

A Model of Monetary Exchange in Over-the-Counter Markets*

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Abstract

We develop a model of monetary exchange in over-the-counter (OTC) markets and use it to study the effects of monetary policy on asset prices and standard measures of financial liquidity, such as the size of bid-ask spreads, trade volume, and the incentives of dealers to supply immediacy, both by participating in the market-making activity, and by holding asset inventories on their own account.

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*This draft is preliminary and incomplete.

1 Introduction

We develop a model of monetary exchange in financial over-the-counter (OTC) markets and use it to study some elementary questions in financial and monetary economics. Specifically, we consider a setting in which a financial asset that yields a dividend flow of consumption goods (e.g., an equity or a real bond) is traded by investors who have time-varying heterogeneous valuations for the dividend. In order to realize the gains from trade that arise from their heterogeneous private valuations, investors participate in a bilateral market with random search that is intermediated by specialized dealers who have access to a competitive interdealer market. In the bilateral market, which has all the stylized features of a typical OTC market structure, investors and dealers seek to trade the financial asset using fiat money as a medium of exchange. Periodically, dealers and investors are able to rebalance their portfolios in a frictionless (Walrasian) market.

We use the theory to study the role that money and monetary policy play in shaping asset prices and the performance of OTC markets. Since money is used as a payment instrument in financial transactions, the quantity of real balances affects the equilibrium allocation of the asset. Anticipated inflation reduces real balances and distorts the asset allocation by causing too many assets to remain in the hands of investors with relatively low valuations. We find that in a monetary equilibrium, the asset price is larger than the expected present discounted value that any agent assigns to the dividend stream. The difference between the transaction price and the highest individual valuation is a “speculative premium” which investors are willing to pay because they anticipate capital gains from reselling the asset to investors with higher valuations in the future. We show that the speculative premium and the asset price depend on the market structure where the asset trades, e.g., both the premium and the price are increasing in the effective bargaining power of investors in the OTC market, as captured by the product of their trading probability and bargaining power in bilateral transactions with dealers. Monetary policy also affects the speculative motives and the resulting speculative premium. Anticipated inflation reduces the real money balances used to finance asset trading, which limits the ability of high-valuation traders to purchase the asset from low-valuation traders. As a result, the speculative premium and the asset price are decreasing in the rate of (expected) inflation. We also use the model to study the effects of monetary policy on standard measures of financial liquidity of OTC markets, such as the size of bid-ask spreads, the volume of trade, and the

incentives of dealers to supply immediacy, both by choosing to participate in the market-making activity, as well as by holding asset inventories on their own account.

2 The model

Time is represented by a sequence of periods indexed by $t = 0, 1, \dots$. Each time-period is divided into two subperiods where different activities take place. There is a continuum of infinitely lived agents called *investors*, each identified with a point in the set $\mathcal{I} = [0, 1]$. There is also a continuum of infinitely lived agents called *dealers*, each identified with a point in the set $\mathcal{D} = [0, v]$, where $v \in \mathbb{R}_+$. At the beginning of every period there is a continuum of productive units (or *trees*) with measure $A^s \in \mathbb{R}_{++}$. A typical tree, i , yields an exogenous quantity $y_t^i \in \{0, y\}$ of a perishable consumption good at the end of the first subperiod of every period t . We assume $y \in \mathbb{R}_{++}$, and refer to y_t^i as the period- t *dividend* of tree i . The motion of y_t^i follows a Markov chain defined by $\Pr(y_{t+1}^i = y | y_t^i = y) = \pi$ and $\Pr(y_{t+1}^i = y | y_t^i = 0) = 0$, where $\pi \in [0, 1]$, with $y_0^i = y$ for all i . Hence each tree yields the same dividend y unless it “fails” (with probability $1 - \pi$) in which case it becomes permanently unproductive. (We assume that failed trees physically disappear, e.g., they depreciate fully.) The time- t dividend profile, i.e., the list y_t^i for all i , becomes known to all agents at the beginning of period t , and at that time each tree that fails is immediately replaced by a new tree that yields dividend y in the initial period, and follows the same Markov process as other trees thereafter. In the second subperiod of every period, every agent has access to a linear production technology that transforms a unit of the agent’s effort into a unit of another kind of perishable homogeneous consumption good.

Each productive tree has outstanding one durable and perfectly divisible equity share that represents the bearer’s ownership of the tree and confers him the right to collect the dividends. At the beginning of every period $t \geq 1$, each investor receives an endowment of $(1 - \pi) A^s$ equity shares corresponding to the new trees created in that period. When a tree fails, its equity share disappears with the tree. There is a second financial instrument, money, which is intrinsically useless (it is not an argument of any utility or production function, and unlike equity, ownership of money does not constitute a right to collect any resources). The stock of money at time t is denoted A_t^m . The initial stock of money, $A_0^m \in \mathbb{R}_{++}$, is given, and $A_{t+1}^m = \gamma A_t^m$, with $\gamma \in \mathbb{R}_{++}$. A monetary authority injects or withdraws money via lump-sum transfers or taxes to investors in the second subperiod of every period. At the beginning of period $t = 0$, each investor is

endowed with a portfolio of equity shares and money. All financial instruments are perfectly recognizable, cannot be forged, and can be traded among agents in every subperiod.

In the second subperiod of every period, all agents can trade the consumption good produced in that subperiod, equity shares, and money, in a spot Walrasian market. In the first subperiod of every period, trading is organized as follows: Investors and dealers can trade equity shares and money in a random bilateral *OTC market*, while dealers can also trade equity shares and money with other dealers in a spot Walrasian *interdealer market*. We use $\alpha \in [0, 1]$ to denote the probability that an individual investor is able to contact another investor in the OTC market. Once the two investors have contacted each other, the pair negotiates a trade involving equity shares and money. We assume that, with probability $\eta \in [0, 1]$, the terms of the trade are chosen by the investor who values the equity dividend the most, and by the other investor with complementary probability.¹ After the transaction has been completed, the investors part ways. Similarly, we use $\delta \in [0, \min(v, 1 - \alpha)]$ to denote the probability that an individual investor is able to make contact with a dealer in the OTC market. The probability that a dealer contacts an investor is $\delta/v \equiv \kappa \in [0, 1]$. Once a dealer and an investor have contacted each other, the pair negotiates the quantity of equity shares that the dealer will buy from, or sell to the investor in exchange for money. We assume that the terms of the trade between an investor and a dealer in the OTC market are chosen by the investor with probability $\theta \in [0, 1]$, and by the dealer with probability $1 - \theta$. After the transaction has been completed, the dealer and the investor part ways.² The timing assumption is that the round of OTC trade between investors and dealers takes place in the first subperiod of a typical period t , and ends before trees yield dividends. Hence equity is traded *cum dividend* in the OTC market (and in the interdealer market) of the first subperiod, but *ex dividend* in the Walrasian market of the second subperiod. We assume that agents cannot make binding commitments, that there is no enforcement, and that histories of actions are private in a way that precludes any borrowing and lending, so any trade must be *quid pro quo*. This assumption and the structure of preferences described below create the need for a medium of exchange.³

¹In the event that both investors value the dividend the same, each gets selected to make a take-it-or-leave-it offer with equal probability.

²See Zhang (2012) for an OTC model with long-term relationships between investors and dealers.

³Notice that under these conditions there cannot exist a futures market for fruit, so an agent who wishes to consume the fruit dividend must be holding the equity share at the time the dividend is paid. A similar assumption is typically made in search models of financial OTC trade, e.g., see Duffie et al. (2005) and Lagos and Rocheteau (2009).

All agents discount payoffs across periods with the same factor, $\beta \in (0, 1)$. An individual dealer's preferences are given by

$$\mathbb{E}_0^d \sum_{t=0}^{\infty} \beta^t (c_{td} - h_{td})$$

where c_{td} is his consumption of the homogeneous good that is produced, traded and consumed in the second subperiod of period t , and h_{td} is the utility cost from exerting h_{td} units of effort to produce this good. The expectation operator \mathbb{E}_0^d is with respect to the probability measure induced by the dividend process and the random trading process in the OTC market. Dealers get no utility from the dividend good.⁴ An individual investor's preferences are given by

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t (\varepsilon_{ti} y_{ti} + c_{ti} - h_{ti})$$

where y_{ti} is the quantity of the dividend good that investor i consumes at the end of the first subperiod of period t , c_{ti} is his consumption of the homogeneous good that is produced, traded and consumed in the second subperiod of period t , and h_{ti} is the utility cost from exerting h_{ti} units of effort to produce this good. The variable ε_{ti} denotes the realization of a preference shock that is distributed independently over time and across agents, with a differentiable cumulative distribution function G on the support $[\varepsilon_L, \varepsilon_H] \subseteq [0, \infty]$, and $\bar{\varepsilon} = \int \varepsilon dG(\varepsilon) < \infty$. Investor i learns his realization ε_{ti} at the beginning of period t , before the OTC trading round. The expectation operator \mathbb{E}_0 is with respect to the probability measure induced by the dividend process, the investor's preference shock and the random trading process in the OTC market.

3 Efficiency

Consider a social planner who wishes to maximize the sum of all agents' expected discounted utilities, subject to the same meeting frictions that agents face in the decentralized formulation. Specifically, in the first subperiod of every period, the planner can only reallocate assets within the pairs of the measure α of investors who have contacted each other directly, and among all dealers and the measure δ of investors chosen at random from the rest of the population. Let $\mathcal{B}_t \subseteq \mathcal{I}$ denote the subset of investors who get a bilateral trading opportunity with another investor in the OTC market of period t . For any $i \in \mathcal{B}_t$, let $b(i) \in \mathcal{B}_t$ denote investor i 's partner

⁴This assumption implies that dealers have no direct consumption motive for holding the equity share. It is easy to relax, but it is the standard benchmark in the literature, e.g., see Duffie et al. (2005) and Lagos and Rocheteau (2009), Lagos, Rocheteau and Weill (2011), and Weill (2007).

in the bilateral meeting. Notice that $\int_{\mathcal{B}_t} di = \alpha$ is the measure of investors who have an OTC meeting with another investor, and $\int_{\mathcal{B}_t} \mathbb{I}_{\{i \leq b(i)\}} di = \alpha/2$ is the total number of direct bilateral transactions between investors in the OTC market. We restrict attention to symmetric allocations (identical agents receive equal treatment). Let c_{tD} and h_{tD} denote a dealer's consumption and production of the homogeneous consumption good in the second subperiod of period t . Let $c_{tI}(\varepsilon)$ and $h_{tI}(\varepsilon)$ denote consumption and production of the homogeneous consumption good in the second subperiod of period t by an investor with idiosyncratic preference type ε . Let \tilde{a}_{tD} denote the beginning-of-period- t (before depreciation) equity holding of a dealer, and let a'_{tD} denote the equity holding of a dealer at the end of the first subperiod of period t (after OTC trade). Let \tilde{a}_{tI} denote the beginning-of-period- t (before depreciation and endowment) asset holding of an investor. Finally, let $\underline{a}_{tij}(\varepsilon_i, \varepsilon_j)$ denote the post-trade equity holding of an investor i with preference type ε_i who has a direct bilateral trade opportunity with an investor j with preference type ε_j , and let a'_{tI} denote a measure on $\mathcal{F}([\varepsilon_L, \varepsilon_H])$, the Borel σ -field defined on $[\varepsilon_L, \varepsilon_H]$. The measure a'_{tI} is interpreted as the distribution of post-trade asset holdings among investors with different preference types who contacted a dealer in the first subperiod of period t . With this notation, the planner's problem consists of choosing a nonnegative allocation,

$$\left\{ \tilde{a}_{tD}, a'_{tD}, c_{tD}, h_{tD}, \tilde{a}_{tI}, a'_{tI}, \left[\left(\underline{a}_{tib(i)}(\varepsilon_i, \varepsilon_{b(i)}) \right)_{i \in \mathcal{B}_t}, c_{tI}(\varepsilon_i), h_{tI}(\varepsilon_i) \right]_{\varepsilon_i, \varepsilon_{b(i)} \in [\varepsilon_L, \varepsilon_H]} \right\}_{t=0}^{\infty}, \quad (1)$$

to maximize

$$\begin{aligned} & \sum_{t=0}^{\infty} \beta^t \left[\delta \int_{[\varepsilon_L, \varepsilon_H]} \varepsilon y a'_{tI}(d\varepsilon) + \int_{\varepsilon_L}^{\varepsilon_H} [(1 - \alpha - \delta) \varepsilon y a_{tI} + c_{tI}(\varepsilon) - h_{tI}(\varepsilon)] dG(\varepsilon) + v(c_{tD} - h_{tD}) \right. \\ & \left. + \int_{\mathcal{B}_t} \int \int \mathbb{I}_{\{i \leq b(i)\}} \left[\varepsilon_i \underline{a}_{tib(i)}(\varepsilon_i, \varepsilon_{b(i)}) + \varepsilon_{b(i)} \underline{a}_{tb(i)i}(\varepsilon_{b(i)}, \varepsilon_i) \right] y dG(\varepsilon_i) dG(\varepsilon_{b(i)}) di \right] \end{aligned}$$

subject to

$$\underline{a}_{tib(i)}(\varepsilon_i, \varepsilon_{b(i)}) + \underline{a}_{tb(i)i}(\varepsilon_{b(i)}, \varepsilon_i) \leq 2a_{tI} \quad (2)$$

$$v\tilde{a}_{tD} + \tilde{a}_{tI} \leq A^s \quad (3)$$

$$va'_{tD} + \delta \int_{[\varepsilon_L, \varepsilon_H]} a'_{tI}(d\varepsilon) \leq va_{tD} + \delta a_{tI} \quad (4)$$

$$\int_{\varepsilon_L}^{\varepsilon_H} c_{tI}(\varepsilon) dG(\varepsilon) + vc_{tD} \leq \int_{\varepsilon_L}^{\varepsilon_H} h_{tI}(\varepsilon) dG(\varepsilon) + vh_{tD} \quad (5)$$

$$a_{tD} = \pi \tilde{a}_{tD} \quad (6)$$

$$a_{tI} = \pi \tilde{a}_{tI} + (1 - \pi) A^s. \quad (7)$$

Proposition 1 *The efficient allocation satisfies the following three conditions for every t : (a) $\tilde{a}_{tD} = (A^s - \tilde{a}_{tI})/v = A^s/v$, (b) $\underline{a}_{tib(i)}(\varepsilon_i, \varepsilon_{b(i)}) = \mathbb{I}_{\{\varepsilon_{b(i)} < \varepsilon_i\}} 2a_{tI} + \mathbb{I}_{\{\varepsilon_{b(i)} = \varepsilon_i\}} a^o$ for all $i \in \mathcal{B}_t$, with $a^o \in [0, 2a_{tI}]$, and (c) $a'_{ti}(E) = \mathbb{I}_{\{\varepsilon_H \in E\}} [\pi/\delta + (1 - \pi)] A^s$, where $\mathbb{I}_{\{\varepsilon_H \in E\}}$ is an indicator function that takes the value 1 if $\varepsilon_H \in E$, and 0 otherwise, for any $E \in \mathcal{F}([\varepsilon_L, \varepsilon_H])$.*

According to Proposition 1, the efficient allocation is characterized by the following three properties: (a) only dealers carry equity between periods, (b) in bilateral direct trades between investors, all the equity shares are allocated to the highest valuation investor, and (c) among those investors who have a trading opportunity with a dealer, only those with the highest preference type hold equity shares at the end of the first subperiod.

