Monetary Policy and Dutch Disease: The Case of Price and Wage Rigidity*

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Abstract

We study a model of a small open economy that specializes in the production of commodities and that exhibits frictions in the setting of both prices and wages. We study the optimal response of monetary and exchange rate policy following a positive (negative) shock to the price of the exportable that generates an appreciation (depreciation) of the local currency. According to the calibrated version of the model, deviations from full price stability can generate welfare gains that are equivalent to almost 0.5% of lifetime consumption, as long as there is a significant degree of rigidity in nominal wages. On the other hand, if the rigidity is concentrated in prices, the welfare gains can be at most 0.1% of lifetime consumption. We also show that a rule - formally defined in the paper - that resembles a "dirty floating" regime can approximate the optimal policy remarkably well.

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1. INTRODUCTION

In this paper, we study optimal monetary and exchange rate policy in a small open economy with both price and wage rigidity, following a shock to the price of an exportable commodity. From a theoretical point of view, and as we show in the paper, the presence of both types of rigidities implies that, to the extent that fiscal policy is unresponsive to shocks, full price stability is not optimal.

This paper is part of a research project that is motivated by the experiences of many small open economies that in the last two decades became—very effectively!—inflation targeters. Many of these economies are commodity producers, and the size of the commodity sector to GDP is very high. For example, in the decade from 2000 to 2010, exports of copper and marine products for Chile were, on average, around 17% of GDP, while total exports of oil and marine products in Norway accounted for 20% of GDP (Hevia and Nicolini, 2013). During the same decade, the real price of copper and oil experienced changes of around 300%. The size of these shocks is orders of magnitude above any business cycle shock we have ever seen.

These shocks have direct effects on the monetary side of these economies. In effect, the correlation between the HP-filtered price of the exportable commodity and the HP-filtered nominal exchange rate—to concentrate our analysis at the business cycle frequency—ranges from $-50\%$ to $-70\%$ for both countries, depending on whether one includes the last three periods of the decade in the sample. That period of time witnessed very large changes in commodity prices, comoving with the peso in Chile and the krone in Norway.\footnote{The series are first logged and then HP-filtered with a smoothing parameter of 1600.} In Figure 1, we plot the HP-filtered data for the nominal exchange rate and the relevant commodity price for both countries, where the correlation is strikingly clear.

Clearly, this correlation cannot be independent of the policy regime. In a currency board or fixed exchange rate regime, that correlation is naturally zero. Thus, one should expect the...
Fig. 1. Nominal exchange rates and main exportable commodity price

correlation between the price of copper and the exchange rate in Chile to be much closer to zero during the early nineties, during which time inflation was driven down from close to 30% to below 10% using a managed exchange rate. But in a very successful inflation-targeting regime, like the ones followed by both central banks during the period, the nominal exchange rate freely floats, so the correlation is determined by market forces. According to the model that follows, the negative correlation is a direct implication of the inflation-targeting regime. Indeed, in the model, following a large increase in the price of the exportable, and given price stability, the nominal exchange rate—and the real, given price stability—suffers a strong appreciation, generating the traditional Dutch disease effect, which is the optimal response of prices and quantities to a relative price shock.

We are interested in studying conditions under which the strict inflation-targeting regime is optimal. Put differently, we want to find conditions under which these Dutch disease episodes are inefficient from the viewpoint of the allocation of resources. In our view, this is one of the main policy questions in countries like Chile. Indeed, during the successful inflation-targeting
period, the central bank deviated from the strict rule twice: once in April 2008 and again in January 2011. On both occasions, the justification for the intervention was essentially the same: the terms of trade were too high and the nominal exchange rate too low.

In a previous paper (Hevia and Nicolini, 2013), we studied an economy with only price frictions and showed that even in a second best environment with distorting taxes, domestic price stability is optimal as long as preferences are of the isoelastic type (which is typically used in the literature), even if fiscal policy cannot respond to shocks. In other words, restrictions on price setting do not necessarily imply that the large and persistent deviations observed in nominal and real exchange rates in countries like Chile are suboptimal, as long as monetary policy is executed so as to stabilize domestic prices. The model in that paper thus fully justifies a pure inflation targeting regime (at a zero rate of domestic price inflation). In other words, the “Dutch disease” is not really a disease, it is just the optimal response of prices and quantities to a relative price shock.

In the conclusion to our previous paper, we pointed out that our results fall apart in the presence of both price and wage frictions. Quantitatively exploring this question is the purpose of this paper. Exploring this policy problem in the context of both price and wage rigidity seems to us a natural step. Most medium-scale models used for monetary policy evaluation nowadays exhibit both types of rigidities, following the work of Christiano, Eichenbaum, and Evans (2005) and Smets and Wouters (2007). How far from pure price stability is the optimal policy once both types of frictions are present? What implications do they have for the “dirty floating” debate in countries that experience large fluctuations in their terms of trade? Answering these questions is the contribution of this paper.

As mentioned, the theory is clear: the presence of both frictions, coupled with inflexible tax instruments, implies that full price stability is not optimal. For completeness, we first generalize the closed economy results of Correia et al. (2013) and the small open economy results of Hevia and Nicolini (2013) with only price frictions to our small open economy model with price and wage frictions and then show that, in general, full price stability is not optimal when taxes cannot
be time and state dependent. But the main exploration of this paper is a quantitative one: on the one hand, we explore numerically how far apart from full inflation targeting the optimal policy is; on the other, we compute the welfare difference between the optimal policy and full inflation targeting. For the calibrated version of our model, we find that the key factor is the degree of wage rigidity. When wages are indeed highly rigid (the Calvo parameter is as high as 0.8) and there is enough price rigidity (a Calvo parameter of at least 0.25), the welfare effect of full price stability relative to the optimal policy can be as high as 0.5% of lifetime consumption. However, if the rigidity is mostly concentrated in prices with some wage rigidity, the welfare effect is bounded above by 0.1% of lifetime consumption. We also show that a dirty floating regime approximates the optimal policy remarkably well.

The model we use is the one explored in Hevia and Nicolini (2013), but in this model, we allow for heterogeneous labor with market power and frictions in wage setting. A virtue of the model is that it is fully consistent with the evidence presented in Figure 1, as we show in the paper. We present the model in Section 2. In Section 3 we describe the calibration and numerical solutions and discuss optimal policy. A final section concludes.

