OPTIMAL TRADING AND SHIPPING OF AGRICULTURAL COMMODITIES

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ABSTRACT. We develop and implement a model for a profit maximizing firm that provides an intermediation service between commodity producers and commodity end-users. We are motivated by the grain intermediation business at Los Grobo—one of the largest commodity-trading firms in South America. Producers and end-users are distributed over a realistic spatial network and trade with the firm through contracts for delivery of grain during the marketing season. The firm owns spatially-distributed storage facilities, and begins the marketing season with a portfolio of prearranged purchase and sale contracts with upstream and downstream counterparts. The firm aims to maximize profits while satisfying all previous commitments, possibly through the execution of new transactions. Under realistic constraints for capacities, network structure and shipping costs, we identify the optimal trading, storing and shipping policy for the firm as the solution of a profit-maximizing optimization problem, encoded as a minimum cost flow problem in a time-expanded network that captures both geography and time. We perform extensive numerical examples and show significant efficiency gains derived from the joint planning of logistics and trading.

KEYWORDS. Agriculture, Commodities, Logistics, Finance, Network Flow Problems.

1. INTRODUCTION

1.1. The Problem.

The study of agriculture and production of commodities has been a traditional topic within the operations research community and there is an ample literature devoted to operational and tactical decision making. Some problems that have been studied include crop selection, crop rotation, application of fertilizers and other chemicals, procurement, processing, trade, logistics and supply chain (Glen, 1987; Lowe and Preckel, 2006; Weintraub and Romero, 2006; Ahumada and Villalobos, 2009).

Secomandi (2010, p. 450) raises important questions on the interdependency of different business units within a commodities firm: "is it always possible to decouple inventory-trading and operational decisions? If this is not the case, what is the likely loss in value of ignoring the interface between trading and operations in practice?" Motivated by the previous questions and by the business activities of Grupo Los Grobo,[1] a large agricultural commodity firm in South America (Regunaga, 2010; Bell and Scott, 2010), we set to study the profit optimization problem for a firm simultaneously considering grain trading and logistics. Grupo Los Grobo, and competitors including large multinational firms such as Bunge, Cargill and Louis Dreyfus, have each traded recently over a million tonnes of grain per year in Argentina by intermediating between inland farmers and waterfront-based exporters, mills and crushing factories. All of these firms respond to the needs of their geographically distributed suppliers and buyers by entering into purchase and sale agreements. Commodity-trading firms can sign contracts of various types. A spot transaction is the sale or purchase of a commodity of certain quality for immediate delivery at a certain location. Firms also engage in transactions that are delayed in time, such as futures and forward contracts (Geman, 2009). These allow counterparties to buy and sell a commodity for delivery on a fixed time in the future, at a price specified at the inception of the contract. However, transactions that arise at the request of upstream sellers and downstream buyers are not usually balanced automatically in the aggregate nor for specific delivery periods and regions. A commodity-trading firm will frequently own an unbalanced trading book that will require additional future transactions to avoid future default. Therefore, the firm simultaneously faces a logistical and a financial problem, as it must ship grain across space and time in the most efficient manner and it must sign new contracts to make the whole plan feasible.

1.2. **Objective of the Paper.**

The objective of this paper is to develop and implement a profit-optimization model for a commodity-trading firm that purchases a bulk commodity over time in different geographic locations and sells it to downstream users also in different geographic locations. The list of objectives achieved by the model in this paper include

- The ability to compute an optimal shipping and storing policy that simultaneously takes into account the main logistical and financial aspects encountered in the operation of the firm. This replaces costly manual planning work.
- The identification of new transactions that must be entered to satisfy previously-existing commitments in an optimal manner.
- The ability to run the model repeatedly and virtually instantly in response to changes in the inputs.
- The automatic generation of global profit sensitivities with respect to new transactions.

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[1] For information about the group please refer to `www.losgrobo.com`.

- The ability to evaluate the profitability of major capital investments such as a proprietary truck fleet or new storage facilities.
- The ability to evaluate conjectural scenarios against an optimal policy, which can be used to compute the risk arising from various strategies and to evaluate insurance products.

Our model spans multiple planning periods within a rolling horizon framework. Typically this would be used with periods ranging from a week to a month, with a horizon of many months ahead to plan the logistics during and after the harvest. We consider vertical differentiation on quality and allow for product conversion.

Decisions are represented by a minimum cost flow problem in a time-expanded network that encodes both geography and time (Ahuja et al., 1993). The constraints are flexible enough for the model to indicate new transactions that guarantee the satisfaction of preexisting commitments. There are, in addition, various constraints on physical capacities. The profit function includes all income derived from selling grain, minus the cost of goods being sold and all relevant transportation and processing costs. Since cash flows occur on multiple periods over time, these are aggregated after properly discounting through interest rates. The solution to the problem is the set of optimal flows and intermediate holdings, which represent the shipping and storage of grain in multiple periods in the future. In order to achieve flow balance and be feasible, an optimal solution typically must include flow associated to additional contracts to be signed by the firm with upstream and downstream counterparts. Overall, our model identifies a globally-optimal logistical and financial solution for the intermediation business of a large commodity trader.

1.3. **Literature Review.** We discuss the literature that relates to our model by connecting it to two different but inter-related streams. There is a first stream on agriculture that focuses on operational and tactical decisions, while there is a second stream in the interface between operations and finance. Both streams are closely related to other types of commodities such as metals, petroleum oil and other related products so we will refer to those commodities as well.

In a seminal paper, Heady (1954) addressed agricultural decision making at the farm level by providing a linear programming solution to the allocation of land between two crops. The work of Symonds (1955) applied operations research techniques to processes related to handling commodities by refineries. The abundant literature on OR for agriculture applied optimization techniques to many problems in the area. Examples include crop selection, crop rotation, application of fertilizers and other chemicals, logistics, and agricultural planning at the regional sector level. Surveys discussing this in detail are e.g., Glen (1987);

Lowe and Preckel (2006); Weintraub and Romero (2006); Ahumada and Villalobos (2009). The focus on agricultural applications in the work of Jones et al. (2003) is related to our model. They proposed and implemented a production optimization model for the seed business at Syngenta, where both the supply and demand of seeds are uncertain due to weather and changing customer needs. Their model considers multiple products and multiple periods where demand is strongly auto-correlated over time. However, unlike our work, Jones et al. do not model physical trading networks nor employ financial contracts in hedging.