4 Equilibrium

We begin with the formulation of the individual dealer's optimization problem during a typical period. Let $V_t^D(\mathbf{a}_t)$ denote the maximum expected discounted payoff of a dealer who enters the OTC round of period t with portfolio $\mathbf{a}_t \equiv (a_t^m, a_t^s)$. Let $W_t^D(\mathbf{a}_t)$ denote the maximum expected discounted payoff of a dealer who is holding portfolio \mathbf{a}_t at the beginning of the second subperiod of period t . Let ϕ_t^m be the real price of money, and ϕ_t^s be the real *ex dividend* price of equity in the second subperiod of period t (both expressed in terms of the second-subperiod consumption good). Then,

$$\begin{aligned} W_t^D(\mathbf{a}_t) &= \max_{c_t, h_t, \tilde{\mathbf{a}}_{t+1}} [c_t - h_t + \beta V_{t+1}^D(\mathbf{a}_{t+1})] & (8) \\ \text{s.t. } c_t + \phi_t \tilde{\mathbf{a}}_{t+1} &\leq h_t + \phi_t \mathbf{a}_t, \\ c_t, h_t &\in \mathbb{R}_+, \tilde{\mathbf{a}}_{t+1} \in \mathbb{R}_+^2 \\ \mathbf{a}_{t+1} &= (\tilde{a}_{t+1}^m, \pi \tilde{a}_{t+1}^s), \end{aligned}$$

where $\phi_t \equiv (\phi_t^m, \phi_t^s)$, $\tilde{\mathbf{a}}_{t+1} \equiv (\tilde{a}_{t+1}^m, \tilde{a}_{t+1}^s)$, and $\phi_t \mathbf{a}_t$ denotes the dot product of ϕ_t and \mathbf{a}_t .

Let $\hat{W}_t^D(\mathbf{a}_t)$ denote the maximum expected discounted payoff of a dealer with portfolio \mathbf{a}_t in the first subperiod of period t , conditional on not having contacted an investor in the OTC market. Since the unmatched dealer can still access the interdealer market,

$$\begin{aligned} \hat{W}_t^D(\mathbf{a}_t) &= \max_{\hat{a}_t^m, \hat{a}_t^s} W_t^D(\hat{a}_t^m, \hat{a}_t^s) & (9) \\ \text{s.t. } \hat{a}_t^m + p_t \hat{a}_t^s &\leq a_t^m + p_t a_t^s, \\ \hat{a}_t^m, \hat{a}_t^s &\in \mathbb{R}_+, \end{aligned}$$

where p_t is the dollar price of equity in the interdealer market of period t .

Next consider the situation of a dealer who enters the OTC round of trade of period t with portfolio \mathbf{a}_{td} , and contacts an investor with portfolio \mathbf{a}_{ti} and preference type ε in the OTC market. With probability θ the terms of trade are determined by a take-it-or-leave-it offer by the investor, and the resulting post-trade portfolios of the investor and the dealer are denoted

$$\begin{aligned} & [\bar{a}_{i^*}^m(\mathbf{a}_{ti}, \mathbf{a}_{td}, \varepsilon; \boldsymbol{\psi}_t), \bar{a}_{i^*}^s(\mathbf{a}_{ti}, \mathbf{a}_{td}, \varepsilon; \boldsymbol{\psi}_t)] \\ & [\bar{a}_d^m(\mathbf{a}_{ti}, \mathbf{a}_{td}, \varepsilon; \boldsymbol{\psi}_t), \bar{a}_d^s(\mathbf{a}_{ti}, \mathbf{a}_{td}, \varepsilon; \boldsymbol{\psi}_t)], \end{aligned}$$

respectively, where $\boldsymbol{\psi}_t \equiv (1/p_t, \phi_t)$. With probability $1 - \theta$ the terms of trade are determined by a take-it-or-leave-it offer by the dealer, and the resulting post-trade portfolios of the investor and the dealer are

$$\begin{aligned} & [\bar{a}_i^m(\mathbf{a}_{ti}, \mathbf{a}_{td}, \varepsilon; \boldsymbol{\psi}_t), \bar{a}_i^s(\mathbf{a}_{ti}, \mathbf{a}_{td}, \varepsilon; \boldsymbol{\psi}_t)] \\ & [\bar{a}_{d^*}^m(\mathbf{a}_{ti}, \mathbf{a}_{td}, \varepsilon; \boldsymbol{\psi}_t), \bar{a}_{d^*}^s(\mathbf{a}_{ti}, \mathbf{a}_{td}, \varepsilon; \boldsymbol{\psi}_t)], \end{aligned}$$

respectively.⁵ We can now write the value function of a dealer who enters the OTC round of trade of period t with portfolio \mathbf{a}_{td} ,

$$\begin{aligned} V_t^D(\mathbf{a}_{td}) &= \kappa\theta \int \hat{W}_t^D [\bar{a}_d^m(\mathbf{a}_{ti}, \mathbf{a}_{td}, \varepsilon; \boldsymbol{\psi}_t), \bar{a}_d^s(\mathbf{a}_{ti}, \mathbf{a}_{td}, \varepsilon; \boldsymbol{\psi}_t)] dH_t(\mathbf{a}_{ti}, \varepsilon) \\ &+ \kappa(1 - \theta) \int \hat{W}_t^D [\bar{a}_{d^*}^m(\mathbf{a}_{ti}, \mathbf{a}_{td}, \varepsilon; \boldsymbol{\psi}_t), \bar{a}_{d^*}^s(\mathbf{a}_{ti}, \mathbf{a}_{td}, \varepsilon; \boldsymbol{\psi}_t)] dH_t(\mathbf{a}_{ti}, \varepsilon) \\ &+ (1 - \kappa) \hat{W}_t^D(\mathbf{a}_{td}), \end{aligned} \tag{10}$$

where H_t is the joint cumulative distribution function over the preference types and portfolios held by the random investor whom the dealer may contact in the OTC market of period t .

We now analyze the investor's problem in a typical period. Let $V_t^I(\mathbf{a}_{ti}, \varepsilon)$ denote the maximum expected discounted payoff of an investor who has preference type ε and is holding portfolio $\mathbf{a}_{ti} \equiv (a_{ti}^m, a_{ti}^s)$ at the beginning of the OTC round of period t . Let $W_t^I(\mathbf{a}_t)$ denote the maximum expected discounted payoff of an investor who is holding portfolio \mathbf{a}_t at the beginning

⁵In what follows, we will sometimes use $\bar{a}_{i^*}^m$ to denote $\bar{a}_{i^*}^m(\mathbf{a}_{ti}, \mathbf{a}_{td}, \varepsilon; \boldsymbol{\psi}_t)$, $\bar{a}_{i^*}^s$ to denote $\bar{a}_{i^*}^s(\mathbf{a}_{ti}, \mathbf{a}_{td}, \varepsilon; \boldsymbol{\psi}_t)$, etc.

of the second subperiod of period t (after the trees have borne dividends). Then,

$$\begin{aligned}
W_t^I(\mathbf{a}_t) &= \max_{c_t, h_t, \tilde{\mathbf{a}}_{t+1}} \left[c_t - h_t + \beta \int V_{t+1}^I(\mathbf{a}_{t+1}, \varepsilon') dG(\varepsilon') \right] \\
\text{s.t. } c_t + \phi_t \tilde{\mathbf{a}}_{t+1} &\leq h_t + \phi_t \mathbf{a}_t + T_t \\
c_t, h_t &\in \mathbb{R}_+, \tilde{\mathbf{a}}_{t+1} \in \mathbb{R}_+^2 \\
\mathbf{a}_{t+1} &= (\tilde{a}_{t+1}^m, \pi \tilde{a}_{t+1}^s + (1 - \pi) A^s),
\end{aligned} \tag{11}$$

where T_t is the real value of the time- t lump-sum monetary transfer (tax, if negative). Since ε is i.i.d. over time, $W_t^I(\mathbf{a}_t)$ is independent of ε and the portfolio that each investor chooses to carry into period $t + 1$ is independent of ε . Consequently, in what follows we can write $dH_t(\mathbf{a}_{ti}, \varepsilon) = dF_t^I(\mathbf{a}_{ti}) dG(\varepsilon)$, where F_t^I is the joint cumulative distribution function of investors' money and equity holdings at the beginning of the OTC round of period- t .

Consider a bilateral meeting in the OTC trading round of period t , between an investor i with portfolio \mathbf{a}_{ti} and preference type ε_i , and an investor j with portfolio \mathbf{a}_{tj} and preference type ε_j . Let $\tilde{\eta}(\varepsilon_i, \varepsilon_j) \equiv \eta \mathbb{I}_{\{\varepsilon_j < \varepsilon_i\}} + (1 - \eta) \mathbb{I}_{\{\varepsilon_i < \varepsilon_j\}} + (1/2) \mathbb{I}_{\{\varepsilon_i = \varepsilon_j\}}$ denote the probability that the investor with preference type ε_i has the power to make a take-it-or-leave-it offer in a bilateral trade with an investor with preference type ε_j . When investor i makes the take-it-or-leave-it offer to investor j , the resulting post-trade portfolios of investors i and j are denoted

$$\begin{aligned}
[\underline{a}_{j^*}^m(\mathbf{a}_{ti}, \mathbf{a}_{tj}, \varepsilon_i, \varepsilon_j; \boldsymbol{\psi}_t), \underline{a}_{j^*}^s(\mathbf{a}_{ti}, \mathbf{a}_{tj}, \varepsilon_i, \varepsilon_j; \boldsymbol{\psi}_t)] \\
[\underline{a}_j^m(\mathbf{a}_{tj}, \mathbf{a}_{ti}, \varepsilon_j, \varepsilon_i; \boldsymbol{\psi}_t), \underline{a}_j^s(\mathbf{a}_{tj}, \mathbf{a}_{ti}, \varepsilon_j, \varepsilon_i; \boldsymbol{\psi}_t)],
\end{aligned}$$

respectively. With probability $1 - \tilde{\eta}(\varepsilon_i, \varepsilon_j)$ the terms of trade are determined by a take-it-or-leave-it offer by investor j , and the resulting post-trade portfolios of investors i and j are

$$\begin{aligned}
[\underline{a}_i^m(\mathbf{a}_{ti}, \mathbf{a}_{tj}, \varepsilon_i, \varepsilon_j; \boldsymbol{\psi}_t), \underline{a}_i^s(\mathbf{a}_{ti}, \mathbf{a}_{tj}, \varepsilon_i, \varepsilon_j; \boldsymbol{\psi}_t)] \\
[\underline{a}_{j^*}^m(\mathbf{a}_{tj}, \mathbf{a}_{ti}, \varepsilon_j, \varepsilon_i; \boldsymbol{\psi}_t), \underline{a}_{j^*}^s(\mathbf{a}_{tj}, \mathbf{a}_{ti}, \varepsilon_j, \varepsilon_i; \boldsymbol{\psi}_t)],
\end{aligned}$$

respectively.⁶ We can now write the value function of an investor who enters the OTC round

⁶In what follows, we will sometimes use $\underline{a}_{ti^*}^m$ to denote $\underline{a}_{ti}^m(\mathbf{a}_{ti}, \mathbf{a}_{tj}, \varepsilon_i, \varepsilon_j; \boldsymbol{\psi}_t)$, \underline{a}_{tj}^s to denote $\underline{a}_j^s(\mathbf{a}_{tj}, \mathbf{a}_{ti}, \varepsilon_j, \varepsilon_i; \boldsymbol{\psi}_t)$, etc.