2. THE MODEL

We study a discrete time model of a small open economy inhabited by households, the government, competitive firms that produce a tradable commodity, competitive firms that produce final goods, and a continuum of firms that produce differentiated intermediate goods. There are two differentiated traded final goods: one produced at home and the other produced in the rest of the world. The small open economy faces a downward-sloping demand for the final good it produces but takes as given the international price of the foreign final good. There are also two commodities—one produced at home, the other imported—used in the production of intermediate goods. These intermediate goods are used to produce the final domestic good.
Households

A representative household has preferences over contingent sequences of two final consumption goods, \( C^h_t \) and \( C^f_t \), and leisure \( L_t \). The utility function is weakly separable between the final consumption goods and leisure and is represented by

\[
E_0 \sum_{t=0}^{\infty} \beta^t U(C_t, L_t),
\]

where \( 0 < \beta < 1 \) is a discount factor, \( C_t = H(C^h_t, C^f_t) \) is a function homogeneous of degree one and increasing in each argument, and \( U(C, L) \) is concave and increasing in both arguments.

Sticky Wages. In order to allow for sticky wages, we assume the single household has a continuum of members indexed by \( h \in [0,1] \), each supplying a differentiated labor input \( n_{ht} \). Preferences of the household are described by (1), where leisure is

\[
L_t = \bar{L} - \int_0^1 n_{ht} dh,
\]

and \( \bar{L} \) is the total amount of time available for work or for leisure.

The differentiated labor varieties aggregate up to total labor input \( N_t \), used in production, according to the Dixit-Stiglitz aggregator

\[
N_t = \left[ \int_0^1 n_{ht} \frac{\theta^w}{\theta^w - 1} dh \right]^{\frac{\theta^w}{\theta^w - 1}}, \quad \theta^w > 1.
\]

Each member of the household, which supplies a differentiated labor variety, behaves under monopolistic competition. The workers set wages as in Calvo (1983), with the probability of being able to revise the wage \( 1 - \alpha^w \). This lottery is \( i.i.d. \) across workers and over time. The workers that are not able to set wages in period 0 all share the same wage \( w_{-1} \). Other prices
are taken as given. There is a complete set of state-contingent assets. We consider an additional tax, a payroll tax on the wage bill paid by firms, $\tau^p_t$.

**Market Structure** Financial markets are complete. We let $B_{t,t+1}$ and $B^*_{t,t+1}$ denote one-period discount bonds denominated in domestic and foreign currency, respectively. These are bonds issued at period $t$ that pay one unit of the corresponding currency at period $t + 1$ on a particular state of the world and zero otherwise.

The household’s budget constraint is given by

$$P_t C_t^h + P_t C_t^f + E_t [Q_t B_{t,t+1} + S_t Q^*_t B^*_{t,t+1}] \leq W_t (1 - \tau^n_t) N_t + B_{t-1,t} + S_t \frac{\hat{B}^*_{t-1,t}}{1 + \tau^*_t},$$

where $S_t$ is the nominal exchange rate between domestic and foreign currency, $W_t$ is the nominal wage rate, $\tau^n_t$ is a labor income tax, $\tau^*_t$ is a tax on the return of foreign-denominated bonds (a tax on capital flows), and $Q_t B_{t,t+1}$ is the domestic currency price of the one-period contingent domestic bond normalized by the conditional probability of the state of the economy in period $t + 1$ conditional on the state in period $t$. Likewise, $Q^*_t B^*_{t,t+1}$ is the normalized foreign currency price of the foreign bond.\(^2\) In this constraint, we assume that dividends are fully taxed and that consumption taxes are zero (we explain these choices later).

Using the budget constraint at periods $t$ and $t + 1$ and rearranging gives the no-arbitrage condition between domestic and foreign bonds:

$$Q_t B_{t,t+1} = Q^*_t B^*_{t,t+1} (1 + \tau^*_t) \frac{S_t}{S_{t+1}}. \tag{5}$$

Working with the present value budget constraint is convenient. To that end, for any $k > 0$, we

\(^2\)We use the notation $\hat{B}^*_{t,t+1}$ instead of simply $B^*_{t,t+1}$ to distinguish foreign bonds held by the household sector from foreign bonds held by the aggregate economy.
let \( Q_{t,t+k} = Q_{t,t+1}Q_{t+1,t+2}...Q_{t+k-1,t+k} \) be the price of one unit of domestic currency at a particular history of shocks in period \( t+k \) in terms of domestic currency in period \( t \); an analogous definition holds for \( Q^*_{t,t+k} \). Iterating forward on (4) and imposing the no-Ponzi condition \( \lim_{t \to \infty} E_0[Q_{0,t}B_t + S_tQ^*_0] \geq 0 \) gives

\[
E_0 \sum_{t=0}^{\infty} Q_{0,t} \left( P^h_t C^h_t + P^f_t C^f_t - W_t (1 - \tau^n_t) N_t \right) \leq 0,
\]

where we have assumed that initial financial wealth is zero, or \( B_{-1,0} = \bar{B}^*_{-1,0} = 0 \).

The household maximizes (1) subject to (6). The optimality conditions are given by

\[
\frac{H_{C^h_t}(C^h_t, C^f_t)}{H_{C^f_t}(C^h_t, C^f_t)} = \frac{P^h_t}{P^f_t},
\]

\[
\frac{U_C(C_t, L_t) H_{C^h_t}(C^h_t, C^f_t)}{P^h_t} = \beta \frac{1}{Q_{t,t+1}} \frac{U_C(C_{t+1}, L_{t+1}) H_{C^h_t}(C^h_t, C^f_t)}{P^h_{t+1}},
\]

plus an optimal wage decision that will be discussed later.

**Government**

The government sets monetary and fiscal policy and raises taxes to pay for exogenous consumption of the home final good, \( G^h_t \).\(^3\) Monetary policy consists of rules for either the nominal interest rate \( R_t \) or the nominal exchange rate \( S_t \). Fiscal policy consists of labor taxes \( \tau^n_t \); payroll taxes \( \tau^n_t \), export and import taxes on foreign goods, \( \tau^h_t \) and \( \tau^f_t \), respectively; taxes on returns of foreign assets \( \tau^*_t \); and dividend taxes \( \tau^d_t \).

The two sources of pure rents in the model are the dividends of intermediate good firms and the profits of commodity producers – equivalently, one can think of the latter as a tax on the rents associated with a fixed factor of production. Throughout the paper, we assume that all

\(^3\)It is straightforward to also let the government consume foreign goods.
rents are fully taxed so that \( \tau^d_t = 1 \) for all \( t \). The reason for this assumption is that if pure rents are not fully taxed, the Ramsey government will use other instruments to partially tax those rents. We deliberately abstract from those effects in the optimal policy problem.

Our description of fiscal policy is for completeness. As is well known,\(^4\) when fiscal policy can respond to shocks and there is a complete set of instruments, price stability is optimal. The taxes described in this section do represent a complete set of instruments. The optimal monetary policy becomes nontrivial once fiscal instruments are exogenously restricted to be unresponsive to shocks.