One important aspect that has received less attention in the literature on agriculture is the relation between operational decisions with contracting and finance, so for this we turn to the supply chain contracting literature in the operations management field. The main goal of this literature is achieving operational efficiency taking into account financial aspects. Early examples of operation management problems for commodities are Cahn (1948); Bellman (1956); Dreyfus (1957); Charnes et al. (1966). They characterize optimal trading policies for warehouse management considering operational constraints but do not consider limits in purchases and sales nor derivatives markets. In Turvey and Baker (1990) future contracts and other derivatives are used taking into account the company's capital structure and correlations between crop yields and future prices. Settings involving spot purchases between one seller and many buyers are considered in Wu et al. (2002), while in Wu and Kleindorfer (2005) the focus is on one buyer and many sellers who can buy spot or futures contracts in two periods. Examples in these articles include electricity, semi-conductors and metals such as aluminium. Some of the elements in our model are present in Martínez de Albéniz and Simón (2008) where commodity traders who can buy in one market and sell in another are studied taking into account operational constraints such as logistics. The article identifies an optimal trading strategy for the kerosene market in the United States. The focus of Secomandi (2010) is on natural-gas processing facilities through a multiperiod model that allows for future contracts, storage and transformations between multiple commodities, taking into account operational constraints on facilities. This model relies on traditional work on inventory management that computes an optimal set of purchases, storage and sales with fluctuating markets. The analysis in Ahumada and Villalobos (2011) analyses short term production and distribution decisions in the fresh produce industry and finds the value of keeping production as flexible as possible but does not include financial decisions as we do in our work. The case of an agricultural firm that sources an input and converts it into two different outputs taking into account prearranged contracts and the spot market is considered in Boyabatlıet al. (2011); Boyabatlı (2015). Results in Kazaz and Webster (2011) focus on risk-averse firms that grow an agricultural product with an uncertain quality and yield, and compare the optimal strategies to those in the setting of the traditional newsvendor problem. The work of Goel and Gutierrez

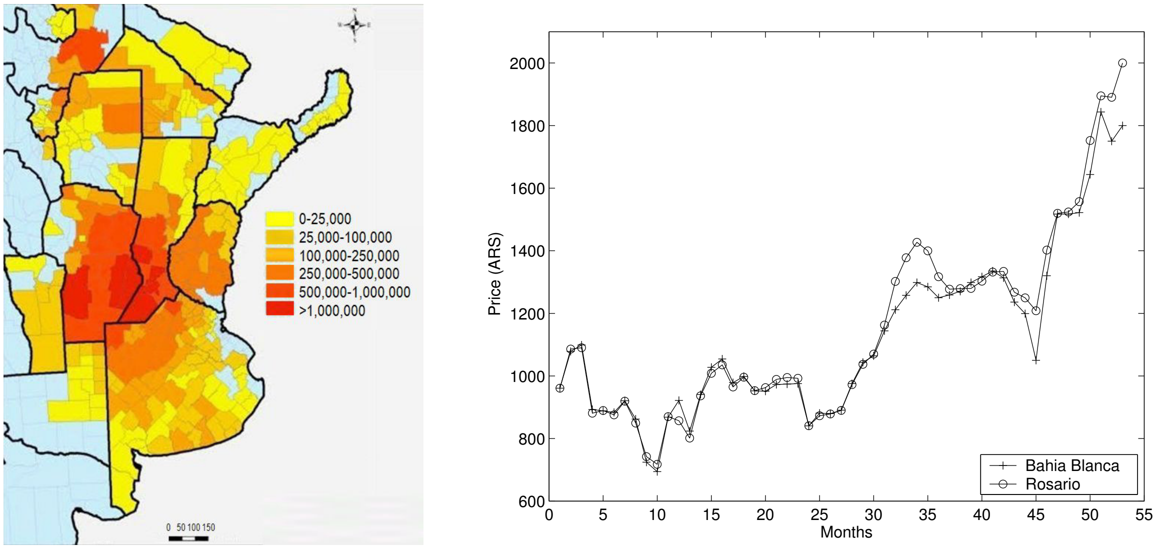
(2011) concentrates on procurement flexibility and the value of using futures prices. The model has a single source and multiple retailers and at any given time there is a tradeoff between buying spot versus buying a future for the next time period. The demand is stochastic and realized after purchasing decisions. The work of Dong et al. (2014) is about operations flexibility for refineries, and focuses on different end-products and the transformations between them. They consider a two-stage stochastic program that captures uncertainty in prices.

Finally, the model closest to our work in its scope of application and objectives is that in Devalkar et al. (2011), which describes how the ITC group improved the procurement of commodities in rural India in year 2000. The e-Choupal initiative included the procurement, processing and trade of multiple commodities in multiple periods under uncertain prices. They consider a firm that buys in the spot market in every period, processes the input and sells the processed output using forward contracts. They include capacity constraints that represent limits to the amount of soybeans that can be converted into soybean meal and soybean oil. Similarly to our model, Devalkar et al. assume that the future value of the spot price is equal to the price of futures seen today. But this work does not consider forward purchases nor logistics and there is no underlying network component, which is the basis of our work. Because we both allow for forward purchases and sales in multiple periods, we have to deal with spatial as well as temporal arbitrage constraints. Unlike Devalkar et al., we find that it may be useful to carry temporary imbalances since there may be opportunities to make beneficial purchases at later periods.

1.4. **Structure of the Paper.** In Section 2, we explain the specifics of our case study. Section 3 contains the mathematical formulation of the model, while Section 4 describes the data used in our experiments. We present the solutions to the model in Section 5 and the conclusions ensuing from our case study in Section 6.

2. THE CASE OF LOS GROBO

The most important business of Los Grobo is trading soybeans, corn and wheat (Regunaga, 2010; Bell and Scott, 2010). The production and intermediation of grain follow seasonal patterns. Soybeans are planted primarily in November and December for harvesting in March and April. The high season for soybean shipping begins right after harvest and lasts from March to August. The main grain-producing region in Argentina, located at the center of the country, covers roughly 600 thousand square kilometres. As Figure 1a shows, the 2007/8 production of soybeans concentrated mostly in the central region of Argentina. In recent years, the highly atomized producers in the region have produced close to an annual 40 and 15 million tonnes of soybeans and wheat respectively (Regunaga, 2010).



(A) Soybean production by region in 2007/2008 (tonnes). Source: SAGPyA

(B) Soybean spot prices per tonne at Rosario and Bahía Blanca ports between January 2008 and May 2012.

FIGURE 1. Soybeans in Argentina

Soybeans are produced almost exclusively to be exported to Europe and Asia. Domestic shipping requires scheduling about 1.3 million truckloads with a transportation cost in late 2012 of 38 USD per tonne of soybeans for the average 300 km distance from farm to ports or mills. Deep-water ports, accessible to transoceanic vessels, are located on the eastern boundary of the main producing region, shaped by the Paraná river and the Atlantic Ocean shore. There are about 10 deep-water ports, including those in the vicinity of the large cities of Rosario, Buenos Aires and Bahía Blanca. An intermediate stop at a storage and drying facility is required if the grain carries too much moisture according to international quality standards. If destinations are over capacity, or if purchase and sale contracts differ in their timing, grains can be stored. Commodity traders operate in the spot market, for immediate delivery, and in the futures market, in which the price is set at time t_0, for the delivery and simultaneous payment of grain at a predetermined later time t. Futures prices for delivery at different locations on various dates are related by arbitrage restrictions based on the possibility of storing and/or shipping the commodity to exploit price differentials across contracts. Nevertheless, the correlation between prices at different locations is not perfect. For example, as the harvest in the northern and southern parts of Argentina do not occur at the same time, northern and southern ports receive supply at different moments of the year and forward prices for these locations reflect that. Figure 1b shows spot soybean prices between 2008 and 2012, at the two main ports in Argentina (Rosario and Bahía Blanca), located on the north and south of the main producing region. In Argentina, forwards prices at ports are liquidly quoted.