of period t with portfolio \mathbf{a}_{it} and preference type ε_i ,

$$\begin{aligned}
V_t^I(\mathbf{a}_{ti}, \varepsilon_i) &= \delta \int \theta \{ \varepsilon_i y \bar{a}_{i*}^s(\mathbf{a}_{ti}, \mathbf{a}_{td}, \varepsilon_i; \boldsymbol{\psi}_t) + \\
&\quad W_t^I [\bar{a}_{i*}^m(\mathbf{a}_{ti}, \mathbf{a}_{td}, \varepsilon_i; \boldsymbol{\psi}_t), \bar{a}_{i*}^s(\mathbf{a}_{ti}, \mathbf{a}_{td}, \varepsilon_i; \boldsymbol{\psi}_t)] \} dF_t^D(\mathbf{a}_{td}) \\
&+ \delta \int (1 - \theta) \{ \varepsilon_i y \bar{a}_i^s(\mathbf{a}_{ti}, \mathbf{a}_{td}, \varepsilon_i; \boldsymbol{\psi}_t) + \\
&\quad W_t^I [\bar{a}_i^m(\mathbf{a}_{ti}, \mathbf{a}_{td}, \varepsilon_i; \boldsymbol{\psi}_t), \bar{a}_i^s(\mathbf{a}_{ti}, \mathbf{a}_{td}, \varepsilon_i; \boldsymbol{\psi}_t)] \} dF_t^D(\mathbf{a}_{td}) \\
&+ \alpha \int \tilde{\eta}(\varepsilon_i, \varepsilon_j) \{ \varepsilon_i y \underline{a}_{i*}^s(\mathbf{a}_{ti}, \mathbf{a}_{tj}, \varepsilon_i, \varepsilon_j; \boldsymbol{\psi}_t) + \\
&\quad W_t^I [\underline{a}_{i*}^m(\mathbf{a}_{ti}, \mathbf{a}_{tj}, \varepsilon_i, \varepsilon_j; \boldsymbol{\psi}_t), \underline{a}_{i*}^s(\mathbf{a}_{ti}, \mathbf{a}_{tj}, \varepsilon_i, \varepsilon_j; \boldsymbol{\psi}_t)] \} dH_t(\mathbf{a}_{tj}, \varepsilon_j) \\
&+ \alpha \int [1 - \tilde{\eta}(\varepsilon_i, \varepsilon_j)] \{ \varepsilon_i y \underline{a}_i^s(\mathbf{a}_{ti}, \mathbf{a}_{tj}, \varepsilon_i, \varepsilon_j; \boldsymbol{\psi}_t) + \\
&\quad W_t^I [\underline{a}_i^m(\mathbf{a}_{ti}, \mathbf{a}_{tj}, \varepsilon_i, \varepsilon_j; \boldsymbol{\psi}_t), \underline{a}_i^s(\mathbf{a}_{ti}, \mathbf{a}_{tj}, \varepsilon_i, \varepsilon_j; \boldsymbol{\psi}_t)] \} dH_t(\mathbf{a}_{tj}, \varepsilon_j) \\
&+ (1 - \alpha - \delta) [\varepsilon_i y a_{ii}^s + W_t^I(\mathbf{a}_{ti})], \tag{12}
\end{aligned}$$

where F_t^D is the cumulative distribution function over portfolios held by the random dealer whom the dealer may contact in the OTC market of period t . Next, we characterize the outcomes of trades in the OTC and the interdealer markets.

The maximization problem (9) represents the portfolio problem of a dealer who did not contact an investor in the OTC market of period t . The solution is summarized as follows:

Lemma 1 *A dealer who enters period t with portfolio \mathbf{a}_t and does not contact an investor, enters the second subperiod of period t with portfolio $(\hat{a}_{td}^m, \hat{a}_{td}^s) \equiv (\hat{a}_d^m(\mathbf{a}_t; \boldsymbol{\psi}_t), \hat{a}_d^s(\mathbf{a}_t; \boldsymbol{\psi}_t))$, given by*

$$\hat{a}_{td}^m = \begin{cases} = 0 & \text{if } p_t \phi_t^m < \phi_t^s \\ \in [0, a_t^m + p_t a_t^s] & \text{if } p_t \phi_t^m = \phi_t^s \\ = a_t^m + p_t a_t^s & \text{if } \phi_t^s < p_t \phi_t^m \end{cases} \quad \text{and} \quad \hat{a}_{td}^s = \begin{cases} a_t^s + \frac{1}{p_t} a_t^m & \text{if } p_t \phi_t^m < \phi_t^s \\ a_t^s + \frac{1}{p_t} (a_t^m - \hat{a}_{td}^m) & \text{if } p_t \phi_t^m = \phi_t^s \\ 0 & \text{if } \phi_t^s < p_t \phi_t^m, \end{cases}$$

and his maximum expected discounted payoff is

$$\hat{W}_t^D(\mathbf{a}_t) = \bar{\phi}_t (a_t^m + p_t a_t^s) + W_t^D(\mathbf{0}) \tag{13}$$

where $\bar{\phi}_t \equiv \max(\phi_t^m, \phi_t^s/p_t)$, and

$$\begin{aligned}
W_t^D(\mathbf{0}) &= \max_{\tilde{\mathbf{a}}_{t+1} \in \mathbb{R}_+^2} [\beta V_{t+1}^D(\mathbf{a}_{t+1}) - \phi_t \tilde{\mathbf{a}}_{t+1}] \\
\text{s.t. } \mathbf{a}_{t+1} &= (\tilde{a}_{t+1}^m, \pi \tilde{a}_{t+1}^s). \tag{14}
\end{aligned}$$

If $p_t \phi_t^m < \phi_t^s$, then a dealer who holds any cash in the interdealer market can use a dollar to buy $1/p_t$ equity shares, and the net return from this trade equals ϕ_t^s/p_t (the real value of the equities in the Walrasian market of the second subperiod of period t) minus ϕ_t^m (the real cost of the trading strategy), which is strictly positive. Hence, it is optimal for the dealer to sell off all his cash for equity if $p_t \phi_t^m < \phi_t^s$. Conversely, if $\phi_t^s < p_t \phi_t^m$, it is optimal for the dealer to sell off any equity holdings he may have and carry only cash into the second subperiod of period t .

Consider the bargaining problem between an investor with preference type ε and portfolio (a_{ti}^m, a_{ti}^s) who contacts a dealer with portfolio (a_{td}^m, a_{td}^s) in the OTC market of period t . With probability θ the investor has the power to make a take-it-or-leave-it offer to the dealer. The investor chooses his offer of post-trade portfolios for himself, $(\bar{a}_{ti}^m, \bar{a}_{ti}^s)$, and for the dealer, $(\bar{a}_{td}^m, \bar{a}_{td}^s)$, by solving

$$\begin{aligned} & \max_{\bar{a}_{ti}^m, \bar{a}_{ti}^s, \bar{a}_{td}^m, \bar{a}_{td}^s} [\varepsilon y \bar{a}_{ti}^s + W_t^I(\bar{a}_{ti}^m, \bar{a}_{ti}^s)] \\ \text{s.t. } & \bar{a}_{ti}^m + \bar{a}_{td}^m + p_t(\bar{a}_{ti}^s + \bar{a}_{td}^s) \leq a_{ti}^m + a_{td}^m + p_t(a_{ti}^s + a_{td}^s) \\ & \hat{W}_t^D(\bar{a}_{td}^m, \bar{a}_{td}^s) \geq \hat{W}_t^D(a_{td}^m, a_{td}^s) \\ & \bar{a}_{ti}^m, \bar{a}_{ti}^s, \bar{a}_{td}^m, \bar{a}_{td}^s \in \mathbb{R}_+. \end{aligned}$$

The first constraint requires that the combined value of the post-trade portfolios held by the investor and the dealer cannot exceed the combined value of their pre-trade portfolios. The second constraint is the dealer's individual rationality constraint.

With complementary probability $1 - \theta$, the dealer has the power to make a take-it-or-leave-it offer to the investor. The dealer chooses his offer of post-trade portfolios for himself, $(\bar{a}_{td}^m, \bar{a}_{td}^s)$, and for the dealer, $(\bar{a}_{ti}^m, \bar{a}_{ti}^s)$, by solving

$$\begin{aligned} & \max_{\bar{a}_{ti}^m, \bar{a}_{ti}^s, \bar{a}_{td}^m, \bar{a}_{td}^s} \hat{W}_t^D(\bar{a}_{td}^m, \bar{a}_{td}^s) \\ \text{s.t. } & \bar{a}_{ti}^m + \bar{a}_{td}^m + p_t(\bar{a}_{ti}^s + \bar{a}_{td}^s) \leq a_{ti}^m + a_{td}^m + p_t(a_{ti}^s + a_{td}^s) \\ & \varepsilon y \bar{a}_{ti}^s + W_t^I(\bar{a}_{ti}^m, \bar{a}_{ti}^s) \geq \varepsilon y a_{ti}^s + W_t^I(a_{ti}^m, a_{ti}^s) \\ & \bar{a}_{ti}^m, \bar{a}_{ti}^s, \bar{a}_{td}^m, \bar{a}_{td}^s \in \mathbb{R}_+. \end{aligned}$$

The first constraint requires that the combined value of the pre-trade portfolios is enough to finance the post-trade portfolios. The second constraint is the investor's individual rationality constraint. The following result summarizes the outcome of the bargaining game between an investor and a dealer.

Lemma 2 Consider the bargaining problem between an investor i with portfolio (a_{ti}^m, a_{ti}^s) and preference type ε , and a dealer d with portfolio (a_{td}^m, a_{td}^s) in the OTC market of period t .

(i) With probability θ the investor chooses the terms of trade, and in this case the investor exits the meeting with post-trade portfolio $(\bar{a}_{ti^*}^m, \bar{a}_{ti^*}^s)$ given by

$$\bar{a}_{ti^*}^m \begin{cases} = 0 & \text{if } \varepsilon_t^* < \varepsilon \\ \in [0, a_{ti}^m + p_t a_{ti}^s] & \text{if } \varepsilon = \varepsilon_t^* \\ = a_{ti}^m + p_t a_{ti}^s & \text{if } \varepsilon < \varepsilon_t^* \end{cases} \quad \text{and} \quad \bar{a}_{ti^*}^s = \begin{cases} a_{ti}^s + \frac{1}{p_t} a_{ti}^m & \text{if } \varepsilon_t^* < \varepsilon \\ a_{ti}^s + \frac{1}{p_t} (a_{ti}^m - \bar{a}_{ti^*}^m) & \text{if } \varepsilon = \varepsilon_t^* \\ 0 & \text{if } \varepsilon < \varepsilon_t^* \end{cases}$$

where

$$\varepsilon_t^* \equiv \frac{p_t \phi_t^m - \phi_t^s}{y}. \quad (15)$$

The dealer's portfolio $(\bar{a}_{td}^m, \bar{a}_{td}^s)$ that results from trading with the investor is given by

$$\bar{a}_{td}^m \in [0, a_{td}^m + p_t a_{td}^s] \quad \text{and} \quad \bar{a}_{td}^s = a_{td}^s + \frac{1}{p_t} (a_{td}^m - \bar{a}_{td}^m).$$

(ii) With probability $1 - \theta$ the dealer chooses the terms of trade, and in this case the investor exits the meeting with post-trade portfolio $(\bar{a}_{ti}^m, \bar{a}_{ti}^s)$ given by

$$\bar{a}_{ti}^m \begin{cases} = 0 & \text{if } \varepsilon_t^* < \varepsilon \\ \in [0, a_{ti}^m + p_t a_{ti}^s] & \text{if } \varepsilon = \varepsilon_t^* \\ = a_{ti}^m + p_t^o(\varepsilon) a_{ti}^s & \text{if } \varepsilon < \varepsilon_t^* \end{cases} \quad \text{and} \quad \bar{a}_{ti}^s = \begin{cases} a_{ti}^s + \frac{1}{p_t^o(\varepsilon)} a_{ti}^m & \text{if } \varepsilon_t^* < \varepsilon \\ a_{ti}^s + \frac{1}{p_t} (a_{ti}^m - \bar{a}_{ti}^m) & \text{if } \varepsilon = \varepsilon_t^* \\ 0 & \text{if } \varepsilon < \varepsilon_t^* \end{cases}$$

where

$$p_t^o(\varepsilon) \equiv \left(\frac{\varepsilon y + \phi_t^s}{\varepsilon_t^* y + \phi_t^s} \right) p_t = \frac{\varepsilon y + \phi_t^s}{\phi_t^m}. \quad (16)$$

The dealer's portfolio $(\bar{a}_{td^*}^m, \bar{a}_{td^*}^s)$ that results from trading with the investor is given by

$$\bar{a}_{td^*}^m \begin{cases} \in [0, a_{td}^m + p_t a_{td}^s + [p_t^o(\varepsilon) - p_t] \frac{a_{ti}^m}{p_t^o(\varepsilon)}] & \text{if } \varepsilon_t^* < \varepsilon \\ \in [0, a_{td}^m + p_t a_{td}^s] & \text{if } \varepsilon = \varepsilon_t^* \\ \in [0, a_{td}^m + p_t a_{td}^s + [p_t - p_t^o(\varepsilon)] a_{ti}^s] & \text{if } \varepsilon < \varepsilon_t^* \end{cases}$$

and

$$\bar{a}_{td^*}^s = \begin{cases} a_{td}^s + \frac{1}{p_t} [a_{td}^m + [p_t^o(\varepsilon) - p_t] \frac{a_{ti}^m}{p_t^o(\varepsilon)} - \bar{a}_{td^*}^m] & \text{if } \varepsilon_t^* < \varepsilon \\ a_{td}^s + \frac{1}{p_t} (a_{td}^m - \bar{a}_{td^*}^m) & \text{if } \varepsilon = \varepsilon_t^* \\ a_{td}^s + \frac{1}{p_t} [a_{td}^m + [p_t - p_t^o(\varepsilon)] a_{ti}^s - \bar{a}_{td^*}^m] & \text{if } \varepsilon < \varepsilon_t^*. \end{cases}$$