**Final good firms**

Perfectly competitive firms produce the domestic final good \( Y^h_t \) by combining a continuum of nontradable intermediate goods indexed by \( i \in (0, 1) \) using the technology

\[
Y^h_t = \left[ \int_0^1 y^\theta_{it} \, di \right]^{\frac{\theta}{\theta - 1}},
\]

where \( \theta > 1 \) is the elasticity of substitution between each pair of intermediate goods. Taking as given the final good price, \( P^h_t \), and the prices of each individual variety of intermediate goods, \( P^h_{it} \) for \( i \in (0, 1) \), the firm’s problem implies the cost minimization condition

\[
y^*_{it} = Y^h_t \left( \frac{P^h_{it}}{P^h_t} \right)^{-\theta}
\]

for all \( i \in (0, 1) \). Integrating this condition over all varieties and using the production function gives a price index relating the final good price and the prices of the individual varieties,

\[
P^h_t = \left( \int_0^1 P^{h_1-\theta}_{it} \, di \right)^{\frac{1}{1-\theta}}.
\]

Minimization of labor costs

Before describing the technologies of the sectors that demand labor, we believe it is useful to describe the labor cost minimization problem. Firms minimize ∫₀¹ wₙhdh, where wₙₜ is the wage of the h-labor, for a given aggregate Nₜ, subject to (3). The demand for nₙₜ is

\[ n_{ht} = \left( \frac{w_{ht}}{W_t} \right)^{-\theta_w} N_t, \tag{11} \]

where \( W_t \) is the aggregate wage level, given by

\[ W_t = \left[ \int_0^1 w_{ht}^{1-\theta_w} dh \right]^{\frac{1}{1-\theta_w}}. \tag{12} \]

It follows that \( \int_0^1 w_{ht} n_{ht} dh = W_t N_t \).

The optimal wage-setting conditions by the monopolistic competitive workers are now

\[ w_t = \frac{\theta_w}{\theta_w - 1} E_t \sum_{j=0}^\infty \eta_{t,j} \frac{U_{t+c_j+1} (t+c_j+1)(w_t)}{U_C (t+c_j) (1 + \tau_{t+j}+1)} (1 - \tau_{t+j}+1) N_{t+j}, \tag{13} \]

with

\[ \eta_{t,j} = \frac{(1 - \tau_{t+j}+1) (\alpha^w \beta)^j \frac{U_C (t+c_j+1)(w_t)}{U_C (t+c_j) (1 + \tau_{t+j}+1)} (w_t)^{\theta_w} N_{t+j}}{E_t \sum_{j=0}^\infty (1 - \tau_{t+j}+1) (\alpha^w \beta)^j \frac{U_C (t+c_j+1)(w_t)}{U_C (t+c_j) (1 + \tau_{t+j}+1)} (w_t)^{\theta_w} N_{t+j}}. \tag{14} \]

The wage level (12) can be written as

\[ W_t = \left[ (1 - \alpha^w) w_t^{1-\theta_w} + \alpha^w W_t^{1-\theta_w} \right]^{-\frac{1}{1-\theta_w}}. \tag{15} \]

Using (11), we can write (2) as

\[ N_t = \left[ \int_0^1 \left( \frac{w_{ht}}{W_t} \right)^{-\theta_w} dh \right]^{-1} (\bar{L} - L_t). \tag{16} \]
From (12), it must be that \( \int_0^1 \left( \frac{w_{ht}}{W_t} \right)^{-\theta} dh \geq 1 \). This means that for a given total time dedicated to work, \( \bar{L} - L_t \), the resources available for production are maximized when there is no wage dispersion.

In equilibrium

\[ \bar{L} - L_t = N_t \sum_{j=0}^{t+1} \bar{\omega}_j^w \left( \frac{w_{t-j}}{W_t} \right)^{-\theta^w} , \]

where \( \bar{\omega}_j^w \) is the share of household members that have set wages \( j \) periods before, \( \bar{\omega}_j^w = (\alpha^w)^j(1 - \alpha^w), j = 0, 2, ..., t \), and \( \bar{\omega}_{t+1}^w = (\alpha^w)^{t+1} \), which is the share of workers that have never set wages and charge the exogenous wage \( w_{-1} \).

**Primary commodity sector**

Two tradable commodities, denoted by \( x \) and \( z \), are used as inputs in the production of intermediate goods. The home economy, however, is able to produce only the commodity \( x \); the commodity \( z \) must be imported. We denote by \( P^x_t \) and \( P^z_t \) the local currency prices of the commodities.

Total output of commodity \( x \), denoted as \( X_t \), is produced according to the technology

\[ X_t = A_t \left( n^x_t \right)^\rho , \quad (17) \]

where \( n^x_t \) is labor, \( A_t \) is the level of productivity, and \( 0 < \rho \leq 1 \). Implicit in this technology is the assumption of a fixed factor of production (when \( \rho < 1 \)), which we broadly interpret as land.

Profit maximization implies

\[ \rho P^x_t A_t \left( n^x_t \right)^{\rho - 1} = W_t(1 + \tau^p_t) . \quad (18) \]
Because the two commodities can be freely traded, the law of one price holds:

\[ P_t^x = S_t P_t^{x*} \]
\[ P_t^z = S_t P_t^{z*} , \]

where \( P_t^{x*} \) and \( P_t^{z*} \) denote the foreign currency prices of the \( x \) and \( z \) commodities.\(^5\)

We can use (19) and (18) to obtain

\[ \rho S_t P_t^{x*} A_t (n_t^x)^{\rho-1} = W_t (1 + \tau_t^p) , \]

which, given values for the exogenous shocks and given an allocation, restricts the feasible values for \( \{S_t, W_t, \tau_t^p\} \).

**Intermediate good firms**

Each intermediate good \( i \in (0, 1) \) is produced by a monopolistic competitive firm that uses labor and the two tradable commodities with the technology

\[ y_{it} = \eta Z_t x_{it}^{\eta_1} z_{it}^{\eta_2} (n_{it}^y)^{\eta_3} , \]

where \( x_{it} \) and \( z_{it} \) are the demand for commodities, \( n_{it}^y \) is labor, \( Z_t \) denotes the level of productivity, \( \eta_j \geq 0 \) for \( j = 1, 2, 3 \), \( \sum_{j=1}^{3} \eta_j = 1 \), and \( \eta = \eta_1^{-\eta_1} \eta_2^{-\eta_2} \eta_3^{-\eta_3} \).

The associated nominal marginal cost function is common across intermediate good firms and given by

\[ MC_t = \frac{(P_t^x)^{\eta_1} (P_t^z)^{\eta_2} W_t^{\eta_3} (1 + \tau_t^p)^{\eta_3}}{Z_t} . \]

Using (18) and (19), the nominal marginal cost can be written as \( MC_t = S_t MC_t^* \), where \( MC_t^* \).