3. A MODEL THAT CAPTURES NEW TRANSACTIONS AND LOGISTICS

3.1. **Definition of the Network.** We consider a generic commodity-trading firm that provides an intermediation service during a *marketing period* of several months. Material events, including signing new contracts, payments, storage and shipments occur in this time interval. For modeling purposes time is partitioned in several shorter periods indexed by $T = \{t_0, t_1, \ldots, t_{\bar{T}}\}$, which can represent months, half-months or weeks, depending on the needs of the user. The spatial structure of the network is encoded in three sets of nodes: origins $O = \{o_1, \ldots, o_{\bar{O}}\}$ representing farms, plants $P = \{p_1, \ldots, p_{\bar{P}}\}$ representing storage facilities, and destinations $D = \{d_1, \ldots, d_{\bar{D}}\}$ representing ports and crushing mills. The time-expanded network contains T copies of each spatial node, one for each time period, totaling $(\bar{O} + \bar{P} + \bar{D})\bar{T}$ nodes. Shipments assigned to a particular period happen instantly at the beginning of the period because the period length is much longer than the actual shipping time. Hence, for each time period, the network includes directed arcs that form a complete set that connects all origins to all plants, all plants to all destinations, and all origins to all destinations. In addition, for each node there are arcs connecting each time period with the next. These arcs represent that grain can be stored in that location. In the basic model we allow storage in plants and also in origins by using silobags.

Arcs that encode shipping are labeled with a shipping cost (in ARS/tonne)[2]. We refer to these values by $c_{od}(o, d)$ (origin to destination), $c_{op}(o, p)$ (origin to plant), and $c_{pd}(p, d)$ (plant to destination), for $o \in O$, $p \in P$, and $d \in D$. Storage arcs are labeled with the cost to store grain (in ARS/tonne). Plants and ports have capacities (measured in tonnes) that impose upper bounds in the amount of grain that can go through them in each period. We refer to plant capacity by $cap_p(p)$ and to destination capacity in ports by $cap_d(d)$.

3.2. **Definition of Purchase and Sale Transactions.** At t_0, the firm owns a book of purchase contracts that specify that the firm is committed to collect a certain amount of soybeans from an origin at some point in the future. The price to be paid in exchange for the grain may already be fixed in the contract, or may be left to be fixed later, in which case the contract specifies the time-frame during which this should happen.

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[2] ARS means Argentinean Pesos. In the second half of 2013, one Argentine peso was roughly equivalent to 0.20 US dollars

In this case the farmer (who sells soybeans to the firm) keeps the right to decide *when* to fix the price at the prevailing market rate.

A sale contract determines when, where, and how much grain must be delivered. As in purchase contracts, there are priced and non-priced contracts. In the latter, the firm owns the right to decide *when* to fix the price, which will be set according to market rates.

The sum of the volumes in all previously-signed contracts for a each origin and destination leads to supply and demand aggregates (in tonnes):

- Total amount purchased with and without price: $\mathit{purch}_{wp}(o,t)$, and $\mathit{purch}_{np}(o,t)$, respectively.
- Total amount sold with and without price: $\mathit{sales}_{wp}(d,t)$, and $\mathit{sales}_{np}(d,t)$, respectively.

The cost and revenue arising from prearranged transactions are fixed prior to running the model so they are just added to the objective function at the corresponding time-period. In addition, initial inventories might exist in farms, storage facilities and ports. We denote them (in tonnes) by $\mathit{InitInv}_o(o)$, $\mathit{InitInv}_p(p)$, and $\mathit{InitInv}_d(d)$, for origins, plants, and destinations, respectively.

The firm can sign spot and forward/futures contracts as necessary. At the time of running the model t_0, forwards prices $pr_{\mathit{fwd},d}(d,t)$ (in ARS/tonne) at a port d to deliver at time t are taken as inputs. Each farm has a *natural destination* defined as the destination that maximizes the net *farmer offered price*, itself computed as the difference between the prevailing market forward price at destination minus standard transportation costs from the farm. The natural destination corresponding to an origin $o \in O$ is

$$\mathit{NatDest}(o,t) = \arg\max_{d \in D} \{pr_{\mathit{fwd},d}(d,t) - c_{od}(o,d)\}.$$

This price can be seen as a lower bound on what farmers may expect to receive in a competitive market, as they could otherwise arrange to ship the grain to the final destination on their own and achieve this price. Although in practice natural destinations tend to be constant, they depend on market conditions since imbalances in prices at ports or changes in transportation costs may have an impact in the optimal location that a farmer would choose as destination for the production. Plants also have a natural destination, defined in an analogous manner. We refer to the subset of origins and plants that share the same natural destination as a *basin* of that destination. Indeed,

$$Basin(d,t) = \{o \in O : \mathit{NatDest}(o,t) = d\} \cup \{p \in P : \mathit{NatDest}(p,t) = d\}.$$

Putting all this together, we compute the forward price at origin o (in ARS/tonne) as

$$pr_{\mathit{fwd},o}(o,t) = pr_{\mathit{fwd},d}(\mathit{NatDest}(o,t),t) - c_{od}(o,\mathit{NatDest}(o,t)).$$

3.3. **Distributional Assumptions.** Pre-existing contracts and new contractual arrangements made at t_0 for subsequent collection, delivery, reception and transportation of grain are assumed to be free of default risk. Interest rates and all other costs are known at t_0. Contracts without fixed price can be valued at t_0 by absence of arbitrage at the prevailing level of forward prices. Therefore, the trading book of the firm can be valued without stochastic modeling of grain prices at later periods. In sum, the model under consideration is fully deterministic. The optimal policy computed at t_0 is assumed to be carried out in later periods with certainty.

3.4. **The Mathematical Program.** Here we delineate the decision variables, the main constraints, and the objective function of the problem at hand. The main decision variables represent physical flows, inventories held in plants, and new transactions (in tonnes):

- Flows $x_{op}(o,p,t)$ (origin to plant), $x_{pd}(p,d,t)$ (plant to destination), and $x_{od}(o,d,t)$ (origin to destination) at time t.
- Grain stored in origin $x_o(o,t)$ and in plant $x_p(p,t)$ at time t.
- New purchases through forward contracts $x_{\mathit{purchFwd}}(o,t)$ at origin o, new purchases through spot contracts $x_{\mathit{purchSpot}}(d,t)$ at destination d, and new sales through forward contracts $x_{\mathit{saleFwd}}(d,t)$ at destination d. These are signed at t_0 for delivery at time t.

Every origin has an upper bound on the amount of grain that can be produced in its vicinity. This bound must be taken into account when planning for new transactions. Ports, mills and crushing plants have delivery limits, which depend on the capacity and loading speed of each specific destination. Intermediate storage and drying facilities also have fixed capacities. Finally, one must consider mass conservation as flows circulate over space and time. These are some of the constraints that we use to characterize feasible flows.