To interpret the results in Lemma 2, observe that (15) defines the preference type of the ‘‘marginal investor.’’ That is, investors with preference type $\varepsilon < \varepsilon_t^*$ want to sell equity for cash, investors with preference type $\varepsilon > \varepsilon_t^*$ want to buy equity with cash, and the marginal investors

with preference type $\varepsilon = \varepsilon_t^*$ are indifferent between buying or selling equity, as they have no gain from trading in the OTC market.⁷ Consider an investor who has drawn preference type ε and meets a dealer in period t . If $p_t \phi_t^m < \varepsilon y + \phi_t^s$, or equivalently, if $\varepsilon_t^* < \varepsilon$, then the real value of a dollar to the investor is ϕ_t^m (the amount of general goods he can buy in the following centralized market), which is smaller than $(\varepsilon y + \phi_t^s) / p_t$, namely the value to the investor of the (cum-dividend) equity position that can be purchased with a dollar in the interdealer market. Naturally, in this case the bargaining outcome is that the investor sells all his cash and uses it to purchase equity, regardless of whether the dealer or the investor has the bargaining power. Formally, in Lemma 2, $\bar{a}_{ti}^m = \bar{a}_{ti}^m = 0$ if $\varepsilon_t^* < \varepsilon$. Analogously, if $\varepsilon < \varepsilon_t^*$, then the investor sells all his equity for cash, both when he makes the offer, and when the dealer makes the offer (i.e., $\bar{a}_{ti}^s = \bar{a}_{ti}^s = 0$ if $\varepsilon < \varepsilon_t^*$ in the lemma). If $\varepsilon = \varepsilon_t^*$, the investor is indifferent between holding equity or cash; there are no gains from trade between him and the dealer.

The quantity of equity shares the investor gets for his cash holdings when $\varepsilon_t^* < \varepsilon$, and the amount of cash that he gets for his equity shares when $\varepsilon < \varepsilon_t^*$, however, do depend on whether the investor or the dealer has the bargaining power. If the investor has the bargaining power, he can effectively trade money for equity at the interdealer market price, p_t , i.e., he pays p_t dollars per share when he buys equity, and gets p_t dollars per share when he sells equity. Formally, in Lemma 2, $\bar{a}_{ti}^s = a_{ti}^s + \frac{1}{p_t} a_{ti}^m$ if $\varepsilon_t^* < \varepsilon$, and $\bar{a}_{ti}^m = a_{ti}^m + p_t a_{ti}^s$ if $\varepsilon < \varepsilon_t^*$. If the dealer has the bargaining power, he offers less favorable terms of trade to the investor. Effectively, the bargaining outcome implies that the dealer lets the investor trade at $p_t^o(\varepsilon)$ dollars per share, rather than p_t dollars per share. Notice that $\partial p_t^o(\varepsilon) / \partial \varepsilon > 0$, so investors with higher preference types face a higher dollar price per share. Also, note that $p_t^o(\varepsilon) > p_t$ if and only if $\varepsilon_t^* < \varepsilon$. Thus the dealer charges $p_t^o(\varepsilon) > p_t$ dollars per share to an investor who wishes to buy equity (i.e., an investor with $\varepsilon_t^* < \varepsilon$), and pays $p_t^o(\varepsilon) < p_t$ dollars per share to an investor who wishes to sell equity (an investor with $\varepsilon < \varepsilon_t^*$). In other words, in a meeting where the dealer has the bargaining power, $p_t^o(\varepsilon)$ is the nominal *bid price* for investors who want to sell equity, or the nominal *ask price* for investors who want to buy equity. In terms of the lemma, this is why $\bar{a}_{ti}^s = a_{ti}^s + \frac{1}{p_t^o(\varepsilon)} a_{ti}^m$ if $\varepsilon_t^* < \varepsilon$, and $\bar{a}_{ti}^m = a_{ti}^m + p_t^o(\varepsilon) a_{ti}^s$ if $\varepsilon < \varepsilon_t^*$.

The indeterminacy in the dealer's portfolio follows from the fact that after having traded with the investor, the dealer can immediately retrade in the interdealer market, so all the dealer

⁷Another way to interpret (15) is that given ε_t^* , $p_t \phi_t^m = \varepsilon_t^* y + \phi_t^s$ is the *cum dividend* real value of equity to the marginal investor in period t .

cares about is the value of his own *combined* post-trade portfolio. In fact, as the following corollary shows, the post-trade value of the dealer and the investor portfolios are uniquely pinned down.

Corollary 1 *Consider the bargaining problem between an investor i with portfolio (a_{ti}^m, a_{ti}^s) and preference type ε , and a dealer d with portfolio (a_{td}^m, a_{td}^s) in the OTC market of period t .*

(i) *With probability θ the investor chooses the terms of trade, and in this case the dollar value of the investor's and the dealer's post-trade portfolios are, respectively,*

$$\begin{aligned}\bar{a}_{ti^*}^m + p_t \bar{a}_{ti^*}^s &= a_{ti}^m + p_t a_{ti}^s \\ \bar{a}_{td}^m + p_t \bar{a}_{td}^s &= a_{td}^m + p_t a_{td}^s.\end{aligned}$$

(ii) *With probability $1 - \theta$ the dealer chooses the terms of trade, and in this case the dollar value of the investor's and the dealer's post-trade portfolios are, respectively,*

$$\begin{aligned}\bar{a}_{ti}^m + p_t \bar{a}_{ti}^s &= \begin{cases} a_{ti}^m + p_t a_{ti}^s - [p_t^o(\varepsilon) - p_t] \frac{a_{ti}^m}{p_t^o(\varepsilon)} & \text{if } \varepsilon_t^* \leq \varepsilon \\ a_{ti}^m + p_t a_{ti}^s - [p_t - p_t^o(\varepsilon)] a_{ti}^s & \text{if } \varepsilon < \varepsilon_t^* \end{cases} \\ \bar{a}_{td}^m + p_t \bar{a}_{td}^s &= \begin{cases} a_{td}^m + p_t a_{td}^s + [p_t^o(\varepsilon) - p_t] \frac{a_{ti}^m}{p_t^o(\varepsilon)} & \text{if } \varepsilon_t^* \leq \varepsilon \\ a_{td}^m + p_t a_{td}^s + [p_t - p_t^o(\varepsilon)] a_{ti}^s & \text{if } \varepsilon < \varepsilon_t^*. \end{cases}\end{aligned}$$

Corollary 1 shows that the dealer extracts a transaction fee from the investor only when he has the bargaining power. For example, when the dealer encounters an investor with $\varepsilon > \varepsilon_t^*$ who wishes to purchase x shares, the dealer extracts $p_t^o(\varepsilon) - p_t = \frac{(\varepsilon - \varepsilon_t^*)y}{\varepsilon_t^* y + \phi_t^s} p_t$ dollars per share purchased by the investor, for a total fee of $\frac{(\varepsilon - \varepsilon_t^*)y}{\varepsilon_t^* y + \phi_t^s} p_t x$ dollars. In Lemma 2 and Corollary 1, $x = \bar{a}_{ti}^s - a_{ti}^s = \frac{1}{p_t^o(\varepsilon)} a_{ti}^m = \left(\frac{\varepsilon_t^* y + \phi_t^s}{\varepsilon y + \phi_t^s} \right) \frac{1}{p_t} a_{ti}^m$, so the total fee equals $\frac{(\varepsilon - \varepsilon_t^*)y}{\varepsilon y + \phi_t^s} a_{ti}^m$ dollars. Similarly, when the dealer encounters an investor with $\varepsilon < \varepsilon_t^*$ who wishes to sell a_{ti}^s shares, the dealer extracts $p_t - p_t^o(\varepsilon) = \frac{(\varepsilon_t^* - \varepsilon)y}{\varepsilon_t^* y + \phi_t^s} p_t$ dollars per share sold by the investor.

Consider a bilateral meeting in the OTC trading round of period t , between an investor i with portfolio a_{ti} and preference type ε_i , and an investor j with portfolio a_{tj} and preference type ε_j . With probability $\tilde{\eta}(\varepsilon_i, \varepsilon_j)$, investor i has the power to make a take-it-or-leave-it offer to investor j , and in that event investor i chooses an offer of post-trade portfolios for himself, $(\underline{a}_{ti^*}^m, \underline{a}_{ti^*}^s)$, and for investor j , $(\underline{a}_{tj}^m, \underline{a}_{tj}^s)$, by solving

$$\max_{\underline{a}_{ti^*}^m, \underline{a}_{ti^*}^s, \underline{a}_{tj}^m, \underline{a}_{tj}^s} [\varepsilon_i y \underline{a}_{ti^*}^s + W_t^I(\underline{a}_{ti^*}^m, \underline{a}_{ti^*}^s)]$$

$$\begin{aligned}
& \text{s.t. } \underline{a}_{ti}^m + \underline{a}_{tj}^m \leq a_{ti}^m + a_{tj}^m \\
& \underline{a}_{ti}^s + \underline{a}_{tj}^s \leq a_{ti}^s + a_{tj}^s \\
& \varepsilon_j y \underline{a}_{tj}^s + W_t^I(\underline{a}_{tj}^m, \underline{a}_{tj}^s) \geq \varepsilon_j y a_{tj}^s + W_t^I(a_{tj}^m, a_{tj}^s) \\
& \underline{a}_{ti}^m, \underline{a}_{ti}^s, \underline{a}_{tj}^m, \underline{a}_{tj}^s \in \mathbb{R}_+.
\end{aligned}$$

The first two constraints imply that in a bilateral meeting the two investors can (only) reallocate money and assets between themselves. The third constraint ensures that it is individually rational for investor j to accept i 's offer. The following result summarizes the outcome of the bargaining game between two investors.

Lemma 3 *Consider the bargaining problem between an investor i with portfolio (a_{ti}^m, a_{ti}^s) and preference type ε_i , and an investor j with portfolio (a_{tj}^m, a_{tj}^s) and preference type ε_j in the OTC market of period t . Suppose that investor i has the power to choose the terms of trade, then his post-trade portfolio is $\underline{a}_{ti}^* = (\underline{a}_{ti}^{m*}, \underline{a}_{ti}^{s*})$ with*

$$\begin{aligned}
\underline{a}_{ti}^{s*} & \begin{cases} = a_{ti}^s + \min \left[\frac{a_{ti}^m}{p_t^o(\varepsilon_j)}, a_{tj}^s \right] & \text{if } \varepsilon_j < \varepsilon_i \\ \in \left[a_{ti}^s - \min \left[\frac{a_{tj}^m}{p_t^o(\varepsilon_j)}, a_{ti}^s \right], a_{ti}^s + \min \left[\frac{a_{ti}^m}{p_t^o(\varepsilon_j)}, a_{tj}^s \right] \right] & \text{if } \varepsilon_j = \varepsilon_i \\ = a_{ti}^s - \min \left[\frac{a_{tj}^m}{p_t^o(\varepsilon_j)}, a_{ti}^s \right] & \text{if } \varepsilon_i < \varepsilon_j \end{cases} \\
\underline{a}_{ti}^{m*} & = \begin{cases} a_{ti}^m - \min \left[p_t^o(\varepsilon_j) a_{tj}^s, a_{ti}^m \right] & \text{if } \varepsilon_j < \varepsilon_i \\ a_{ti}^m + p_t^o(\varepsilon_j) (a_{ti}^s - \underline{a}_{ti}^{s*}) & \text{if } \varepsilon_j = \varepsilon_i \\ a_{ti}^m + \min \left[p_t^o(\varepsilon_j) a_{ti}^s, a_{tj}^m \right] & \text{if } \varepsilon_i < \varepsilon_j, \end{cases}
\end{aligned}$$

and investor j 's post-trade portfolio is $\underline{a}_{tj} = (\underline{a}_{tj}^m, \underline{a}_{tj}^s)$, with $\underline{a}_{tj}^s = a_{tj}^s + a_{ti}^s - \underline{a}_{ti}^{s*}$ and $\underline{a}_{tj}^m = a_{tj}^m + a_{ti}^m - \underline{a}_{ti}^{m*}$.

In Lemma 3, if $\varepsilon_j < \varepsilon_i$, then investor i wishes to purchase all of investor j 's equity. Since he has all the bargaining power, investor i sets the terms of trade at $p_t^o(\varepsilon_j)$ dollars per equity share, i.e., the dollar price of equity that makes investor j just indifferent between selling equity for dollars or not. The quantity of equity that investor i is able to purchase will depend on his money holdings, a_{ti}^m . If $a_{ti}^m \geq p_t^o(\varepsilon_j) a_{tj}^s$, then he buys all of investor j 's equity holdings, a_{tj}^s , in exchange for $p_t^o(\varepsilon_j) a_{tj}^s$ dollars. If $a_{ti}^m < p_t^o(\varepsilon_j) a_{tj}^s$, then i gives investor j all his money holdings, a_{ti}^m , in exchange for $\frac{a_{ti}^m}{p_t^o(\varepsilon_j)}$ equity shares. Conversely, if $\varepsilon_i < \varepsilon_j$, then investor i wishes to sell all of his equity to investor j . Similarly, the quantity of equity that investor i will sell to investor j depends on j 's money holdings, a_{tj}^m . If $a_{tj}^m \geq p_t^o(\varepsilon_j) a_{ti}^s$, then j buys all of investor i 's equity

holdings, a_{ti}^s , in exchange for $p_t^o(\varepsilon_j)a_{ti}^s$ dollars. If $a_{tj}^m < p_t^o(\varepsilon_j)a_{ti}^s$, then j gives investor i all his money holdings, a_{tj}^m , in exchange for $\frac{a_{tj}^m}{p_t^o(\varepsilon_j)}$ equity shares.