\(^5\)We could also allow for tariffs on the intermediate inputs. However, these tariffs are redundant instruments in this environment.
the marginal cost measured in foreign currency, is given by

\[ MC_t^* = \frac{(P_t^x)^{1-n_2} (P_t^z)^{n_2} (\rho A_t (n^x_t)^{\rho-1} (1 + \tau^p_t))^{n_3}}{Z_t}. \]  \hspace{1cm} (20)

That is, the marginal cost in foreign currency depends on the international commodity prices, on technological factors, and on the equilibrium allocation of labor in the commodities sector.

In addition, cost minimization implies that final intermediate good firms choose the same ratio of inputs,

\[
\frac{x_{it}}{n^y_{it}} = \frac{\eta_1}{\eta_3} \rho A_t (n^x_t)^{\rho-1} (1 + \tau^p_t) 
\]

\[
\frac{z_{it}}{n^y_{it}} = \frac{\eta_2}{\eta_3} \frac{P_t^x}{P_t^z} \rho A_t (n^x_t)^{\rho-1} (1 + \tau^p_t) \quad \text{for all } i \in (0,1), \]  \hspace{1cm} (21)

where we have used (18) in the second equation.

Introducing (21) into the production function gives

\[ y_{it} = n^y_{it} Z_t (\rho A_t (n^x_t)^{\rho-1} (1 + \tau^p_t))^{1-n_3} (P_t^x)^{n_2} (P_t^z)^{-n_2}. \]  \hspace{1cm} (22)

Each monopolist \( i \in (0,1) \) faces the downward-sloping demand curve (9). We follow the standard tradition in the New Keynesian literature and impose Calvo price rigidity. Namely, in each period, intermediate good firms are able to reoptimize nominal prices with a constant probability \( 0 < \alpha^p < 1 \). Those that get the chance to set a new price will set it according to

\[ p_{it}^h = \frac{\theta}{\theta - 1} \frac{E_t \sum_{j=0}^{\infty} \chi_{t,j} (P^x_{t+j})^{n_1} (P^z_{t+j})^{n_2} [W_{t+j}(1 + \tau_{t+j}^p)]^{n_3}}{Z_{t+j}}, \]  \hspace{1cm} (23)

where

\[ \chi_{t,j} = \frac{\alpha^p q_{t,j}^h (P_{t+j}^h)^{\theta} Y_{t+j}^h}{E_t \sum_{j=0}^{\infty} \alpha^j q_{t,j} (P_{t+j}^h)^{\theta} Y_{t+j}^h}. \]  \hspace{1cm} (24)
The price level in (10) can be written as

\[ P_t^h = \left( 1 - \alpha^P \right) \left( P_t^h \right)^{1-\theta} + \alpha^P \left( P_{t-1}^h \right)^{1-\theta} \frac{1}{1-\theta}. \] (25)

**Foreign sector and feasibility**

We assume an isoelastic foreign demand for the home final good of the form

\[ C_t^{hs} = (K_t^*)^\mu \left( P_t^{hs} \right)^{-\mu}, \] (26)

where \( \gamma > 1 \), \( P_t^{hs} \) is the foreign currency price of the home final good, and \( K_t^* \) is a stochastic process that transforms units of foreign currency into domestic consumption goods.\(^6\)

The government imposes a tax \( (1 + \tau_t^h) \) on final goods exported to the rest of the world and a tariff \( (1 + \tau_t^f) \) to final good imports. The law of one price on domestic and foreign final goods then requires

\[ P_t^h (1 + \tau_t^h) = S_t P_t^{hs} \] (27)
\[ P_t^f = S_t P_t^{fs} (1 + \tau_t^f), \]

where \( P_t^{fs} \) is the foreign currency price of the foreign final good.

Net exports measured in foreign currency are given by

\[ m_t^* = P_t^{hs} C_t^{hs} - P_t^{fs} C_t^f + P_t^{zx} \left[ X_t - \int_0^1 x_{it} di \right] - P_t^{zs} \int_0^1 z_{it} di. \] (28)

\(^6\)We allow for the final goods to be traded, so a particular case of our model (the one with \( A = 0 \) and \( \eta_1 = \eta_2 = 0 \)) without commodities is the one typically analyzed in the small open economy New Keynesian literature.
Thus, the net foreign assets of the country, denoted by $B_{t,t+1}^*$, evolve according to

$$B_{t-1,t}^* + m_t^* = E_t B_{t,t+1}^* Q_{t,t+1}^*.$$ \hfill (29)

Solving this equation from period 0 forward, and assuming zero initial foreign assets, gives the economy foreign sector feasibility constraint measured in foreign currency at time 0:

$$E_0 \sum_{t=0}^{\infty} Q_{0,t}^* m_t^* = 0.$$ \hfill (30)

In addition, market clearing in domestic final goods requires

$$Y_t^h = C_t^h + C_t^{hs} + G_t^h,$$ \hfill (31)

and labor market feasibility is given by

$$N_t = \int_0^1 n_{yt}^z di + n_t^x.$$ \hfill (32)

**Fiscal and monetary policies**

We now show how a flexible exchange rate system, coupled with a flexible payroll tax, can jointly stabilize domestic prices and wages. First, using the law of one price for the commodities,

$$P_t^x = S_t P_t^{xx*}$$

$$P_t^z = S_t P_t^{zx*},$$
we can write the cost minimization condition in the commodity sector (18) and the marginal
cost for the intermediate good firm as

$$\rho S_t P_t^x A_t (n_t^x)^{\rho - 1} = W_t (1 + \tau_t^p)$$

$$MC_t = S_t \frac{(P_t^{x*})^{1 - \eta_2} (P_t^{z*})^{\eta_2} (\rho A_t (n_t^x)^{\rho - 1})^{\eta_3}}{Z_t}.$$ 

Because domestic prices are proportional to marginal costs, they will be constant once marginal
costs are constant, which implies

$$MC = S_t \frac{(P_t^{x*})^{1 - \eta_2} (P_t^{z*})^{\eta_2} (\rho A_t (n_t^x)^{\rho - 1})^{\eta_3}}{Z_t},$$

so the nominal exchange rate moves to absorb productivity and commodity price shocks. Note
that the negative correlation between the nominal exchange rate and the prices of the exportable
commodity, presented in Figure 1, follows as a direct result of price stability. We can then use
this implied equilibrium relationship to solve for the nominal exchange rate and use it on the
cost minimization condition of the commodity sector to obtain

$$\rho^{1 - \eta_3} MC \left( \frac{P_t^{x*}}{P_t^{z*}} \right)^{\eta_2} Z_t A_t (n_t^x)^{(\rho - 1)(1 - \eta_3)} = W_t (1 + \tau_t^p).$$

So, to stabilize wages, the payroll tax must move according to

$$(1 + \tau_t^p) = \frac{1}{W} \rho^{1 - \eta_3} MC \left( \frac{P_t^{x*}}{P_t^{z*}} \right)^{\eta_2} Z_t A_t (n_t^x)^{(\rho - 1)(1 - \eta_3)}.$$