- Conservation of mass at origin o at time t: $\mathit{purch}_{wp}(o,t)+\mathit{purch}_{np}(o,t)+x_{\mathit{purchFwd}}(o,t)+x_o(o,t) = \sum_p x_{op}(o,p,t) + \sum_d x_{od}(o,d,t) + x_o(o,t+1)$.
- Conservation of mass at plant p at time t: $\sum_o x_{op}(o,p,t)+x_p(p,t) = \sum_d x_{pd}(p,d,t)+x_p(p,t+1)$.
- Conservation of mass at destination d at time t: $\sum_o x_{od}(o,d,t)+\sum_p x_{pd}(p,d,t)+x_{\mathit{purchSpot}}(d,t) = \mathit{sales}_{wp}(d,t) + \mathit{sales}_{np}(d,t) + x_{\mathit{saleFwd}}(d,t)$.
- Bounded capacity at plants: $\sum_o x_{op}(o,p,t) + x_p(p,t) \leq \mathit{cap}_p(p)$. Those for ports are similar.

To simplify terms in the objective function, we group some parameters and variables for purchases and sales as follows (all in ARS).

- Total prearranged purchase contracts:
  $\mathit{TotPrearrPurch}(t) = \mathit{PrearrPurch}_{wp}(t) + \sum_o pr_{\mathit{fwd},o}(o,t)\mathit{purch}_{np}(o,t)$, where $\mathit{PrearrPurch}_{wp}(t)$ is the cost of prearranged purchases with price.
- Total new purchase contracts: $\mathit{NewPurch}(t) =$
  $\sum_o pr_{\mathit{fwd},o}(o,t)x_{\mathit{purchFwd}}(o,t) + \sum_o (pr_{\mathit{fwd},d}(d,t) + \epsilon)x_{\mathit{purchSpot}}(d,t)$, where ϵ is the bid-ask spread.
- $\mathit{ShippingCost}(t) = \sum_{o,d} c_{od}(o,d)x_{od}(o,d,t) + \sum_{o,p} c_{op}(o,p)x_{op}(o,p,t) + \sum_{p,d} c_{pd}(p,d)x_{pd}(p,d,t)$.
- $\mathit{StorageCost}(t) = \sum_p \mathit{UnitStoragePlant}(p)x_p(p,t) + \sum_o \mathit{LoadOrigin}(o)x_o^+(o,t)$, where $x_o^+(o,t)$ represents the amount of grain that is first stored in period t. This modification, achieved using a standard network transformation, captures the cost of buying silobags.
- $\mathit{LoadingCost}(t) = \sum_{o,p} \mathit{LoadingPlant}_p(p)x_{op}(o,p,t)$ is the total cost of loading grain into plants.

The objective of the problem is to maximize the total profit accrued over all periods discounted at an interest rate r:

$$\max \sum_t \big(\mathit{TotPrearrSales}(t) + \mathit{NewSales}(t) - \mathit{TotPrearrPurch}(t) - \mathit{NewPurch}(t) \\ - \mathit{ShippingCost}(t) - \mathit{StorageCost}(t) - \mathit{ProcessingCost}(t) - \mathit{LoadingCost}(t)\big)e^{-rt}.$$

In addition to the basic model outlined above we implemented two features, detailed in the Appendix, that increase its realism. First, it is desirable to limit the number of new contracts to what is feasible in practice. We have strived to find an *appropriately small* number of new contracts, in the sense that the bulk of the physical flow should be due to pre-existing contracts rather than a byproduct of the model. Hence, we look for the minimum number of additional contracts that make the flow balanced in each basin and for each time period. Second, harvested soybeans sometimes contain humidity in excess of 13.5% of mass which must be dried before the grain can be sold to international exporters. We encode this in the model using two parallel physical networks for the stocks and flows of dry and humid grain. Dry grain can be shipped directly from farms to ports but humid grain must go through processing plants to be dried.

3.5. **Implementation.** The model has been written using AMPL, a high level mathematical programming language, and was optimized by an open-source LP solver called *lpsolve*. The time-expanded networks for the instances we considered resulted in medium-size problems. The example in Section 4 contains 5851 variables and 5204 constraints and optimal solutions could be found in less than a second. The low computational cost means that it can be run on demand and in real time. Once fully integrated into the back-end systems of a commodity-trading firm, we expect several daily runs for various alternative scenarios to be routine, even if the outcome of the model is interpreted as a set of actions on a coarser time grid. The output includes the detailed routing of trucks in space and time, the specifications of new transactions that the sales force should aim to achieve, and various maps that include sensitivity analysis information.

4. DATA

The examples in this section are inspired[3] on the business activities of Los Grobo for the marketing season of grain harvested in Argentina between March and April of 2012. The marketing period begins several months prior to the harvest date and continues for several months after it. In this example, we consider the baseline time of March 2012 as t_0 in our model to compute the optimal logistic solution for the contracts signed before that month. The planning season consists of periods $t_1, \ldots, t_5$ given by April to August 2012. We geocoded all relevant locations and computed distances in kilometres between any two locations using actual roads and a GIS system. Distances were converted into transportation costs using the CATAC[4] table, a price list published by the trucking industry of Argentina.

As of March 2012 Los Grobo owned an initial inventory of 216 purchase contracts for 506,719 tonnes with an already-set price and a present value of ARS 834 million. In this example, the buyer pays the seller shortly after the grain is shipped. We obtained all present values by discounting cash flows at the 15% annual interest rate prevailing then in Argentina. The majority of the purchase contracts specified a delivery date on April and May 2012. As of March 2012, all contracts already had the price fixed. At that time, Los Grobo held 19 sale contracts for 371,712 tonnes with a present value of ARS 610 million, out of which 260,621 tonnes had the price fixed, and 111,091 tonnes had an unspecified price to be fixed later. The relatively small ratio of sale contracts vs. purchase contracts is due to the atomized nature of the farming sector and the concentrated nature of the exporting business. Because of the imbalance between prearranged purchase and sale contracts held by Los Grobo, new sale contracts had to be signed to make the solution feasible and reduce the residual inventory at the end of the season.

We took an upward sloping forward curve for the examples in this paper. In our runs we assume that 10% of the tonnage of soybean collected in most of the purchase contracts is humid. This grain must go through plants to be dried, in addition to grain that may be stored for other reasons.

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[3] The data presented here has been distorted to preserve confidentiality.

[4] Confederación Argentina del Transporte Automotor de Cargas. See `www.catac.org.ar` for details.

5. RESULTS

This section presents the optimal solution for the case study, an analysis of how the solution changes when new contracts are not the optimal ones or when interest rates increase, and a comparison between the optimal solution and a heuristic that emulates the actual behavior of the firm.

5.1. **The Optimal Solution.** Figure 2 displays the optimal logistics. Origins, plants and final destinations are denoted by o, p, and d respectively. The arrows in the figure are physical flows between origins, plants and ports for each period. The width of an arrow is proportional to the volume shipped. We code the time of the flow by shades of grey: darker flows occur later. Next to each origin we report the aggregate amount (in tonnes) of new purchases that should be signed at that location. Next to each destination we report the aggregate amount (in tonnes) of new spot purchases (top) and new sales (bottom) there. Next to each plant we report its inventory turnover. All reported numbers represent sums for all periods. Many flows represented by thin arrows depict the need to ship humid grain to plants. These are sometimes further away than the nearest destination, but there is no option since the grain must be dried before it can be sold. We can clearly see how basins tend to emerge from the optimal logistics since in most cases grain is delivered to the nearest destination. However, sometimes there are flows from one location to more than one destination or vice-versa. These happen because of geographic dispersion in supply and demand, because the relative price changes between different ports and other destinations, and because of capacity issues.