The bargaining outcomes can be substituted in the value functions (10) and (12) to obtain the following result.

Lemma 4 *Let A_{It}^m and A_{It}^s denote the quantity of money and shares held by all investors at the beginning of the OTC round of period t , respectively, i.e., $A_{It}^m = \int a_{ti}^m dF_t^I(\mathbf{a}_{ti})$ and $A_{It}^s = \int a_{ti}^s dF_t^I(\mathbf{a}_{ti})$.*

(i) *The value function of a dealer who enters the OTC round of period t with portfolio $\mathbf{a}_{td} = (a_{td}^m, a_{td}^s)$ is given by*

$$V_t^D(a_{td}^m, a_{td}^s) = \bar{\phi}_t(a_{td}^m + p_t a_{td}^s) + V_t^D(\mathbf{0}) \quad (17)$$

where

$$V_t^D(\mathbf{0}) \equiv \kappa(1 - \theta) \bar{\phi}_t \left[A_{It}^m \int_{\varepsilon_t^*}^{\varepsilon_H} \frac{(\varepsilon - \varepsilon_t^*)y}{\varepsilon y + \phi_t^s} dG(\varepsilon) + p_t A_{It}^s \int_{\varepsilon_L}^{\varepsilon_t^*} \frac{(\varepsilon_t^* - \varepsilon)y}{\varepsilon_t^* y + \phi_t^s} dG(\varepsilon) \right] + W_t^D(\mathbf{0}).$$

(ii) *The value function of an investor who enters the OTC round of period t with portfolio $\mathbf{a}_{ti} = (a_{ti}^m, a_{ti}^s)$ and preference type ε_i is given by*

$$\begin{aligned} V_t^I(a_{ti}^m, a_{ti}^s, \varepsilon_i) &= \phi_t^m a_{ti}^m + (\varepsilon_i y + \phi_t^s) a_{ti}^s + W_t^I(\mathbf{0}) \\ &+ \delta \theta \mathbb{I}_{\{\varepsilon_t^* \leq \varepsilon_i\}} \frac{(\varepsilon_i - \varepsilon_t^*)y}{\varepsilon_t^* y + \phi_t^s} \phi_t^m a_{ti}^m + \delta \theta \mathbb{I}_{\{\varepsilon_i < \varepsilon_t^*\}} (\varepsilon_t^* - \varepsilon_i) y a_{ti}^s \\ &+ \alpha \int \mathbb{I}_{\{\varepsilon_j \leq \varepsilon_i\}} \eta \frac{(\varepsilon_i - \varepsilon_j)y}{\varepsilon_j y + \phi_t^s} \min[\phi_t^m a_{ti}^m, (\varepsilon_j y + \phi_t^s) A_{It}^s] dG(\varepsilon_j) \\ &+ \alpha \int \mathbb{I}_{\{\varepsilon_i < \varepsilon_j\}} (1 - \eta) \frac{(\varepsilon_j - \varepsilon_i)y}{\varepsilon_j y + \phi_t^s} \min[\phi_t^m A_{It}^m, (\varepsilon_j y + \phi_t^s) a_{ti}^s] dG(\varepsilon_j) \quad (18) \end{aligned}$$

where $\mathbb{I}_{\{\varepsilon_t^* \leq \varepsilon_i\}}$ is an indicator function that takes the value 1 if $\varepsilon_t^* \leq \varepsilon_i$, and 0 otherwise.

Notice that the first term on the right side of $V_t^D(\mathbf{0})$ is the expected fee earned by a dealer in the OTC market of period t (the term inside the square bracket on the right side of $V_t^D(\mathbf{0})$ is the expected fee earned by a dealer when he makes an offer to an investor whose preference type, ε , is a random draw from G). To interpret (18), notice that the first line represents the value to the investor of holding the portfolio of money and equity until the end of the period. The remaining four terms represent the expected net gains from trading. For example, the

factor that multiplies the indicator function $\mathbb{I}_{\{\varepsilon_i^* \leq \varepsilon_i\}}$, i.e., $\delta\theta \frac{(\varepsilon_i - \varepsilon_i^*)y}{\varepsilon_i^*y + \phi_i^s} \phi_t^m a_{ti}^m$, is the expected net gain to the investor from exchanging money for shares in a trade with a dealer in the OTC market, and the factor that multiplies the indicator function $\mathbb{I}_{\{\varepsilon_i < \varepsilon_i^*\}}$, i.e., $\delta\theta (\varepsilon_i^* - \varepsilon_i) y a_{ti}^s$, is the expected net gain to the investor from exchanging shares for money in a trade with a dealer in the OTC market.⁸ The last two terms on the right side of (18) represent the expected net gains from trading with another investor in the OTC market.⁹

The following result uses Lemma 4 to characterize the solutions to the portfolio problems that a typical dealer and a typical investor solve in the second subperiod of period t .

Lemma 5 *Let $(\tilde{a}_{t+1d}^m, \tilde{a}_{t+1d}^s)$ and $(\tilde{a}_{t+1i}^m, \tilde{a}_{t+1i}^s)$ denote the portfolios chosen by a dealer and an investor, respectively, in the second subperiod of period t . The first-order necessary and sufficient conditions for optimization that these portfolios must satisfy are*

$$\phi_t^m \geq \beta \max(\phi_{t+1}^m, \phi_{t+1}^s/p_{t+1}) \quad (19)$$

$$\phi_t^s \geq \beta\pi \max(p_{t+1}\phi_{t+1}^m, \phi_{t+1}^s) \quad (20)$$

⁸Conditional on having drawn $\varepsilon_i \geq \varepsilon_i^*$, the investor contacts a dealer with probability δ and has the bargaining power with probability θ . In this case the investor sells a_{ti}^m dollars for $\frac{1}{p_t} a_{ti}^m$ shares, and his net payoff from this transaction is

$$(\varepsilon_i y + \phi_i^s) \frac{1}{p_t} a_{ti}^m - \phi_t^m a_{ti}^m = (\varepsilon_i y + \phi_i^s) \frac{\phi_t^m}{\varepsilon_i^* y + \phi_t^s} a_{ti}^m - \phi_t^m a_{ti}^m = \frac{(\varepsilon_i - \varepsilon_i^*) y}{\varepsilon_i^* y + \phi_t^s} \phi_t^m a_{ti}^m.$$

Conditional on having drawn $\varepsilon_i < \varepsilon_i^*$, the investor contacts a dealer with probability δ and has the bargaining power with probability θ . In this case the investor sells a_{ti}^s shares for p_t dollars each. The investor's net payoff from this transaction is $\phi_t^m p_t a_{ti}^s - (\varepsilon_i y + \phi_i^s) a_{ti}^s = (\varepsilon_i^* - \varepsilon_i) y a_{ti}^s$.

⁹Consider the penultimate term: with probability α the investor contacts another investor in the OTC market, if the other investor's preference type, ε_j , is smaller than ε_i , then the investor with preference type ε_i has the bargaining power with probability η and he spends $\min[a_{ti}^m, p_t^o(\varepsilon_j) A_{It}^s]$ dollars purchasing $\min[a_{ti}^m/p_t^o(\varepsilon_j), A_{It}^s]$ equity shares from the other investor, for a net gain from trade equal to

$$(\varepsilon_i y + \phi_i^s) \min[a_{ti}^m/p_t^o(\varepsilon_j), A_{It}^s] - \phi_t^m \min[a_{ti}^m, p_t^o(\varepsilon_j) A_{It}^s] = \frac{(\varepsilon_i - \varepsilon_j) y}{\varepsilon_j y + \phi_j^s} \min[\phi_t^m a_{ti}^m, (\varepsilon_j y + \phi_j^s) A_{It}^s].$$

Similarly, with probability α the investor contacts another investor in the OTC market, and if the other investor's preference type, ε_j , is larger than ε_i , then the investor with preference type ε_i has the bargaining power with probability $1 - \eta$ and he sells $\min[A_{It}^m/p_t^o(\varepsilon_j), a_{ti}^s]$ equity shares in exchange for $\min[A_{It}^m, p_t^o(\varepsilon_j) a_{ti}^s]$ dollars, for a net gain from trade equal to

$$\phi_t^m \min[A_{It}^m, p_t^o(\varepsilon_j) a_{ti}^s] - (\varepsilon_i y + \phi_i^s) \min[A_{It}^m/p_t^o(\varepsilon_j), a_{ti}^s] = \frac{(\varepsilon_j - \varepsilon_i) y}{\varepsilon_j y + \phi_j^s} \min[\phi_t^m A_{It}^m, (\varepsilon_j y + \phi_j^s) a_{ti}^s].$$

and

$$\begin{aligned} \phi_t^m \geq & \beta \left[1 + \delta \theta \int_{\varepsilon_{t+1}^*}^{\varepsilon_H} \frac{(\varepsilon_i - \varepsilon_{t+1}^*)y}{\varepsilon_{t+1}^* y + \phi_{t+1}^s} dG(\varepsilon_i) \right. \\ & \left. + \alpha \eta \int_{\left[\frac{\phi_{t+1}^m a_{t+1}^m}{A_{t+1}^s} - \phi_{t+1}^s \right] \frac{1}{y}}^{\varepsilon_H} \frac{(\varepsilon_i - \varepsilon_j)y}{\varepsilon_j y + \phi_{t+1}^s} dG(\varepsilon_i) dG(\varepsilon_j) \right] \phi_{t+1}^m \end{aligned} \quad (21)$$

$$\begin{aligned} \phi_t^s \geq & \beta \pi \left[\phi_{t+1}^s + \left(\bar{\varepsilon} + \delta \theta \int_{\varepsilon_L}^{\varepsilon_{t+1}^*} (\varepsilon_{t+1}^* - \varepsilon_i) dG(\varepsilon_i) \right. \right. \\ & \left. \left. + \alpha (1 - \eta) \int_{\varepsilon_L}^{\left[\frac{\phi_{t+1}^m A_{t+1}^m}{a_{t+1}^s} - \phi_{t+1}^s \right] \frac{1}{y}} \int_{\varepsilon_L}^{\varepsilon_j} (\varepsilon_j - \varepsilon_i) dG(\varepsilon_i) dG(\varepsilon_j) \right) y \right] \end{aligned} \quad (22)$$

where (19) holds with “=” if $\tilde{a}_{t+1d}^m > 0$, (20) holds with “=” if $\tilde{a}_{t+1d}^s > 0$, (21) holds with “=” if $\tilde{a}_{t+1i}^m > 0$, and (22) holds with “=” if $\tilde{a}_{t+1i}^s > 0$.

Condition (19) is a dealer’s Euler equation for money. The left side is the real cost of purchasing a dollar (in terms of the homogeneous good) in the second subperiod of period t . The right side is the discounted gain from this marginal dollar in the following period, i.e., the dealer can choose to hold on to the dollar until the second subperiod of $t + 1$ to obtain ϕ_{t+1}^m homogeneous consumption goods, or he can sell the dollar in the interdealer market of the following OTC round for $1/p_{t+1}$ equity shares, each of which will be worth ϕ_{t+1}^s homogeneous goods in the second subperiod of $t + 1$. Naturally, the dealer will choose the best of these two trading strategies. The dealer holds no money overnight if the left side of (19) exceeds the right side. Condition (20) is a dealer’s Euler equation for equity shares. The left side is the real cost of purchasing a share (in terms of the homogeneous good) in the second subperiod of t . The right side is the discounted expected gain from this marginal share in the following period, i.e., the tree remains productive with probability π , and in that event the dealer can sell the share in the interdealer market of the following OTC round for p_{t+1} dollars, each of which will be worth ϕ_{t+1}^m homogeneous goods in the second subperiod of $t + 1$, or he can choose to hold on to the share until the second subperiod of period $t + 1$ to obtain ϕ_{t+1}^s homogeneous consumption goods. The dealer holds no equity overnight if the left side of (20) exceeds the right side.