Clearly, to the extent that fiscal policy cannot be jointly used with monetary policy, there is
a trade-off between eliminating the distortion in prices and eliminating the distortion in wages.
The numerical analysis of that question is addressed in the next section.
3. CALIBRATION AND NUMERICAL ANALYSIS OF MONETARY POLICY

Before we start, clarifying one issue is important. So far, we have been silent with respect to the implementation of particular equilibria through policy. Since the work of Sargent and Wallace (1975), a vast literature has developed that analyzed the problem of unique implementation using particular policy targets. To briefly summarize that literature, in general, when central banks use money or the interest rate as the policy instrument, typically multiple equilibria are consistent with a single policy rule. On the contrary, if the exchange rate is pegged, uniqueness typically arises. Multiple solutions have been offered. The most popular, in the context of interest rate rules, is to only consider a bounded equilibrium and to assume rules that satisfy the Taylor principle. We fully abstain from the issue of implementation and simply assume that policy can successfully target a nominal variable (or a combination of two of them), such as prices of domestic goods, \( P_h \), the nominal wage, \( W_t \), or the nominal exchange rate, \( S_t \).

We consider the following utility function:

\[
U (C, L) = \frac{C^{1-\gamma}}{1-\gamma} - \zeta \frac{(\bar{L} - L)^{1+\psi}}{1+\psi},
\]

where \( \gamma, \zeta, \) and \( \psi \) are positive parameters. The subutility function between domestic and foreign final goods is of the constant elasticity of substitution form,

\[
C = H (C^h, C^f) = \left[ (1 - \varpi)^{1/\phi} (C^h)^{\frac{\phi-1}{\phi}} + \varpi^{1/\phi} (C^f)^{\frac{\phi-1}{\phi}} \right],
\]

where \( \phi \) is the elasticity of substitution between home and foreign goods, and \( \varpi \) is the share parameter associated with the foreign good. As is common in the literature, \( \varpi \) can be interpreted as the degree of openness of the economy.

Each time period in the model represents one quarter. Most of the parameters that we use for calibrating the model are standard and reported in Table 1 in the appendix. We choose
so that the discount factor is 0.95 on an annualized basis and set a standard risk aversion parameter of \( \gamma = 2 \). The parameter \( \psi \) is the reciprocal of the Frisch elasticity of labor supply. We set \( \psi = 1 \), which lies between the micro and macro estimates of this elasticity (Chetty et al., 2011). Furthermore, this number is standard in the literature (see, for example, Catao and Chang, 2013). The parameters \( \zeta \) and \( \bar{L} \) define units of measurement and are not important for the quantitative results of the paper; we set \( \zeta = 1 \) and choose \( \bar{L} \) so that in the steady state, workers allocate one-third of their total available time to market activities.

The parameter \( \phi \) measures the Armington elasticity of substitution between home and foreign final goods. Estimates of the Armington elasticity using microeconomic data tend to be much higher than those based on macroeconomic data. We set \( \phi = 1.5 \), which is a common number used in the international business cycles literature (Backus, Kehoe, and Kydland, 1994). This value is also consistent with the macro estimates of the Armington elasticity reported in Feenstra et al. (2014). We set the share parameter at \( \varpi = 0.2 \). This value is consistent with the observed home bias in consumption (Obstfeld and Rogoff, 2001) and is similar to that used in Catao and Chang (2013).\(^7\)

The production function of the home intermediates is characterized by the three share parameters, \( \eta_1, \eta_2, \eta_3 \), and by the level of productivity \( Z_t \). We set the share parameters at \( \eta_1 = 0.1, \eta_2 = 0.4, \) and \( \eta_3 = 0.5 \). A labor share of about 50\% is a standard parameterization. We set \( \eta_1 = 0.1 \) to capture the observation that the home commodity is not used intensively in the production of home goods. The share of imported intermediate inputs \( \eta_2 = 0.4 \) is not intended to capture the import of a single commodity, such as oil in the case of Chile, but of a large array of intermediate inputs and commodities used in the production of goods in the small open economy. We normalize the long-run level of productivity to \( \bar{Z} = 1 \).

Regarding the technology to produce the home commodity, we set a small labor share of

\(^7\)Gali and Monacelli (2005) and de Paoli (2009) use \( \varpi = 0.4 \). Quantitative results are similar if we set \( \varpi \) to 0.4 instead of 0.2.
\(\rho = 0.1\), to capture that the production of commodities is either land or capital intensive, and set the steady-state level of technology, \(\bar{A}\), at \(0.2\). With this calibration, the steady-state share of labor in the commodities sector is about \(0.15\). This is the target number used in Hevia, Neumeyer, and Nicolini (2013) using a broad definition of the commodity sector and an input-output matrix for Chile (see the discussion in that paper for more details).

For the parameterization of the foreign demand of the home final good in equation (26), we assume an elasticity of \(\mu = 1.5\) and set \(K^*\) to a constant value of \(0.1\). The foreign demand does not play an important role in the simulations that we discuss later and, thus, these parameters are almost irrelevant.

The parameters \(\alpha^p\) and \(\alpha^w\) determine the average number of periods between price and wage adjustments. We follow Christiano, Eichenbaum, and Rebelo (2011) and set \(\alpha^w = 0.85\). The parameter \(\alpha^p\) is set at 0.5, which implies an expected price duration of two quarters. This is consistent with the evidence in Klenow and Malin (2010).\(^8\) Finally, as is common in the literature, we consider an efficient steady state. This amounts to imposing a constant labor subsidy that eliminates the monopolistic distortions, and a constant tariff that extracts the monopolistic rents in the trade of the home final good. This follows because the small open economy faces a downward-sloping foreign demand for the final good.

We now consider the calibration of the stochastic processes for the different shocks. We assume that both productivity parameters, \(A_t\) and \(Z_t\), follow autoregressive processes of the form

\[
\log \left( \frac{A_t}{\bar{A}} \right) = \rho_A \log \left( \frac{A_{t-1}}{\bar{A}} \right) + \varepsilon_{At} \\
\log \left( \frac{Z_t}{\bar{Z}} \right) = \rho_Z \log \left( \frac{Z_{t-1}}{\bar{Z}} \right) + \varepsilon_{Zt},
\]

where \(\varepsilon_{At}\) and \(\varepsilon_{Zt}\) are independent mean zero shocks with a standard deviation of \(\sigma_A\) and \(\sigma_Z\).

---

\(^8\)The value used in Christiano, Eichenbaum, and Rebelo (2011) is \(\alpha^P = 0.85\). As we show, the results change very little if we use that value.
respectively. We values of these parameters are set at $\rho_A = \rho_Z = 0.95$ and $\sigma_A = \sigma_Z = 0.013$. These are the standard values used in the literature of business cycles in small open economies (Neumeyer and Perri, 2005).