TABLE 1. Optimal transactions and tonnage for each period

	Periods					
Transactions (in ARS)	**1**	**2**	**3**	**4**	**5**	**Total**
Prearranged purchases	-124,593,427	-616,025,502	-71,712,169	-19,888,346	-1,451,202	-833,670,646
Forward purchases at origins	0	-23,907,818	0	0	0	-23,907,818
Spot purchases at destinations	-20,390,560	-55,542,232	0	0	0	-75,932,792
Prearranged sales	242,663,379	320,723,096	27,405,951	8,379,537	11,257,587	610,429,550
Forward and spot sales at destinations	0	0	0	0	397,361,351	397,361,351
Storage costs at origins and destinations	0	-2,886,719	-2,571,974	-439,881	-169,810	-6,068,385
Loading costs at processing plants	19,282	18,066	3,151	1,234	711	42,444
Storage costs at processing plants	0	-217,257	-288,355	-329,290	-359,497	-1,194,399
Processing costs	38,564	36,133	6,301	2,469	1,421	84,888
Shipping fees paid by farmers	36,111,113	36,421,707	5,985,210	2,130,496	1,301,762	81,950,288
Real shipping fees	-22,011,101	-25,799,274	-2,930,253	-1,220,481	-26,939,117	-78,900,226
Profit	**111,837,250**	**-367,179,800**	**-44,102,137**	**-11,364,262**	**381,003,205**	**70,194,256**
Total sales	**242,663,379**	**320,723,096**	**27,405,951**	**8,379,537**	**408,618,938**	**1,007,790,901**
Tonnage	**1**	**2**	**3**	**4**	**5**	**Total**
Prearranged purchases	235,039	222,874	30,246	11,800	6,760	506,719
Forward purchases at origins	0	15,211	0	0	0	15,211
Spot purchases at destinations	11,082	32,341	0	0	0	43,423
Prearranged sales	148,122	193,590	16,000	6,000	8,000	371,712
Forward and spot sales at destinations	0	0	0	0	192,700	192,700

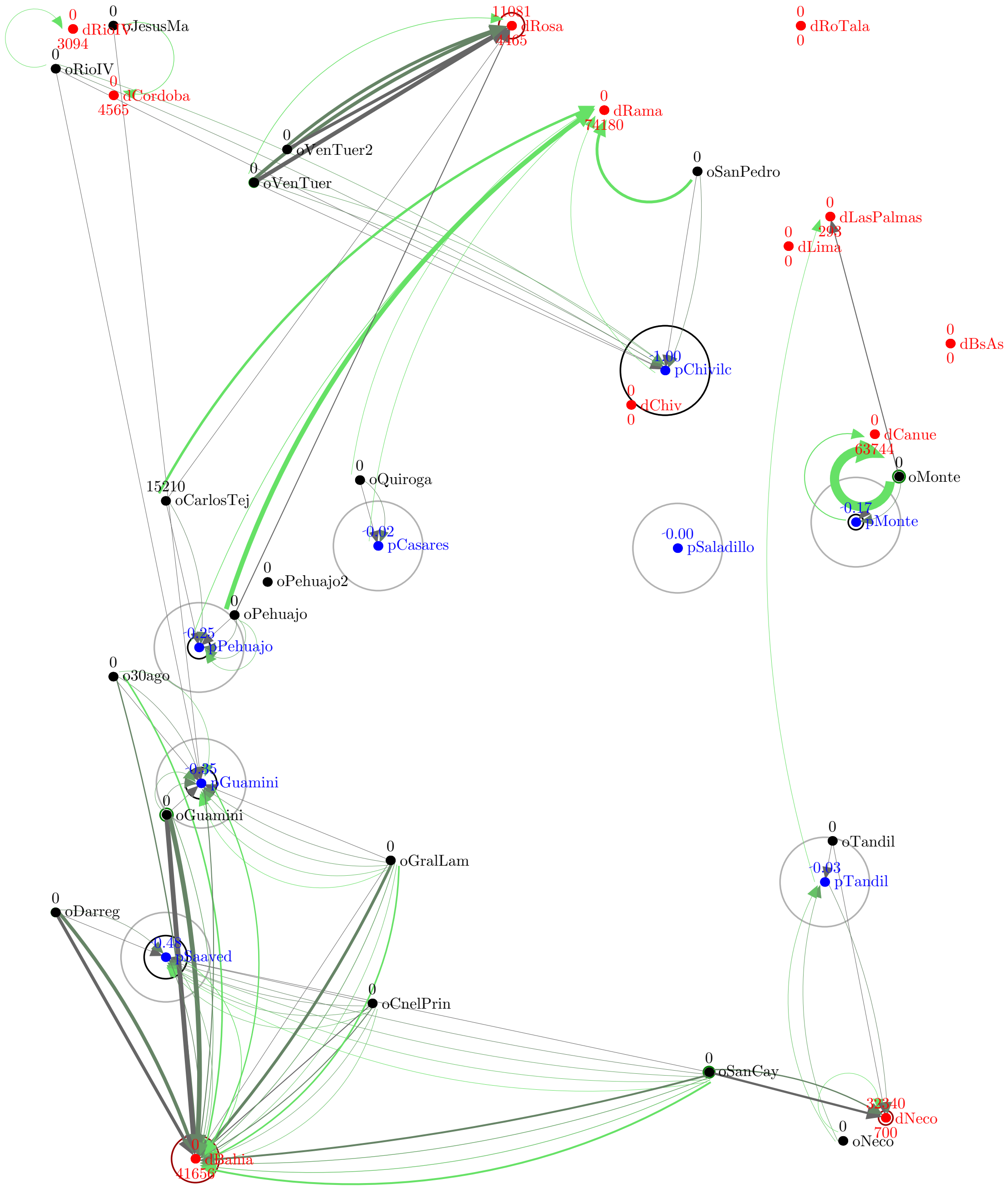


FIGURE 2. Optimal flows over time (periods 1 to 5)

Table 1 summarizes the transactions and fees (in ARS), and physical volumes (in tonnes) per period. The upper panel represents the cashflows at different times discounted to March 2012, hence amounts in this panel can be added to each other. Positive numbers represent income to Los Grobo. The bottom panel in Table 1 represents physical flows per period and type of transaction. Rows *Prearranged purchases* and *Prearranged sales* in the bottom panel indicate that most existing transactions are roughly equally split between periods 1 and 2, with some residual income in later periods. Comparing both, we see that Los Grobo is already committed to purchasing more grain than it sells in periods 1 to 4. Conservation of mass implies that the remaining inventory must be sold in period 5. This is achieved by new transactions reported in *Forward and spot sales at destinations*.

In this example we allowed the model to add new contracts to balance flow on each basin, alleviating the need of long distance shipping. It is optimal to purchase 58,634 additional tonnes of soybeans in periods 1 and 2 (reported in *Forward purchases at origins* and *Spot purchases at destinations* in both panels), even though prearranged purchases already exceeded prearranged sales. These additional purchases in specific locations optimally balance flows per basin because in this case it is more profitable to cover imbalances locally rather than using long hauls.