Condition (21) is the investor’s Euler equation for money. The left side is the real cost of purchasing a dollar in the second subperiod of period t . The right side is the discounted expected benefit from carrying this additional dollar into the following period, which consists of

three components: (i) the expected benefit from holding the dollar until the second subperiod of period $t + 1$ (i.e., if the investor does not spend the dollar in the OTC market), (ii) the expected gain from using the dollar to purchase equity from a dealer in the OTC market of period $t + 1$, and (iii) the expected gain from using the dollar to purchase equity from another investor in the OTC market of period $t + 1$. To interpret (21), it is useful to rewrite it as

$$\begin{aligned} \phi_t^m \geq & \beta \left\{ \phi_{t+1}^m + \delta\theta [1 - G(\varepsilon_{t+1}^*)] \mathbb{E} \left[\frac{\varepsilon_i y + \phi_{t+1}^s}{p_{t+1}} - \phi_{t+1}^m \middle| \varepsilon_i \geq \varepsilon_{t+1}^* \right] \right. \\ & \left. + \alpha \omega_{t+1}^m(a_{t+1i}^m, A_{t+1}^s) \eta \mathbb{E} \left[\frac{\varepsilon_i y + \phi_{t+1}^s}{p_{t+1}^o(\varepsilon_j)} - \phi_{t+1}^m \middle| (\varepsilon_i, \varepsilon_j) \in \Omega_{t+1}^m(a_{t+1i}^m, A_{t+1}^s) \right] \right\} \end{aligned}$$

where $\mathbb{E}[\cdot|\cdot]$ denotes the conditional expectation of $(\varepsilon_i, \varepsilon_j)$, and for any $(a_{t+1i}^m, a_{t+1j}^s) \in \mathbb{R}_+^2$,

$$\Omega_{t+1}^m(a_{t+1i}^m, a_{t+1j}^s) = \left\{ (\varepsilon_i, \varepsilon_j) \in [\varepsilon_L, \varepsilon_H]^2 : \varepsilon_j < \varepsilon_i \text{ and } a_{t+1i}^m < p_{t+1}^o(\varepsilon_j) a_{t+1j}^s \right\}$$

and $\omega_{t+1}^m(a_{t+1i}^m, a_{t+1j}^s) \equiv \int \int \mathbb{I}_{\{(\varepsilon_i, \varepsilon_j) \in \Omega_{t+1}^m(a_{t+1i}^m, a_{t+1j}^s)\}} dG(\varepsilon_i) dG(\varepsilon_j)$. With probability $\delta\theta[1 - G(\varepsilon_{t+1}^*)]$, the investor contacts a dealer in the OTC market, has bargaining power, and wishes to purchase equity. In this event, he uses the marginal dollar worth ϕ_{t+1}^m to purchase $\frac{1}{p_{t+1}}$ equity shares each of which yields expected utility from the dividend equal to $\mathbb{E}[\varepsilon_i | \varepsilon_i \geq \varepsilon_{t+1}^*] y$, and a resale value of ϕ_{t+1}^s homogeneous goods in the second subperiod of $t + 1$. With probability α , investor, call him i , contacts another investor, e.g., investor j . Then $\omega_{t+1}^m(a_{t+1i}^m, A_{t+1}^s) \eta$ denotes the joint probability that i 's preference type is higher than j 's (so i acts as a buyer of equity), and i has bargaining power (which happens with conditional probability η), and the bilateral gains from trade are constrained by i 's money holdings (which given i 's money holdings, a_{t+1i}^m , and j 's equity holdings, A_{t+1}^s , at the time of the trade, occurs if the bilateral dollar price of equity is large enough, i.e., if j 's individual valuation of equity, ε_j , is large enough). In this event, carrying an additional dollar into period $t + 1$ helps investor i reap gains from trade in the bilateral trade with the other investor, and i 's expected gain from trading the marginal dollar is the (conditional expected) value of the additional equity he purchases, i.e., $\frac{1}{p_{t+1}^o(\varepsilon_j)}$ equity shares each worth $\varepsilon_i y + \phi_{t+1}^s$, minus the value of the dollar, ϕ_{t+1}^m .

Condition (22) is the investor's Euler equation for equity. To interpret this condition it is useful to rewrite it as

$$\begin{aligned} \phi_t^s \geq & \beta \pi \left\{ \bar{\varepsilon} y + \phi_{t+1}^s + \delta\theta G(\varepsilon_{t+1}^*) \mathbb{E} [p_{t+1} \phi_{t+1}^m - (\varepsilon_i y + \phi_{t+1}^s) | \varepsilon_i \leq \varepsilon_{t+1}^*] \right. \\ & \left. + \alpha \omega_{t+1}^s(a_{t+1i}^s, A_{t+1}^m) (1 - \eta) \mathbb{E} [p_{t+1}^o(\varepsilon_j) \phi_{t+1}^m - (\varepsilon_i y + \phi_{t+1}^s) | (\varepsilon_i, \varepsilon_j) \in \Omega_{t+1}^s(a_{t+1i}^s, A_{t+1}^m)] \right\} \end{aligned}$$

where for any $(a_{t+1i}^s, a_{t+1j}^m) \in \mathbb{R}_+^2$,

$$\Omega_{t+1}^s(a_{t+1i}^s, a_{t+1j}^m) = \left\{ (\varepsilon_i, \varepsilon_j) \in [\varepsilon_L, \varepsilon_H]^2 : \varepsilon_i < \varepsilon_j \text{ and } p_{t+1}^o(\varepsilon_j) a_{t+1i}^s < a_{t+1j}^m \right\}$$

and $\omega_{t+1}^s(a_{t+1i}^s, a_{t+1j}^m) \equiv \int \int \mathbb{I}_{\{(\varepsilon_i, \varepsilon_j) \in \Omega_{t+1}^s(a_{t+1i}^s, a_{t+1j}^m)\}} dG(\varepsilon_i) dG(\varepsilon_j)$. The left side is the real cost of purchasing an additional equity share in the second subperiod of t . The right side is the discounted expected benefit from carrying an additional equity share into the following period, which consists of three terms. First, $\bar{\varepsilon}y + \phi_{t+1}^s$, the expected benefit of holding the equity share until the end of period $t + 1$ (i.e., if the investor does not sell the equity in the OTC market). Second, with probability $\delta\theta G(\varepsilon_{t+1}^*)$, the investor contacts a dealer in the OTC market, has bargaining power, and wishes to sell equity. In this event, he obtains $p_{t+1}\phi_{t+1}^m$ dollars for selling the marginal equity share which he expects to value $\mathbb{E}[\varepsilon_i y + \phi_{t+1}^s | \varepsilon_i \leq \varepsilon_{t+1}^*]$. Finally, with probability α investor i contacts another investor j in the OTC market. Then $\omega_{t+1}^s(a_{t+1i}^s, A_{It+1}^m) (1 - \eta)$ denotes the joint probability that i 's preference type is lower than j 's (so i acts as a seller of equity), and i has bargaining power (which happens with conditional probability $1 - \eta$), and the bilateral gains from trade are constrained by i 's equity holdings (which given i 's equity holdings, a_{t+1i}^s , and j 's money holdings, A_{It+1}^m , at the time of the trade, occurs if the bilateral dollar price of equity is low enough, i.e., if j 's individual valuation of equity, ε_j , is low enough). In this event, an additional equity share helps investor i reap gains from trade in the bilateral trade with the other investor, and i 's expected gain from trading the marginal share is the (conditional expected) value of the real balances he receives, i.e., $p_{t+1}^o(\varepsilon_j) \phi_{t+1}^m$, minus the (conditional expected) value of the equity share he sells, i.e., $\varepsilon_i y + \phi_{t+1}^s$.

Let A_{Dt+1}^m and A_{Dt+1}^s denote the quantities of money and equity shares, respectively, held by all dealers at the beginning of the OTC round of period $t+1$, i.e., $A_{Dt+1}^m = v \int a_{t+1d}^m dF_{t+1}^D(\mathbf{a}_{t+1d})$, and $A_{Dt+1}^s = v \int a_{t+1d}^s dF_{t+1}^D(\mathbf{a}_{t+1d})$. Let \tilde{A}_{Dt+1}^m and \tilde{A}_{Dt+1}^s denote the total quantities of money and shares held by all dealers at the end of period t , i.e., $A_{Dt+1}^m = \tilde{A}_{Dt+1}^m$ and $A_{Dt+1}^s = \pi \tilde{A}_{Dt+1}^s$. Similarly, let \tilde{A}_{It+1}^m and \tilde{A}_{It+1}^s denote the total quantities of money and shares held by all investors at the end of period t , i.e., $A_{It+1}^m = \tilde{A}_{It+1}^m$ and $A_{It+1}^s = \pi \tilde{A}_{It+1}^s + (1 - \pi) A^s$. Let \bar{A}_{Dt}^m and \bar{A}_{Dt}^s denote the quantity of money and shares held after the OTC round of trade of period t by all the dealers who are able to trade in the first subperiod. Similarly, let \bar{A}_{It}^m and \bar{A}_{It}^s denote the quantity of money and shares held after the OTC round of trade of period t by all

the investors who are able to trade in the first subperiod. For $k = s, m$,

$$\begin{aligned}\bar{A}_{Dt}^k &= \kappa v \theta \int \hat{a}_d^k [\bar{\mathbf{a}}_d(\mathbf{a}_{ti}, \mathbf{a}_{td}, \varepsilon; \boldsymbol{\psi}_t); \boldsymbol{\psi}_t] dF_t^D(\mathbf{a}_{td}) dF_t^I(\mathbf{a}_{ti}) dG(\varepsilon) \\ &\quad + \kappa v (1 - \theta) \int \hat{a}_d^k [\bar{\mathbf{a}}_{d^*}(\mathbf{a}_{ti}, \mathbf{a}_{td}, \varepsilon; \boldsymbol{\psi}_t); \boldsymbol{\psi}_t] dF_t^D(\mathbf{a}_{td}) dF_t^I(\mathbf{a}_{ti}) dG(\varepsilon) \\ &\quad + (1 - \kappa) v \int \hat{a}_d^k(\mathbf{a}_{td}; \boldsymbol{\psi}_t) dF_t^D(\mathbf{a}_{td})\end{aligned}$$

and

$$\bar{A}_{It}^k = \delta \int \left[\theta \bar{a}_{i^*}^k(\mathbf{a}_{ti}, \mathbf{a}_{td}, \varepsilon; \boldsymbol{\psi}_t) + (1 - \theta) \bar{a}_i^k(\mathbf{a}_{ti}, \mathbf{a}_{td}, \varepsilon; \boldsymbol{\psi}_t) \right] dF_t^D(\mathbf{a}_{td}) dF_t^I(\mathbf{a}_{ti}) dG(\varepsilon)$$

where $\bar{\mathbf{a}}_d(\mathbf{a}_{ti}, \mathbf{a}_{td}, \varepsilon; \boldsymbol{\psi}_t) \equiv (\bar{a}_d^m(\mathbf{a}_{ti}, \mathbf{a}_{td}, \varepsilon; \boldsymbol{\psi}_t), \bar{a}_d^s(\mathbf{a}_{ti}, \mathbf{a}_{td}, \varepsilon; \boldsymbol{\psi}_t))$. We are now ready to define equilibrium.

Definition 1 *An equilibrium is a sequence of terms of trade in the OTC market*

$$\{\langle \bar{\mathbf{a}}_{td}, \bar{\mathbf{a}}_{td^*}, \hat{\mathbf{a}}_{td} \rangle_{d \in \mathcal{D}}, \langle \bar{\mathbf{a}}_{ti^*}, \bar{\mathbf{a}}_{ti}, \underline{\mathbf{a}}_{ti^*}, \underline{\mathbf{a}}_{ti} \rangle_{i \in \mathcal{I}}\}_{t=0}^\infty,$$

as given in Lemma 1, Lemma 2, and Lemma 3, together with a sequence of asset holdings

$$\{\langle \mathbf{a}_{t+1d}, \tilde{\mathbf{a}}_{t+1d} \rangle_{d \in \mathcal{D}}, \langle \mathbf{a}_{t+1i}, \tilde{\mathbf{a}}_{t+1i} \rangle_{i \in \mathcal{I}}\}_{t=0}^\infty$$

and prices $\{\boldsymbol{\psi}_t\}_{t=0}^\infty \equiv \{1/p_t, \phi_t^m, \phi_t^s\}_{t=0}^\infty$, such that for all t , (i) the asset allocation solves the individual optimization problems (8) and (11) taking prices as given, and (ii) prices are such that all Walrasian markets clear, i.e., $\tilde{A}_{Dt+1}^s + \tilde{A}_{It+1}^s = A^s$ (the end-of-period- t Walrasian market for equity), $\tilde{A}_{Dt+1}^m + \tilde{A}_{It+1}^m = A_{t+1}^m$ (the end-of-period- t Walrasian market for money), and $\bar{A}_{Dt}^k + \bar{A}_{It}^k = A_{Dt}^k + \delta A_{It}^k$ for $k = s, m$ (the period- t OTC interdealer market for equity and money). An equilibrium is “monetary” if $\phi_t^m > 0$ for all t , and “nonmonetary” otherwise.

In what follows, we specialize the analysis to stationary equilibria where all real variables are constant and nominal variables grow at the same rate as the money supply. Specifically, for all t , $A_{Dt}^s = A_D^s$, $A_{It}^s = A_I^s$, $\phi_t^s = \phi^s$, $p_t \phi_t^m \equiv \bar{\phi}^s$, $\varepsilon_t^* = \frac{\bar{\phi}^s - \phi^s}{y} \equiv \varepsilon^*$, $\phi_t^m A_{It}^m \equiv Z$, $\phi_t^m A_{Dt}^m \equiv Z_D$, and $\phi_t^m / \phi_{t+1}^m = p_{t+1} / p_t = A_{t+1}^m / A_t^m = \gamma$. Throughout the analysis we maintain the assumption $\gamma > \beta$, but the following proposition considers the limiting case $\gamma \rightarrow \beta$.