It remains to calibrate the price processes. Commodity prices tend to be correlated among them. One possibility is to calibrate the price processes by running a vector autoregression (VAR) with the exportable and importable commodity prices. The problem with this approach, however, is that it is not obvious how to identify the importable commodity. Indeed, while exportable commodities are easily identified, importable commodities are not concentrated in a few goods. We thus proceed as follows. We calibrate the price process of the home commodity by running a first-order autoregression using HP-filtered world prices of copper deflated by the U.S. consumer price index over the period 2000–2014:

$$\log \left( \frac{P^{xz}_t}{\bar{P}^{xz}} \right) = \rho_x \log \left( \frac{P^{xz}_{t-1}}{\bar{P}^{xz}} \right) + \varepsilon_{xt},$$

where $\varepsilon_{xt} \sim N(0, \sigma^2_x)$. The estimation delivers $\rho_x = 0.72$ and $\sigma_x = 0.016$. We next impose a VAR structure of the form

$$\begin{bmatrix} \log \left( \frac{P^{xz}_t}{\bar{P}^{xz}} \right) \\ \log \left( \frac{P^{z}_t}{\bar{P}^{z}} \right) \end{bmatrix} = \begin{bmatrix} \rho_x & \zeta \\ \zeta & \rho_z \end{bmatrix} \begin{bmatrix} \log \left( \frac{P^{xz}_{t-1}}{\bar{P}^{xz}} \right) \\ \log \left( \frac{P^{z}_{t-1}}{\bar{P}^{z}} \right) \end{bmatrix} + \begin{bmatrix} \varepsilon_{xt} \\ \varepsilon_{zt} \end{bmatrix},$$

where $\rho_z = \rho_x$, so that the importable commodity is as persistent as the home commodity, but $\sigma_z = \sigma_x/2$, reflecting that shocks to a bundle of commodities will be less volatile than shocks to a single commodity. Finally, we set the free parameter $\zeta$ so that the model is able to replicate the correlation between the home commodity price and the nominal exchange observed in Chile over the sample period discussed earlier. Setting $\zeta = 0.18$ implies a correlation between the nominal exchange rate and the commodity price of $-0.63$ under a policy of price stability. As a reference, if we set $\zeta = 0$, the latter correlation drops to $-0.20$.  

20
In what follows, and given the policy rule we now discuss, we simulate the model by shutting down all shocks except the commodity price \( P_t^x \). To approximate the solution of the model, we use the quadratic perturbation method around the steady state developed by Schmitt-Grohé and Uribe (2004).

**The policy rule**

In order to capture our interpretation of the recent Chilean experience, one could consider a regime in which price stability is the main stated objective, but with some interventions to reduce the volatility of the nominal exchange rate (similar to the interventions in 2008 and 2011).

Note that from the solution for the marginal cost, we can write

\[ MC_t = S_t MC_t^*, \]

where

\[ MC_t^* = \frac{(P_t^x)^{1-\eta_2} (P_t^z)^{\eta_2} (\rho A_t (n_t^x)^{\rho-1})^{\eta_3}}{Z_t}. \]

Clearly, \( MC_t^* \) – the marginal cost in foreign currency – is a function of the underlying shocks. As we mentioned earlier, full price stability implies constant marginal costs in local currency, so

\[ S_t = \frac{MC}{MC_t^*}. \]

We then allow for a general rule in which the deviations of the log of the nominal exchange rate adjust a fraction of the deviations in the log of the marginal costs in foreign currency, or

\[ d\ln S_t = -\nu d\ln MC_t^*. \]  \hspace{1cm} (33)

Thus, when \( \nu = 1 \), we have pure inflation targeting, when \( \nu = 0 \) we have a currency peg, and
by letting $\nu \in (0,1)$ we can have all intermediate cases: the lower the value for $\nu$, the lower the volatility of the nominal exchange rate and the larger the volatility of domestic inflation. With the policy rule so specified, the model can be solved numerically.

The policy trade-off implied by the previous rule may reflect the one implied by dirty floating regimes, in which some intervention in foreign exchange markets is allowed. According to the theory, dampening the movements in the nominal exchange rate implies increasing the volatility of marginal costs and therefore the volatility of the price level. However, what seems a more natural trade-off, given the nature of the two distortions, is stabilizing prices versus stabilizing nominal wages. In the theory section, we showed how a payroll tax can be used together with the nominal exchange rate to stabilize both prices and wages. Once the payroll tax cannot be used, the nominal exchange rate can be used to stabilize either of them but not both. Thus, let

$$w_t^h \equiv \frac{W_t}{P_t^h}.$$  

Then, we can define a policy where

$$d \ln W_t = \nu d \ln w_t^h.$$  

Thus, if $\nu = 0$, nominal wages are fully stabilized, whereas $\nu = 1$ implies full price stability. The optimal policy is given by the value of $\nu$ that maximizes welfare, given the process for the exogenous shock—the price of the exportable commodity in this case. The earlier discussion suggests that the optimal policy will indeed have $\nu \in (0,1)$. This conjecture will be verified numerically later. In the appendix we discuss how we perform the welfare comparisons across the different rules.

In view of the preceding discussion, we first consider the case in which the policy trades off price stability with nominal wage stability, which is the trade-off that will deliver the optimal
policy. Then, we discuss how the rule that trades off price stability with nominal exchange rate stability behaves, particularly compared with the optimal policy. We believe exploring this a priori suboptimal rule is interesting for two reasons. First, it is the one that best approximates, in our view, the dirty floating policy debate, as our discussion of the recent Chilean experience suggests. Second, while stabilizing wages in this simple model is trivial, it is much less so in a real economy with so many sectors and so many different types of labor. It does seem to us that focusing the policy debate on a single, extremely visible price is much more attractive.

The price/wage trade-off

We will first discuss the results using the general policy rule (34). To begin, we present simulations for the model with the baseline calibration, except that we set the rigidity in wages to be zero. The advantage of this case is that when prices are fully stabilized ($\nu = 1$), we obtain the optimal allocation, which we use as a benchmark. In Figure 2 we show the impulse responses for output, the real wage, the real exchange rate, and labor following a one-standard-deviation positive shock to the price of the exportable, for several values of $\nu$. Throughout the paper, output (GDP) is computed as the sum of the value added evaluated at the steady-state prices.

As expected, at the efficient allocation ($\nu = 1$), there is a redistribution of labor toward the exportable sector (labor increases by 14% in the commodity sector and drops by 0.8% in the home good sector). Consumption of the home good becomes very expensive, so it goes down, increasing total labor supply. This lowers the real wage and firms hire more labor overall, so GDP goes up by almost 0.35%. Since prices are stable, as the nominal exchange rate goes down, so does the real exchange rate. When nominal wages are stabilized ($\nu = 0$), the same equilibria would obtain by increases in the price level, if prices were fully flexible. But they are not, so the drop in the real wage is lower in this case. Because the price of final goods does not go up as much, demand for the final consumption good is relatively higher, so the drop in labor at the
Fig. 2. Economy with no wage rigidities

home good sector is smaller and the increase in GDP is higher (although the effect is small). Because the price level does not increase enough, the real exchange rate does not drop very much.