All the extra inventory (grain from purchase contracts in the portfolio plus new forwards, net of sale contracts in the portfolio) is sold and delivered in the last period. Available forward prices in this example made it optimal to buy new grain in earlier periods and sell it for delivery in later periods, even after considering the cost of holding inventory. Hence, all new sale contracts are for delivery and payment in period 5. The cost faced by Los Grobo for storing grain at origins is customarily delayed by one month therefore reported zero for period 1 in row *Storage costs at origins*. *Storage costs at processing plants* are much lower for Los Grobo than the cost of storing at origins because the farmer pays the cost associated to drying. However, Los Grobo faces a cost if it decides to store grain to wait for more favorable market conditions. Most of the grain processed at plants in this example was humid, which led to extra revenue (see row *Processing costs*).

We have assumed that transportation is arranged by Los Grobo as a service to the farmer, who pays the cost of shipping to a fixed destination specified by the contract. This includes a stop at a processing plant if the grain is humid. However, Los Grobo retains the right to change the routing of the grain at its own expense. The row *Shipping fee paid by farmers* represents the fees for transportation agreed upon in the contract while the row *Real shipping fees* represents the total cost incurred by the firm to execute the actual shipping. Because sometimes grain is shipped to locations different from what is priced in the contract, fees paid by farmers can be lower than the cost to Los Grobo.

Total profit and margin per period are highly variable over time. Adding up all profits along the season, the final margin is 6.97%, which is typical of this business. Margins in commodity trading are thin even when transactions and logistics occur as planned. The model assumes no chance of failure in the logistical processes planned from $t = 0$. In reality, however, a large operation such as Los Grobo's often encounters violations to this assumption. For instance, sudden labor strikes, logistical mishaps and counterpart defaults.

TABLE 2. Sensitivity Analysis: profit change per tonne bought in three origins and each period

Origin \ Period	1	2	3	4	5
San Cayetano	-95.67	-11.64	-20.57	-148.75	-258.60
Tandil	-95.67	0	0	-181.77	-258.47
Necochea	-95.67	0	-76.90	-197.82	-258.60

By their short term nature, these aspects are not captured by our model but they tend to have a relatively minor effect. The computational tool presented in this paper allows the firm to optimize its most important processes at a tactical level.

5.2. **Sensitivity to New Transactions.** Another useful output of the model is the sensitivity of global profit with respect to the new transactions suggested by the optimization model. This measure is the reduction in profit if one were to sign an additional purchase contract at a given location and time, considering that the new purchase contract optimally replaces some of the new transactions returned by the optimal solution to the problem. Technically, these values are the reduced costs of the variables $x_{purchFwd}(o, t)$ representing new forward contracts at an origin o at time t. These sensitivities are non-positive because the optimal solution includes the best possible new purchase contracts. Table 2 provides such estimates for selected locations in the optimal solution discussed in the previous section. This analysis is useful in practice because the recommendations encoded in the optimal solution cannot always be carried on due to unforeseen restrictions. The analysis helps the firm make decisions regarding the profitability of transactions at different locations or times. The zeros in the table imply the existence of multiple, alternative optimal solutions to the problem.

5.3. **Sensitivity to Inputs.** It is also relevant to evaluate changes in the optimal solution for changes in inputs to the model. We focus on the sensitivity of the solution with respect to changes in the interest rate. This sensitivity is important in the highly volatile macroeconomic context of Argentina. The interest rate in March 2012, the moment in which the model was assumed to be run in Section 4, was close to 15%. By comparison, at the time of writing this paper in late 2014, interest rates are in excess of 25%. Therefore, we recompute the optimal solution assuming that this higher level of interest rates prevailed in March 2012. Results are shown in Table 3. Several conclusions emerge from the comparison of Table 1 and Table 3. First, the pecuniary amounts displayed in both tables are already discounted to March 2012. Therefore, even for prearranged transactions with fixed prices, present values are smaller in Table 3 than in Table 1. Second, a significant portion of profits in Table 1 was due to purchases in earlier periods balanced with sales and delivery in later periods. This strategy is less profitable in a context of higher interest rates. The total profit in Table 3 is smaller than in Table 1. This also has an impact on new transactions. It is noticeable that certain tonnage is bought in periods 1 and 2 in Table 1, but this is largely deferred to period 3 in Table 3.

TABLE 3. Optimal transactions and tonnage for each period when interest rate is 25%

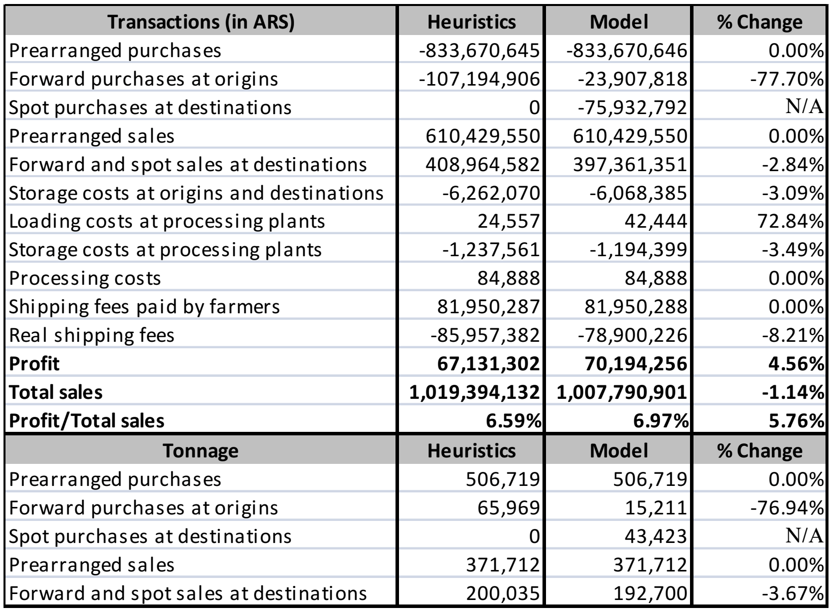
Interest Rate of 25%	**Periods**					
Transactions (in ARS)	**1**	**2**	**3**	**4**	**5**	**Total**
Prearranged purchases	-124,593,427	-610,913,287	-70,526,871	-19,397,301	-1,403,626	-826,834,511
Forward purchases at origins	0	0	-40,969,954	0	0	-40,969,954
Spot purchases at destinations	0	-55,081,303	0	0	0	-55,081,303
Prearranged sales	242,663,379	318,061,509	26,952,971	8,172,646	10,888,520	606,739,023
Forward and spot sales at destinations	0	0	0	0	384,334,296	384,334,296
Storage costs at origins and destinations	0	-2,502,947	-2,031,453	-1,410,685	-170,566	-6,115,651
Loading costs at processing plants	19,440	18,206	3,173	1,242	714	42,774
Storage costs at processing plants	0	-218,935	-290,360	-331,214	-361,097	-1,201,607
Processing costs	38,881	36,412	6,345	2,483	1,428	85,548
Shipping fees paid by farmers	36,111,113	36,119,453	5,886,283	2,077,894	1,259,085	81,453,829
Real shipping fees	-23,420,267	-25,585,174	-2,881,820	-1,190,347	-27,742,972	-80,820,580
Profit	**130,819,117**	**-340,066,065**	**-83,851,686**	**-12,075,283**	**366,805,781**	**61,631,864**
Total sales	**242,663,379**	**318,061,509**	**26,952,971**	**8,172,646**	**395,222,816**	**991,073,319**
Tonnage	**1**	**2**	**3**	**4**	**5**	**Total**
Prearranged purchases	235,039	222,874	30,246	11,800	6,760	506,719
Forward purchases at origins	0	0	26,293	0	0	26,293
Spot purchases at destinations	0	32,341	0	0	0	32,341
Prearranged sales	148,122	193,590	16,000	6,000	8,000	371,712
Forward and spot sales at destinations	0	0	0	0	192,700	192,700