Proposition 2 *The allocation implemented by the stationary monetary equilibrium converges to the symmetric efficient allocation as $\gamma \rightarrow \beta$, i.e., the Friedman rule is efficient.*

4.1 Pure-dealer OTC market

In this section we consider the case with $\alpha = 0$, i.e., a market in which all OTC trade is intermediated by dealers and there is no direct bilateral trade among investors. For the analysis that follows, it is convenient to define

$$\hat{\gamma} \equiv \beta \left[1 + \frac{(1 - \delta\theta)(1 - \beta\pi)(\hat{\varepsilon} - \bar{\varepsilon})}{\hat{\varepsilon}} \right] \quad \text{and} \quad \bar{\gamma} \equiv \beta \left[1 + \frac{\delta\theta(1 - \beta\pi)(\bar{\varepsilon} - \varepsilon_L)}{\beta\pi\bar{\varepsilon} + (1 - \beta\pi)\varepsilon_L} \right] \quad (23)$$

where $\hat{\varepsilon} \in [\bar{\varepsilon}, \varepsilon_H]$ is the unique solution to

$$\bar{\varepsilon} - \hat{\varepsilon} + \delta\theta \int_{\varepsilon_L}^{\hat{\varepsilon}} G(\varepsilon) d\varepsilon = 0. \quad (24)$$

Lemma 6 (in the appendix) establishes that $\hat{\gamma} < \bar{\gamma}$. The following proposition summarizes the equilibrium set.

Proposition 3 *Assume $\alpha = 0$. (i) A nonmonetary equilibrium exists for any parametrization. (ii) There is no stationary monetary equilibrium if $\gamma \geq \bar{\gamma}$. (iii) In the nonmonetary equilibrium, $A_I^s = A^s - A_D^s = A^s$ (only investors hold equity shares), there is no trade in the OTC market, and the equity price in the Walrasian market is*

$$\phi^s = \frac{\beta\pi}{1 - \beta\pi} \bar{\varepsilon} y.$$

(iv) If $\gamma \in (\beta, \bar{\gamma})$, then there is one stationary monetary equilibrium; asset holdings of dealers and investors at the beginning of the OTC round of period t are $A_{Dt}^m = A_t^m - A_{It}^m = 0$ and

$$A_D^s = A^s - A_I^s \begin{cases} = \pi A^s & \text{if } \beta < \gamma < \hat{\gamma} \\ \in [0, \pi A^s] & \text{if } \gamma = \hat{\gamma} \\ = 0 & \text{if } \hat{\gamma} < \gamma < \bar{\gamma}, \end{cases}$$

and asset prices are

$$\phi^s = \begin{cases} \frac{\beta\pi}{1 - \beta\pi} \varepsilon^* y & \text{if } \beta < \gamma \leq \hat{\gamma} \\ \frac{\beta\pi}{1 - \beta\pi} \left[\bar{\varepsilon} + \delta\theta \int_{\varepsilon_L}^{\varepsilon^*} G(\varepsilon) d\varepsilon \right] y & \text{if } \hat{\gamma} < \gamma < \bar{\gamma} \end{cases} \quad (25)$$

$$Z = \frac{A_D^s + \delta G(\varepsilon^*) A_I^s}{\delta\theta [1 - G(\varepsilon^*)] \frac{1}{\varepsilon^* y + \phi^s} + \delta(1 - \theta) \int_{\varepsilon^*}^{\varepsilon_H} \frac{1}{\varepsilon y + \phi^s} dG(\varepsilon)} \quad (26)$$

$$\bar{\phi}^s = \varepsilon^* y + \phi^s \quad (27)$$

$$\phi_t^m = \frac{Z}{A_t^m} \quad (28)$$

$$p_t = \frac{\bar{\phi}^s}{Z} A_t^m, \quad (29)$$

where, for any $\gamma \in (\beta, \bar{\gamma})$, $\varepsilon^* \in (\varepsilon_L, \varepsilon_H)$ is the unique solution to

$$\frac{(1 - \beta\pi) \int_{\varepsilon^*}^{\varepsilon_H} [1 - G(\varepsilon)] d\varepsilon}{\varepsilon^* + \beta\pi \left[\bar{\varepsilon} - \varepsilon^* + \delta\theta \int_{\varepsilon_L}^{\varepsilon^*} G(\varepsilon) d\varepsilon \right] \mathbb{I}_{\{\hat{\gamma} < \gamma\}}} - \frac{\gamma - \beta}{\beta\delta\theta} = 0. \quad (30)$$

(v) (a) As $\gamma \rightarrow \bar{\gamma}$, $\varepsilon^* \rightarrow \varepsilon_L$ and $\phi^s \rightarrow \frac{\beta\pi}{1-\beta\pi} \bar{\varepsilon}y$. (b) As $\gamma \rightarrow \beta$, $\varepsilon^* \rightarrow \varepsilon_H$ and $\phi^s \rightarrow \frac{\beta\pi}{1-\beta\pi} \varepsilon_H y$.

In the nonmonetary equilibrium, dealers are inactive and equity shares are held only by investors. With no valued money, investors and dealers cannot exploit the gains from trade that arise from the heterogeneity in preference types in the first subperiod of every period, and the equilibrium real asset price, $\phi^s = \frac{\beta\pi}{1-\beta\pi} \bar{\varepsilon}y$, is equal to the expected discounted value of the dividend stream since the equity share is not traded. (Shares can be traded in the Walrasian market of the second subperiod, but gains from trade at that stage are nil.) The stationary monetary equilibrium exists only if the inflation rate is not too high, i.e., if $\gamma < \bar{\gamma}$. In the monetary equilibrium, the marginal preference type, ε^* , which according to Lemma 2 partitions the set of investors into those who buy and those who sell the asset when they meet a dealer in the OTC market, is characterized in part (iv) of Proposition 3. In the OTC market, investors with $\varepsilon < \varepsilon^*$ who contact dealers sell all their equity holdings for money, while investors with $\varepsilon > \varepsilon^*$ who contact dealers spend all their money buying equity. Thus unlike what happens in the nonmonetary equilibrium, the OTC market is active in the monetary equilibrium, and it is easy to show that the marginal type, ε^* , is strictly decreasing in the rate of inflation, i.e., $\frac{\partial \varepsilon^*}{\partial \gamma} < 0$ both for $\gamma \in (\beta, \hat{\gamma})$, and for $\gamma \in (\hat{\gamma}, \bar{\gamma})$ (see Corollary 2 in the appendix). Intuitively, the real value of money falls as γ increases, and the marginal investor type, ε^* , decreases, reflecting the fact that under the higher inflation rate, the investor that was marginal under the lower inflation rate is no longer indifferent between carrying cash and equity out of the OTC market (he prefers equity).

According to Proposition 3, $\phi^s < \phi^s + \varepsilon^*y = p_t \phi_t^m$ in the monetary equilibrium, so Lemma 1 implies that dealers hold no equity shares at the end of the OTC round: all equity is held by investors, in particular, by those investors who carried equity into the period but were unable to contact a dealer, and by those investors who purchased equity shares from dealers. After the round of OTC trade, all the money supply is held by the investors who carried cash into the period but were unable to contact a dealer, by the investors who sold equity shares to dealers, and by those dealers who had bargaining power in the OTC negotiations or carried equity into the OTC market. A feature of the monetary equilibrium is that dealers never hold money

overnight: at the beginning of every period t , the money supply is all in the hands of investors, i.e., $A_{Dt}^m = 0$ and $A_{It}^m = A_t^m$.¹⁰ The reason is that access to the interdealer market allows dealers to intermediate assets without having to carry cash. Whether it is investors or dealers who hold the equity overnight, depends on the inflation rate: if it is low, i.e., if $\gamma \in (\beta, \hat{\gamma})$, then only dealers hold equity shares overnight, that is, $\tilde{A}_{Dt+1}^s = A^s$ and $\tilde{A}_{It+1}^s = 0$ for all t . Conversely, if the inflation rate is high, i.e., if $\gamma \in (\hat{\gamma}, \bar{\gamma})$, then at the end of every period t , all equity shares are in the hands of investors, i.e., $\tilde{A}_{Dt+1}^s = 0$ and $\tilde{A}_{It+1}^s = A^s$. To understand this result, it is useful to inspect the Euler equations for equity shares. In a stationary equilibrium, (20) reduces to

$$1 \geq \beta\pi R_d^s(\varepsilon^*), \text{ “} = \text{” if } \tilde{A}_{Dt+1}^s > 0, \quad (31)$$

where

$$R_d^s(\varepsilon^*) \equiv \frac{\varepsilon^* y + \phi^s}{\phi^s} = \frac{\bar{\phi}^s}{\phi^s}.$$

Dealers do not wish to hold equity overnight if (31) holds with strict inequality. The equilibrium return to a dealer from holding equity overnight, $R_d^s(\varepsilon^*)$, consists of the expected capital gain from purchasing equity in the second subperiod and reselling it in the OTC market of the following period. Similarly, in a stationary equilibrium (22) reduces to

$$1 \geq \beta\pi R_i^s(\varepsilon^*), \text{ “} = \text{” if } \tilde{A}_{It+1}^s > 0, \quad (32)$$

where

$$\begin{aligned} R_i^s(\varepsilon^*) &\equiv G(\varepsilon^*) \left\{ \delta\theta \frac{\bar{\phi}^s}{\phi^s} + [(1-\delta) + \delta(1-\theta)] \frac{\bar{\varepsilon}^{*l} y + \phi^s}{\phi^s} \right\} + [1 - G(\varepsilon^*)] \frac{\bar{\varepsilon}^{*h} y + \phi^s}{\phi^s} \\ &= \frac{\bar{\varepsilon} y + \delta\theta G(\varepsilon^*) (\varepsilon^* - \bar{\varepsilon}^{*l}) y + \phi^s}{\phi^s}, \end{aligned}$$

$\bar{\varepsilon}^{*l} \equiv \int_{\varepsilon_L}^{\varepsilon^*} \varepsilon \frac{dG(\varepsilon)}{G(\varepsilon^*)}$, and $\bar{\varepsilon}^{*h} \equiv \int_{\varepsilon^*}^{\varepsilon_H} \varepsilon \frac{dG(\varepsilon)}{1-G(\varepsilon^*)}$. Investors do not wish to hold equity overnight if (32) holds with strict inequality. The equilibrium expected return to an investor from holding equity overnight, $R_i^s(\varepsilon^*)$, can be thought of as a weighted average of four gross returns. The first, $\frac{\bar{\phi}^s}{\phi^s}$, is the capital gain of an investor who sells equity in the OTC market at the interdealer market price ($\bar{\phi}^s$), which is what occurs with probability $G(\varepsilon^*) \delta\theta$, i.e., when the investor draws a preference type lower than ε^* (so he wishes to sell in the OTC market), contacts a dealer, and has the bargaining power. The second, $\frac{\bar{\varepsilon}^{*l} y + \phi^s}{\phi^s}$, is the expected equity return to an investor who

¹⁰In a stationary equilibrium, (19) becomes $\gamma \bar{\phi}^s > \beta \max(\bar{\phi}^s, \phi^s) = \beta \bar{\phi}^s$.

wishes to sell in the OTC market but fails to contact a dealer, which occurs with probability $G(\varepsilon^*)(1 - \delta)$. In this case the expected equity payoff consists of the expected value of the period dividend conditional on wanting to sell, $\bar{\varepsilon}^{*l}y$, and the resale value of the equity in the following Walrasian round of trade, ϕ^s . The third, also $\frac{\bar{\varepsilon}^{*l}y + \phi^s}{\phi^s}$, is the expected capital gain of an investor who sells equity in the OTC market at the dealer's expected bid price, $\bar{\varepsilon}^{*l}y + \phi^s$, that (in expected value) reaps all the gains from trade from an investor who wishes to sell, an event that occurs with probability $G(\varepsilon^*)\delta\theta$, i.e., when the investor draws a preference type lower than ε^* (so he wishes to sell in the OTC market), contacts a dealer, and the dealer has the bargaining power. The fourth, $\frac{\bar{\varepsilon}^{*h}y + \phi^s}{\phi^s}$, is the expected equity return of an investor who does not wish to sell in the OTC market and therefore keeps the equity share for a full period, which occurs with probability $1 - G(\varepsilon^*)$. In this case the expected equity payoff consists of the expected value of the period dividend conditional on not wanting to sell, $\bar{\varepsilon}^{*h}y$, and the resale value of the equity in the following Walrasian round of trade, ϕ^s .

From (31) and (32), dealers hold all equity shares overnight (i.e., $\tilde{A}_{Dt+1}^s = A^s$ and $\tilde{A}_{It+1}^s = 0$) if and only if $R_i^s(\varepsilon^*) < R_d^s(\varepsilon^*)$, i.e., if and only if $\bar{\varepsilon} + \delta\theta G(\varepsilon^*)(\varepsilon^* - \bar{\varepsilon}^{*l}) < \varepsilon^*$. This condition is equivalent to

$$\bar{\varepsilon} + \delta\theta \int_{\varepsilon_L}^{\varepsilon^*} G(\varepsilon) d\varepsilon < \varepsilon^*,$$

which is in turn equivalent to $\hat{\varepsilon} < \varepsilon^*$, where $\hat{\varepsilon}$ is defined by (24). Intuitively then, all equity is held by dealers overnight if the marginal preference type that partitions investors between buyers and sellers is large enough (larger than $\hat{\varepsilon}$), else all equity is held by investors overnight, and strictly speaking, dealers only provide brokerage services in the OTC market.¹¹

Given the marginal preference type, ε^* , part (iv) of Proposition 3 gives all asset prices in closed form. The real price of equity (in terms of the homogeneous consumption good) in the Walrasian round of trade, ϕ^s , is given by (30). The dollar price of equity in the OTC market,

¹¹To get some more intuition, notice that

$$\frac{\partial R_i^s(\varepsilon^*)}{\partial \varepsilon^*} = \delta\theta G(\varepsilon^*) \frac{y}{\phi^s} < \frac{y}{\phi^s} = \frac{\partial R_d^s(\varepsilon^*)}{\partial \varepsilon^*}$$

and

$$\frac{\varepsilon_L y + \phi^s}{\phi^s} = R_d^s(\varepsilon_L) < R_i^s(\varepsilon_L) = \frac{\bar{\varepsilon} y + \phi^s}{\phi^s} < \frac{[\delta\theta\varepsilon_H + (1 - \delta\theta)\bar{\varepsilon}]y + \phi^s}{\phi^s} = R_i^s(\varepsilon_H) < R_d^s(\varepsilon_H) = \frac{\varepsilon_H y + \phi^s}{\phi^s}.$$

This reasoning is in terms of ε^* while Proposition 3 is stated in terms of the parameter γ . However, there is a monotonic relationship between γ and ε^* , and as it is shown in the proof of the proposition, $\hat{\varepsilon} < \varepsilon^*$ if and only if $\gamma < \hat{\gamma}$.

p_t , is given by (29). The real price of money (in terms of the homogeneous consumption good) in the Walrasian round of trade, ϕ_t^m , is given by (28). The real price of equity (in terms of the homogeneous consumption good) in the OTC market, $p_t \phi_t^m = \bar{\phi}^s$ is given by (27).