An interesting feature that arises from Figure 2 is that the effect of the policy regime (from a fixed exchange rate to a fully floating one in which prices are stabilized) does not have a very big impact on the transmission mechanism of a commodity price shock, even with a relatively high value for the Calvo parameter ($\alpha^p = 0.5$). The larger differences are in the movements of the real wage and the real exchange rate, but not on the real allocation, which is what matters for welfare.

The effect of the policy regime is much more dramatic for the benchmark calibration (with $\alpha^w = 0.85$), in which the optimal allocation cannot be implemented because of the presence of both price and wage rigidity. We show the relevant impulse responses in Figure 3. When policy fully stabilizes nominal wages, the behavior of labor and output is relatively similar to
the efficient allocation: total output goes up by about 0.35%, total labor goes up by about 1.7%, and the labor reallocation is very similar. Note, however, that fully stabilizing nominal prices delivers a very different outcome: GDP *falls* by 0.3% and total labor by 0.2%.

![Graph](image)

**Fig. 3.** Baseline economy

We also solved the model by setting the share of foreign goods to 0.01 and 0.4 (the benchmark is 0.2) and the degree of price stickiness to 0.25 and 0.85 (the benchmark is 0.5). The results, presented in the appendix, are roughly similar.

Finally, we switched the degree of rigidity between prices and wages, relative to the benchmark. That is, we increased the degree of price rigidity to $\alpha^p = 0.85$ and reduced the degree of wage rigidity to $\alpha^w = 0.5$. Results are depicted in Figure 4. Now, the choice of the policy regime is much less relevant than in the benchmark case. As can be seen, full price stability delivers outcomes that are very similar to the optimal allocation: an increase in output a bit below 0.35%, an increase in total labor close to 1.6%, and a very similar labor reallocation. This is natural,
Fig. 4. Economy with higher price and lower wage rigidity
given that it is in the setting of prices where we have the largest friction. However, notice that
the effects of a regime that fully stabilizes nominal wages do not affect the allocation very much:
it generates an inefficiently larger expansion, but it is small nonetheless (0.4% instead of 0.35%).
For a better visual comparison, in Figure 5 we plot the impulse responses of output, the real
wage, and labor in the final good sector for the benchmark case ($\alpha^p = 0.5$ and $\alpha^w = 0.85$) and
for this last case analyzed, in which the degrees of rigidity between prices and wages have been
switched ($\alpha^p = 0.85$ and $\alpha^w = 0.5$) using the same scales. The difference is remarkable.

The welfare analysis is in line with the previous discussion. We show in Figure 6 the welfare
gain, in units of lifetime consumption, of alternative values of $\nu \in [0, 1]$, relative to the regime
$\nu = 0$, which is equivalent to full wage stability. Naturally, for the case in which $\alpha^w = 0$, full price
stability is optimal, which is reflected in the fact that the line with circles is always increasing.
Note, however, that the opposite policy, the one that fully stabilizes wages ($\nu = 0$), entails a cost
of only 0.1% of lifetime consumption. On the other hand, for our baseline parameterization, the optimal policy is slightly below \( \nu = 0.1 \), which amounts to almost full nominal wage stability. Notice that in this case, which exhibits a high degree of wage rigidity, the welfare cost of full price stability is over 0.45% of lifetime consumption, almost five times more. This is in line with our previous discussion: when there is a high degree of wage rigidity and some degree of price rigidity, the choice of the policy regime becomes more relevant. Notice that lowering the degree of price rigidity to \( \alpha^p = 0.25 \) makes the optimal policy be even closer to full nominal wage stability. Still, the effect of the policy regime (the value for \( \nu \)) is very relevant. Finally, when the wage rigidity is lowered to \( \alpha^w = 0.5 \), the optimal regime becomes close to \( \nu = 0.5 \), and the effect of full price stability becomes lower than 0.1% of lifetime consumption.

Overall, our results imply that the policy regime is much more relevant when there is a substantial degree of wage rigidity, coupled with some rigidity in the setting of prices. On the other hand, if there is a high degree of price stickiness, coupled with some degree of wage rigidity, the
choice of the policy regime is relatively less important. The reason lies behind the logic of the mechanism at the optimal allocation discussed earlier: the drop in the real wage, which increases total labor and generates an expansion. When prices are stabilized, the nominal wage must fall. If nominal wages are very rigid, real wages do not fall and firms do not hire much labor, which creates the usual recession observed in models with wage rigidity. On the other hand, when prices are very rigid but wages are not, if wages are stabilized, the adjustment must be realized by an increase in prices. As before, if prices are very rigid, they do not increase and the real wage does not fall, creating a recession as before. But contrary to the previous case, since prices do not increase, consumption is relatively cheaper, and demand goes up. This demand effect, also common in models with price rigidity, partially compensates for the lack of adjustment in the real wage. Thus, the more rigid the prices, the lower the adjustment in the real wage, but the larger the demand effect. Therefore, the price rigidity is less relevant than the wage rigidity.

Fig. 6. Welfare comparisons of different wage rules
The price/exchange rate trade-off

Admittedly, the notion of nominal wage stability is much simpler in the model than in actual economies. Thus, we now consider a restricted optimal policy problem in which we choose the best value for \( \nu \) but use the rule (33) that trades off price versus nominal exchange rate stability. Figure 7 shows impulse responses following a one-standard-deviation increase in the price of the exportable commodity for the baseline calibration. In Figure 8, we compare the impulse responses of the benchmark case for output, the real wage, and labor in the final good sector with the case in which we reverse the degree of rigidity \((\alpha^p = 0.85 \ and \ \alpha^w = 0.5)\). As before, the policy regime matters more when wages are more rigid than prices.

In Figure 9, we show the welfare effect, in units of lifetime consumption, of alternative values of \( \nu \ in [0, 1] \), relative to the regime \( \nu = 0 \), which is equivalent to full exchange rate stability. Notice that for the baseline calibration, the optimal value for \( \nu \) is close to 0.5, which means a substantial degree of dirty floating. Figure 9 reveals three interesting features. The first is
that, as before, the welfare cost of implementing the wrong regime is higher when the friction is concentrated in wages rather than prices (although the difference is not as big as before).

The second is that the best policy for the baseline calibration is about 0.45\% percent of lifetime consumption, relative to price stability. This is very interesting, since it is very similar to the welfare gain of using the optimal policy, as described in the previous subsection, again relative to full price stability. This means that welfare at the best dirty floating regime is very close to welfare at the optimal policy. To the extent that a policy aimed at stabilizing nominal wages is hard to implement in practice, this result suggests that the best dirty floating regime may be almost as good in terms of implementing good allocations.