5.4. **Estimation of Gains Achieved by Methodology.** In this section we compare the financial results from the optimal logistical strategy with a simpler heuristic that captures Los Grobo's operations in recent years. A precise record of the actual shipping arrangements that followed from the transactions signed by the firm for previous marketing seasons was not available to the authors. However, conversations with senior management at Los Grobo allowed us to identify a representation of the logistical policy followed by the firm in terms of a set of restrictions imposed in our model. Then, the efficiency gains due to adoption of the model in this paper can be measured as the difference in profit between the optimal strategy in the previous section and the heuristic used to represent the actions of the firm. In reality, however, having a fast optimization model provides additional benefits, as already mentioned in Section 1. These include the ability to rerun the model virtually instantly in response to changes in inputs, the estimation of risk measures through the aggregation of multiple scenarios and the ability to evaluate certain infrastructure investments.

To explain the heuristic that mimics the actions taken by Los Grobo, we note that each prearranged purchase had a cost-efficient destination at the time of writing the contract. These destinations were used by the firm to compute the shipping fee to be charged to the farmer and were recorded as part of the contract. Los Grobo then classified contracts into sub-books defined according to their initially-assigned destination. Finally, the firm optimized the logistics and inter-temporal opportunities within each individual sub-book.

This rule is not necessarily optimal as the cost-efficient destination for a certain contract at the time of running the model (March 2012) might differ from the destination that was cost efficient at the time of signing the contract (e.g., December 2011). For instance, this might be the case for certain changes in forward prices at ports. To mimic the firm's decisions, we solved a collection of optimization problems, one for each sub-book. Each of these potentially required new local transactions since shipping to other ports was not allowed. Then, the total profit for this heuristic was calculated as the sum of profits for each sub-book. Figure 3 shows the solution resulting from this procedure. In Table 4 we compare the results of the heuristic to those associated with the optimal rule of Section 4. A difference in profit of 4.56% is economically significant. In absolute terms it is a gain of slightly more than 3 million ARS, which exceeds the one-off cost of implementing the model in terms of consulting fees, software development and hardware. Moreover, the model is likely to add similar value for subsequent years, without demanding a sizable implementation cost.

TABLE 4. Comparison of model's optimal solution and Los Grobo's heuristic



Transactions (in ARS)	**Heuristics**	**Model**	**% Change**
Prearranged purchases	-833,670,645	-833,670,646	0.00%
Forward purchases at origins	-107,194,906	-23,907,818	-77.70%
Spot purchases at destinations	0	-75,932,792	N/A
Prearranged sales	610,429,550	610,429,550	0.00%
Forward and spot sales at destinations	408,964,582	397,361,351	-2.84%
Storage costs at origins and destinations	-6,262,070	-6,068,385	-3.09%
Loading costs at processing plants	24,557	42,444	72.84%
Storage costs at processing plants	-1,237,561	-1,194,399	-3.49%
Processing costs	84,888	84,888	0.00%
Shipping fees paid by farmers	81,950,287	81,950,288	0.00%
Real shipping fees	-85,957,382	-78,900,226	-8.21%
Profit	**67,131,302**	**70,194,256**	**4.56%**
Total sales	**1,019,394,132**	**1,007,790,901**	**-1.14%**
Profit/Total sales	**6.59%**	**6.97%**	**5.76%**
Tonnage	**Heuristics**	**Model**	**% Change**
Prearranged purchases	506,719	506,719	0.00%
Forward purchases at origins	65,969	15,211	-76.94%
Spot purchases at destinations	0	43,423	N/A
Prearranged sales	371,712	371,712	0.00%
Forward and spot sales at destinations	200,035	192,700	-3.67%

The previous examples are indicative of the complexity of the problem faced by any commodity-trading firm that sources a bulk commodity distributed in time and space and delivers it at multiple locations. Given the outcomes of the previous examples it is clear that the flows between the origins, plants and destinations are not fully predetermined by geography.

