factor is likely to be correlated with the regression residual. In effect, equation (4) omits principal components that are relevant to explain current yields (perhaps with a small but still significant coefficient), and those missing principal components are left in the
residual. This omission would not be a problem in a regression of yields on the first three principal components, since all principal components are orthogonal. But if we include the return forecasting factor in the regression, which is constructed with the same data from which we extract the principal components, it is likely that it will be correlated with the missing principal components and, therefore, with the residual.\textsuperscript{7} And second, the right hand variables in the regression are very persistent and conventional heteroskedasticity and autocorrelation consistent (HAC) standard errors are known to work poorly with strongly dependent data (Müller, 2014).

To deal with the potential bias in estimated standard errors we use the robust t-tests developed by Ibragimov and Müller (2010) and recommended by Bauer and Hamilton (2017) in the context of bond return regressions. The procedure consists of splitting the sample into $q$ subsamples and estimating regression (4) separately in each of them.\textsuperscript{8} If the estimates across subsamples are approximately independent and Gaussian, we can test hypothesis about individual coefficients using a t-test with $q - 1$ degrees of freedom.\textsuperscript{9} However, because the principal components and the return forecasting factor are constructed using all the data, it is unlikely that the independence assumption holds for any partition of the sample. Therefore, to implement the test we divide the sample into $q = 8$ equally sized subsamples and, for each of them, we extract the principal components and estimate the return forecasting factor separately. We then estimate regression (4) for each maturity and subsample, and perform the proposed t-test to test the null hypothesis that the coefficient on the return forecasting factor is zero.

The results of the Ibragimov-Müller robust t-tests are displayed in Figure 4. Now we cannot reject the hypothesis that the coefficient on the return forecasting factor $\lambda^{(n)}$ is zero for all the available maturities. We conclude from this exercise that the return

\textsuperscript{7}Using Fama-Bliss data, Cochrane and Piazzesi (2005) found that the fourth principal components of yields explains about 24 percent of the variance of the return forecasting factor. Using the GSW data we also found that the return forecasting factor is correlated with the missing principal components.

\textsuperscript{8}Müller (2014) and Bauer and Hamilton (2017) use $q = 8$ in their estimations.

\textsuperscript{9}Müller (2014) provide evidence showing that this test has excellent size and power in settings where standard HAC inference is seriously distorted.
The figure shows the robust t-statistics of Ibragimov and Müller (2010) (solid lines) and $(0.025, 0.975)$ critical values (dashed lines) of the null hypothesis that the coefficients of the one-year-ahead return forecasting factor and of the first three principal components are zero in regression (4).

forecasting factor does not help to explain the cross-section of yields.

4 Concluding remarks

Cochrane and Piazzesi (2005) document that one-year expected excess returns on bonds of any maturity move proportionally to a single, return forecasting, factor. Furthermore, this single factor contains information unrelated to the traditional level, slope, and curvature factors extracted from bond yields. Consequently, Cochrane and Piazzesi (2005, 2008) propose to augment the traditional three-factor term structure model by adding the return
forecasting factor as an additional risk factor that drives variations in all excess returns. In this paper we evaluate whether the return forecasting factor has more predictive power than the first three principal components of bond yields in regressions of future excess bond returns for holding periods shorter and longer than one year, which is the standard holding period used in empirical work.

We found that return forecasting factor is dramatically outperformed by the first three principal components extracted from bond yields for holding periods longer than one year. Furthermore, we showed that there is no single factor structure that is able to capture all the economically relevant variations in excess bond returns when we consider holding periods different from (and including) one year. This result suggests imposing that a single factor that drives fluctuations in all excess returns is an invalid dimension reduction of the data.

Furthermore, if the return forecasting factor is to be added as an additional state variable into the standard three-factor term structure model, except for knife-edge restrictions on the parameters of the model, the return forecasting factor should also be significantly contemporaneously correlated with current yields even after controlling for the first three principal components. Using the robust t-test proposed by Ibragimov and Müller (2010), we cannot reject the null hypothesis that the slope coefficient of the return forecasting factor is zero for yields of any maturity.
References