The third feature is unrelated to the discussion so far but is still very interesting. A recent paper (Schmitt-Grohé and Uribe, 2012) argues that the cost of a fixed exchange rate regime can be very high relative to a full inflation-targeting one. Our results provide an unintended example in which the result is exactly the opposite: when the wage rigidity is larger than the price rigidity, a policy that fully stabilizes prices is worse than one that fixes the nominal exchange rate. In
our case, the difference can be up to 0.4% of lifetime consumption. Exploring the robustness of this result and using the different experiences of Chile (which targets low inflation) and Ecuador (which dollarizes) in the last 15 years is left for further research.

CONCLUSIONS

From a theoretical viewpoint, the presence of price and wage rigidity implies that full inflation targeting is not the optimal policy. In commodity export countries, which are subject to very large changes in commodity prices that generate very large swings in the real exchange rate, this could be a serious concern. Thus, the question of real exchange rate stabilization has become a central issue in policy debates.

In this paper, we studied a small open economy model that is able to reproduce the large swings in nominal and real exchange rates and which exhibits price and wage frictions. We first showed that if fiscal policy instruments (payroll taxes, for instance) can be made as flexible as
monetary policy, then price stability is the optimal policy. But if fiscal instruments cannot, a trade-off between stabilizing domestic prices or nominal wages is involved. We showed that this trade-off is particularly important for policy design when there is a high degree of nominal wages (Calvo parameter higher than 0.8) and some degree of price rigidity (Calvo parameter higher than 0.25). In this case, the wrong regime can cost as much as 0.45% of lifetime consumption, relative to the optimal rule. On the other hand, if the rigidity in prices is the most severe, the wrong regime can cost at most 0.1% of lifetime consumption. In our benchmark calibration, based on models for the United States, wage rigidity is indeed the one that is the most severe. To the extent that this is a reasonable calibration for small open economies, this means that flexible inflation-targeting regimes that let domestic prices move somewhat may be better than pure price stabilization regimes.

Although implementing a rule that trades off price versus wage stability is very simple in the model, given the heterogeneity of wages in actual economies, the discussion in terms of inflation and exchange rate stabilization seems much more useful. Thus, we also considered such a rule and showed that it can approximate the optimal policy remarkably well. Our paper therefore suggests that strong wage rigidity, coupled with some price rigidity, can justify a dirty floating regime, where policy partially stabilizes the nominal (and real) exchange rate.
REFERENCES


Appendix

Welfare comparisons

This appendix elaborates on the welfare comparisons discussed in the text. Suppose that there is a baseline policy, denoted by $b$, associated with an equilibrium allocation of consumption, aggregate labor, and labor distortions $\{C^b_t, N^b_t, \Delta^w_{t+j} \}$. The proposed policy delivers the level welfare $V^b_t$ at time $t$, given the state of the economy, which we denote by $x_t$,

$$V^b_t = E_t \sum_{j=0}^{\infty} U \left( C^b_{t+j}, \Delta^w_{t+j} N^b_{t+j} \right) \equiv V^{C,b}_t - V^{N,b}_t,$$

where

$$V^{C,b}_t = E_t \left[ \sum_{j=0}^{\infty} \beta^j (C^b_{t+j})^{1-\gamma} \right],$$

$$V^{N,b}_t = E_t \left[ \sum_{j=0}^{\infty} \beta^j \left( \frac{\Delta^w_{t+j} N^b_{t+j}}{1 + \psi} \right)^{1+\psi} \right].$$

Now consider an alternative policy, $a$, with associated allocation $\{C^a_t, N^a_t, \Delta^w_{t+j} \}$ and utility level

$$V^a_t = V^{C,a}_t - V^{N,a}_t.$$

Our objective is to measure the welfare gain of policy $a$ relative to policy $b$ in terms of consumption units. For this, we ask by what fraction the consumption path associated with policy $b$ should be increased (or decreased) forever to achieve the same level of utility as under the alternative policy $a$. In particular, we find the value $\lambda_t$ that satisfies

$$V^a_t = E_t \sum_{j=0}^{\infty} U \left( 1 + \lambda_t \right) C^b_{t+j}, \Delta^w_{t+j} N^b_{t+j} \right) = (1 + \lambda_t)^{1-\gamma} V^{C,b}_t - V^{N,b}_t.$$

Solving for $\lambda_t$ gives

$$\lambda_t = \left( \frac{V^{C,a}_t - V^{N,a}_t + V^{N,b}_t}{V^{C,b}_t} \right)^{1/(1-\gamma)} - 1.$$

The recursive structure of the model implies that the values $V^{C,j}_t$ and $V^{N,j}_t$ for $j = a, b$ are
time-invariant functions of the state $x_t$,

$$V_t^{C,j} = V^{C,j}(x_t)$$
$$V_t^{N,j} = V^{N,j}(x_t),$$

which, in turn, implies that the welfare gain is also a time-invariant function of $x_t$, $\lambda_t = \lambda(x_t)$. We thus can write

$$\lambda(x_t) = \left(\frac{V^{C,a}(x_t) - V^{N,a}(x_t) + V^{N,b}(x_t)}{V^{C,b}(x_t)}\right)^{\frac{1}{1-\gamma}} - 1.$$

We report the average value of $\lambda(x_t)$ under the time-invariant distribution of state $x_t$. This average value is obtained by computing a simulation of 1,200 periods in the model starting from the steady-state condition, dropping the first 200 simulated values, and then computing the average along the simulated sample path. For the welfare comparisons, it is crucial to perform an approximation of the policy functions of degree higher than one (second order in our case); otherwise, all policies deliver the same level of utility.
Table and Additional Figures

Table 1. Baseline parameters

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<th>Parameter</th>
<th>Description</th>
<th>Value</th>
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<tr>
<td>$\nu$</td>
<td>Policy parameter</td>
<td>Varies</td>
</tr>
</tbody>
</table>
This figure displays the impulse responses of GDP, aggregate labor, and the real wage under different values of the share of foreign goods in the home composite good $\bar{\omega}$. The figures in the first row represent an economy with a very low share of foreign final goods in consumption ($\bar{\omega} = 0.01$), the figures in the second row are those of the baseline economy ($\bar{\omega} = 0.2$), and the figures in the third row represent an economy with a higher share of foreign final goods in consumption ($\bar{\omega} = 0.4$).
FIG. 11. Wage Rule: Impulse responses to commodity price shock for different values of price stickiness $\alpha^p$

This figure displays the impulse responses of GDP, aggregate labor, and the real wage under different values of the share of price stickiness parameter $\alpha^p$. The figures in the first row represent an economy with a relatively low degree of price stickiness ($\alpha^p = 0.25$), the figures in the second row are those of the baseline economy ($\alpha^p = 0.5$), and the figures in the third row represent an economy with a higher degree of price stickiness ($\alpha^p = 0.85$).