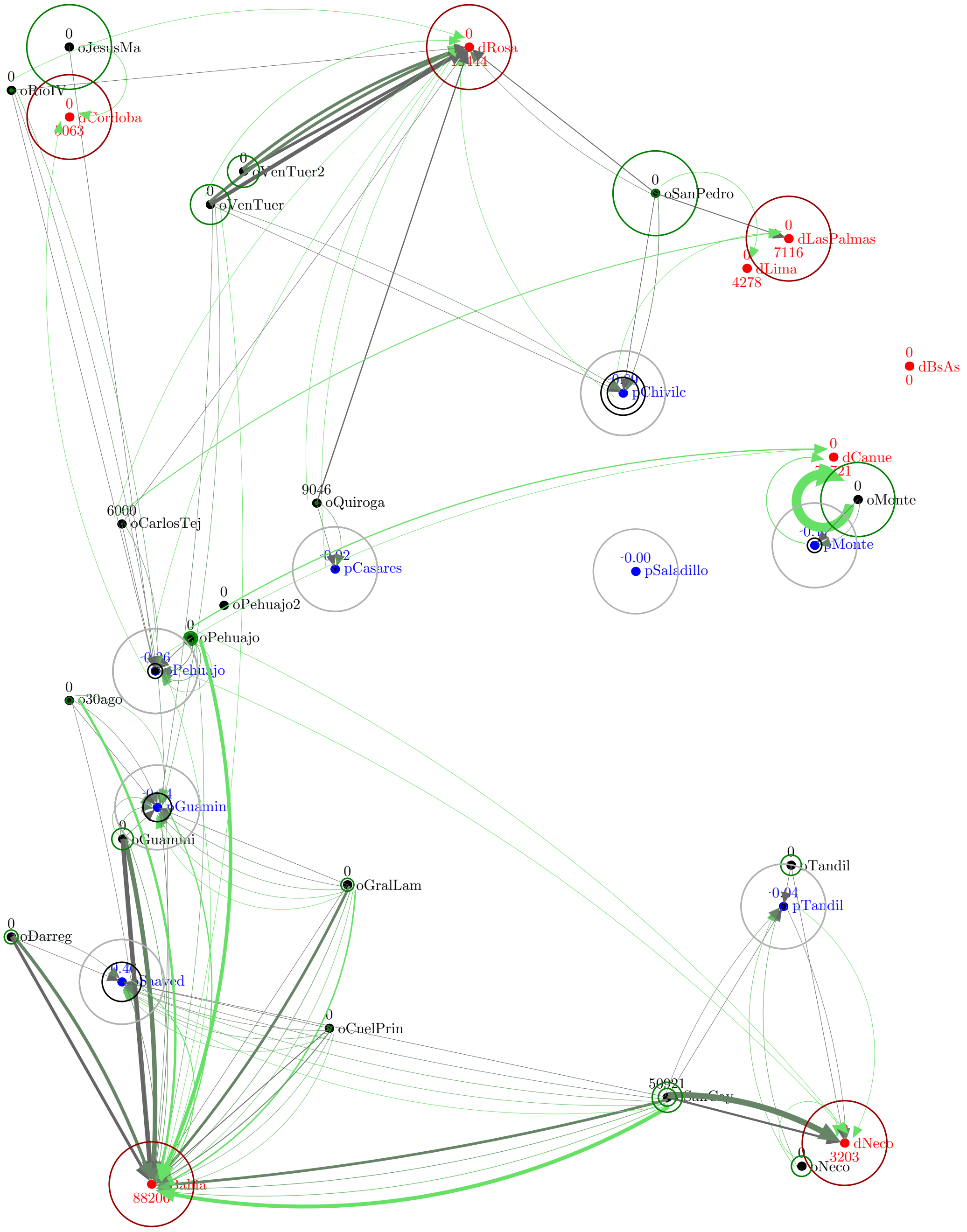


FIGURE 3. Heuristic used to mimic the policy followed by Los Grobo (periods 1 to 5)

6. CONCLUSIONS

Motivated by the grain intermediation business at one of the largest commodity-trading firms in South America we have developed and implemented a deterministic model for a profit-maximizing firm that provides an intermediation service between commodity producers and commodity end users, distributed over a realistic spatial network. Inputs are a set of previously arranged contracts, a term structure of forward prices for new transactions, a realistic physical network, and various shipping and processing costs. We have identified the optimal trading, storing and shipping policy for the firm as the solution of a profit-maximizing optimization problem, encoded as a minimum cost flow problem in a time-expanded network that captures both geography and time. Our work is a contribution to the growing literature on operations and finance. The methodological novelty in our approach is the juxtaposition of a physical trading network with a rich temporal structure of financial contracts. The resulting optimization problem is of linear form, which allows for very fast and efficient computational solutions.

We have presented realistic examples generated under various constraints on the amount and type of additional transactions allowed by the model. We found profit gains relative to a heuristic that represents the actions taken by the firm in the case study and identified several additional advantages of having a model of this kind. A more precise comparison between prior firm's policy and the optimal solution was unfeasible due to the absence of historical shipping records. By valuing all future commitments through spot and forward prices known when running the model and assuming no default risk in the subsequent logistics we leave no stochasticity within the model. It would be interesting to lift this assumption in future work and take into account the real options embedded into the possibility of rerouting decisions to be taken as a function of prices revealed on later periods, or in response to unforeseen logistical problems or changes in the quality or humidity of grains. Such a model would require an explicit stochastic description of grain prices, quality, rain, disruptions, etc.

Due to poor economic conditions in Argentina since 2011, the adoption of this model by industry has been delayed. We are confident that models of this kind will be embraced in the near future in the region.

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7. APPENDIX

We outline some additional details of the model that were not included in Section 3.

7.1. **Quality of grain.**

We also consider a model with two levels of quality: dry and humid grain. We split the flow variables that go from origins to plants in two classes, using super-indices d and h, respectively.

- Dry flow from origin to plant (tonnes): $x^d_{op}(o, p, t)$.
- Humid flow from origin to plant (tonnes): $x^h_{op}(o, p, t)$.

To link back to the original variables, we use the constraint $x^d_{op}(o, p, t) + x^h_{op}(o, p, t) = x_{op}(o, p, t)$. Other flow variables consist of dry grain exclusively since the only locations where humid grain can be transformed into dry one are in the plants, and humid grain cannot be delivered to destinations. We assume that each origin provides a given percentage of humid and dry grain, depending on the location and period, as follows.

- Percentage of humid grain (%): $\mathit{PercHum}(o, t)$.
- Total dry supply from prearranged contracts (tonnes):
  $\mathit{DryGrains}(o, t) = \{\mathit{purch}_{wp}(o, t) + \mathit{purch}_{np}(o, t)\}(1 - \mathit{PercHum}(o, t))$.
- Total humid supply from prearranged contracts (tonnes):
  $\mathit{HumidGrains}(o, t) = \{\mathit{purch}_{wp}(o, t) + \mathit{purch}_{np}(o, t)\}\mathit{PercHum}(o, t)$.

These constraints encode the balance of mass for grain of both qualities, dry and humid:

- Dry grain can be shipped to plants or to destinations:
  $\textit{DryGrains}(o,t) + x_{\textit{purchFwd}}(o,t) = \sum_p x^d_{op}(o,p,t) + \sum_d x_{od}(o,d,t)$. In agreement with market practices, forward contracts are written on dry grain.
- Humid grain must go to plants for processing: $\textit{HumidGrains}(o,t) = \sum_p x^h_{op}(o,p,t)$. Notice that this grain cannot be stored in the origin.
- Aggregating the humid grain, we compute the total cost of drying (ARS) as $\textit{ProcessingCost}(t) = \sum_{o,p} \textit{UnitDryingCost}(p) x^h_{op}(o,p,t)$.

7.2. **Bounds for new transactions.** The model allows new transactions up to the minimal amount required to compensate imbalances *per basin*, with no restriction regarding the locations associated to those new contracts. We aggregate the transaction volume in *each basin* (purchases – sales) computing the net inventory for each period. When the net inventory is negative (purchases – sales < 0)—meaning that in that particular basin and period new purchases are needed to fulfil prearranged sales— new transactions are allowed by an amount up to minus the net inventory.

- Total prearranged contracts for the basin corresponing to destination d (tonnes):
  $\textit{PurchBasin}(d,t) = \sum_{o:\textit{NatDest}(o,t)=d} (\textit{purch}_{wp}(o,t) + \textit{purch}_{np}(o,t))$,
  $\textit{SalesBasin}(d,t) = \textit{sales}_{wp}(d,t) + \textit{sales}_{np}(d,t)$.
- Inventory in basin corresponding to destination d and period $t > t_1$:
  $\textit{Inventory}(d,t) = \max(\textit{Inventory}(d,t-1),0) + \textit{PurchBasin}(d,t) - \textit{SalesBasin}(d,t)$.
- Inventory in basin corresponding to destination d and period $t = t_1$:
  $\textit{Inventory}(d,t_1) = \sum_{o:NatDest(o,t_1)=d} \textit{InitInv}_o(o) + \sum_{p:NatDest(p,t_1)=d} \textit{InitInv}_p(p) + \textit{PurchBasin}(d,t_1) - \textit{SalesBasin}(d,t_1)$.
- Constraint that limits new contracts: $\sum_o x_{\textit{purchFwd}}(o,t) + \sum_d x_{\textit{purchSpot}}(d,t) \leq \sum_{d,t} \textit{Inventory}(d,t)^-$, where $\textit{Inventory}(d,t)^- = -\min(\textit{Inventory}(d,t),0)$ is the negative part of the inventory .