Finally, part (v)(a) states that as the rate of money creation rises toward $\bar{\gamma}$, ε^* approaches the lower bound of the type distribution, ε_L , so no investor wishes to sell equity in the OTC market, and as a result the allocations and prices of the monetary equilibrium approach those of the nonmonetary equilibrium. Part (v)(b) states that as the rate of money creation falls toward β , ε^* increases toward the upper bound of the type distribution, ε_H , so only investors with the highest preference type purchase equity in the OTC market (all other investors wish to sell it).

4.2 Non-intermediated OTC market

In this section we consider the case with $\delta = 0$, i.e., a market in which there are no specialized dealers and all OTC trade is conducted bilaterally among investors. Let

$$\tilde{\gamma} \equiv \beta \left[1 + \alpha \eta \int_{\varepsilon_L}^{\varepsilon_H} \int_{\varepsilon_j}^{\varepsilon_H} \frac{\varepsilon_i - \varepsilon_j}{\varepsilon_j + \frac{\beta\pi}{1-\beta\pi} \bar{\varepsilon}} dG(\varepsilon_i) dG(\varepsilon_j) \right], \quad (33)$$

and define the function $\varphi : [\varepsilon_L, \varepsilon_H] \rightarrow \mathbb{R}$ by

$$\varphi(\varepsilon) \equiv \int_{\varepsilon_L}^{\varepsilon} \int_{\varepsilon_L}^{\varepsilon_j} (\varepsilon_j - \varepsilon_i) dG(\varepsilon_i) dG(\varepsilon_j).$$

Proposition 4 *Assume $\delta = 0$. (i) Dealers are inactive in any equilibrium, and a nonmonetary equilibrium exists for any parametrization. (ii) There is no stationary monetary equilibrium if $\gamma \geq \tilde{\gamma}$. (iii) In the nonmonetary equilibrium there is no trade in the OTC market, and the equity price in the Walrasian market is*

$$\phi^s = \frac{\beta\pi}{1-\beta\pi} \bar{\varepsilon} y.$$

(iv) If $\gamma \in (\beta, \tilde{\gamma})$, then there is one stationary monetary equilibrium and asset prices are

$$\phi^s = \frac{\beta\pi}{1-\beta\pi} [\bar{\varepsilon} + \alpha(1-\eta)\varphi(\varepsilon^c)] y \quad (34)$$

$$Z = (\varepsilon^c y + \phi^s) A^s \quad (35)$$

$$\phi_t^m = \frac{Z}{A_t^m},$$

where, for any $\gamma \in (\beta, \tilde{\gamma})$, $\varepsilon^c \in (\varepsilon_L, \varepsilon_H)$ is the unique solution to

$$\int_{\varepsilon^c}^{\varepsilon_H} \int_{\varepsilon_j}^{\varepsilon_H} \frac{(1 - \beta\pi)(\varepsilon_i - \varepsilon_j)}{(1 - \beta\pi)\varepsilon_j + \beta\pi[\bar{\varepsilon} + \alpha(1 - \eta)\varphi(\varepsilon^c)]} dG(\varepsilon_i) dG(\varepsilon_j) - \frac{\gamma - \beta}{\beta\alpha\eta} = 0. \quad (36)$$

(v) (a) As $\gamma \rightarrow \tilde{\gamma}$, $\varepsilon^c \rightarrow \varepsilon_L$ and $\phi^s \rightarrow \frac{\beta\pi}{1-\beta\pi}\bar{\varepsilon}y$. (b) As $\gamma \rightarrow \beta$, $\varepsilon^c \rightarrow \varepsilon_H$ and $\phi^s \rightarrow \frac{\beta\pi}{1-\beta\pi}[\bar{\varepsilon} + \alpha(1 - \eta)\varphi(\varepsilon_H)]y$.

The interdealer market is inactive when $\delta = 0$, and so are dealers. A stationary monetary equilibrium does not exist if the inflation rate is too large, i.e., if $\gamma \geq \tilde{\gamma}$, and in this case there is no equity trade and the equity price is equal to the expected discounted present value of the dividend. If the inflation rate is low enough, i.e., $\gamma \in (\beta, \tilde{\gamma})$, then a unique stationary equilibrium exists. In this case an investor i starts every period t with a portfolio of money and equity, and he is randomly matched with another investor j during the OTC round of trade. If i 's preference sock is larger than j 's, i.e. if $\varepsilon_j < \varepsilon_i$, then i will want to purchase all of j 's equity holdings. Whether he is able to do so depends the quantity of assets that j holds, which in equilibrium equals A^s , and the dollar price that i has to pay for the equity. In turn, the dollar price will depend on whether investor i or investor j has the bargaining power. If i has the power (this happens with probability η), then the dollar price he pays j for each unit of equity is $p_t^o(\varepsilon_j) = \frac{\varepsilon_j y + \phi^s}{\phi_t^m}$, and since i holds M_t dollars in equilibrium, he can afford to buy all of j 's equity only if $\frac{\varepsilon_j y + \phi^s}{\phi_t^m} A^s \leq M_t$, or equivalently, if $(\varepsilon_j y + \phi^s) A^s \leq Z$ which using (35) can be rewritten as $\varepsilon_j \leq \varepsilon^c$. Thus as γ falls, real balances and ε^c increase, and the investor who wishes to buy equity and has the bargaining power, is cash constrained in a smaller fraction of the bilateral meetings. The value of money depends on the gains from trade of the relatively high valuation investors who buy equity. The equity price (34) on the other hand, reflects the gains from trade of the relatively low valuation investors who sell equity in bilateral transactions ($\alpha(1 - \eta)\varphi(\varepsilon^c)y$ is the expected gain from selling equity to another investor in the OTC round).

5 Asset prices

In this section we study the asset prices reported in Proposition 3 and Proposition 4. In particular, we focus on how they depend on monetary policy, and the degree of OTC frictions as captured by the parameters that regulate trading frequency and the relative bargaining strengths of the various traders.

5.1 Inflation

In a pure-dealer OTC market, the real price of equity in a monetary equilibrium is in part determined by the option available to low-valuation investors to resell the equity to high-valuation investors. As γ increases, equilibrium real money balances fall and the marginal investor type, ε^* , decreases reflecting the fact that under the higher inflation rate, the investor type that was marginal under the lower inflation rate is no longer indifferent between carrying cash and equity out of the OTC market (he prefers equity). Since the marginal investor who prices the equity in the OTC market has a lower valuation, the resale option is smaller, which in turn makes the real equity price (both ϕ^s and $\bar{\phi}^s$) smaller. As expected, the real value of money, ϕ_t^m , also declines with the rate of inflation. These arguments are formalized in the following proposition.

Proposition 5 *Consider the formulation with $\alpha = 0$. In the stationary monetary equilibrium: (i) $\partial\phi^s/\partial\gamma < 0$, (ii) $\partial\bar{\phi}^s/\partial\gamma < 0$, (iii) $\partial Z/\partial\gamma < 0$ and $\partial\phi_t^m/\partial\gamma < 0$.*

In a non-intermediated OTC market, the real price of equity in a monetary equilibrium is also in part determined by the option available to low-valuation investors to resell the equity to high-valuation investors. A higher inflation rate causes real money balances to decline. This reduction in real balances enlarges the set of joint realizations of preference types in bilateral meetings in which the cash constraint binds for high-valuation buyers. In turn, this reduces the value of the marginal preference type, ε^c , of the buyer who is just able to purchase all of the equity from a seller in a bilateral meeting in which the seller has the bargaining power. The result is that ex ante, the period before the OTC round, investors anticipate that the expected gains from selling equity in the OTC market are smaller, and this manifests itself as a smaller equity price in the centralized round of trade. The following proposition formalizes this intuition.

Proposition 6 *Consider the formulation with $\delta = 0$. In the stationary monetary equilibrium: (i) $\partial\phi^s/\partial\gamma < 0$, (ii) $\partial Z/\partial\gamma < 0$ and $\partial\phi_t^m/\partial\gamma < 0$.*

5.2 OTC frictions: trading delays and market power

In a pure-dealer OTC market, $\delta\theta$ is an investor's effective bargaining power in negotiations with dealers. A larger $\delta\theta$ implies a larger gain from trade for low-valuation investors when they off-load the asset onto dealers in the OTC market. This in turn makes investors more willing to

hold equity shares in the centralized market of the previous period, since they anticipate larger gains from selling the equity in case they were to draw a relatively low preference type in the following OTC round. As a result, real equity prices, ϕ^s and $\bar{\phi}^s$, are increasing in δ and θ . If δ increases, money becomes more valuable (both Z and ϕ_t^m increase), provided we focus on a regime in which only investors carry equity overnight.¹² The following proposition formalizes these ideas.

Proposition 7 *Consider the formulation with $\alpha = 0$. In the stationary monetary equilibrium: (i) $\partial\phi^s/\partial(\delta\theta) > 0$, (ii) $\partial\bar{\phi}^s/\partial(\delta\theta) > 0$, (iii) $\partial Z/\partial\delta > 0$ and $\partial\phi_t^m/\partial\delta > 0$, for $\gamma \in (\hat{\gamma}, \bar{\gamma})$.*

The following proposition establishes that in a non-intermediated OTC market, the value of holding equity increases with the bilateral meeting probability, as this increases the probability that the investor may find an opportunity to sell the asset to another investor with higher valuation. Similarly, the value of money increases with α as this increases the probability the investor may be able to use money to buy equity if he were to meet a counterparty with lower valuation.

Proposition 8 *Consider the formulation with $\delta = 0$. In the stationary monetary equilibrium: (i) $\partial\phi^s/\partial\alpha > 0$, (ii) $\partial Z/\partial\alpha > 0$ and $\partial\phi_t^m/\partial\alpha > 0$.*

6 Financial liquidity

In this section we use the theory to study the determinants of standard measures of market liquidity: liquidity provision by dealers, trade volume, and bid-ask spreads. Broker-dealers in OTC markets provide liquidity (*immediacy*) to investors by finding them counterparties for trade, and/or by trading with them out of their own account, effectively becoming their counterparty. Trade volume is a manifestation of the ability of the OTC market to reallocate assets across investors. Bid-ask spreads constitute the main out-of-pocket transaction cost that investors bear in OTC markets. Section 6.1 focuses on dealers' decisions to hold asset inventories with the purpose of becoming trade counterparties for investors. Section 6.2 focuses on the determinants of trade volume. Bid-ask spreads are analyzed in Section 6.3.

¹²Real balances can actually fall with δ for $\gamma \in (\beta, \hat{\gamma})$.

6.1 Liquidity provision by dealers

To simplify the exposition, here we focus on the formulation with $\alpha = 0$. The following result characterizes the effect of inflation on dealers' provision of liquidity by accumulating assets.

Proposition 9 *Consider the formulation with $\alpha = 0$. In the stationary monetary equilibrium: (i) dealers' provision of liquidity by accumulating assets, i.e., A_D^s , is nonincreasing in the inflation rate. (ii) For any γ close to β , dealers' provision of liquidity by accumulating assets is nonmonotonic in $\delta\theta$, i.e., $A_D^s = 0$ for $\delta\theta$ close to 0 and close to 1, but $A_D^s > 0$ for intermediate values of $\delta\theta$.*

To understand part (i) of Proposition 9, recall the discussion that followed Proposition 3. The expected return from holding equity is larger for investors than for dealers with high inflation ($\gamma > \hat{\gamma}$) because in that case the expected resale value of equity in the OTC market is relatively low, and dealers only buy equity to resell in the OTC market, while investors also buy it with the expectation of getting utility from the dividend flow. For low inflation ($\gamma < \hat{\gamma}$), dealers value equity more than investors because the OTC resale value is high and they have a higher probability of making capital gains from reselling than investors, and this trading advantage more than compensates for the fact that investors enjoy the additional utility from the dividend flow. Part (ii) of Proposition 9 states that given a low enough rate of inflation, dealers' incentives to hold equity inventories overnight depend nonmonotonically on the degree of OTC frictions as measured by $\delta\theta$. In particular, dealers will not hold inventories if $\delta\theta$ is either very small or very large. If $\delta\theta$ is close to zero, few investors contact the interdealer market, and this makes the equity price in the OTC market very low, which in turn implies too small a capital gain to induce dealers to hold equity overnight. Conversely, if $\delta\theta$ is close to one, then the dealer is getting no gains from trade in the OTC market and consequently he does not find it worthwhile to invest in equity to resell in the OTC market.

6.2 Volume

First consider the case with $\alpha = 0$. According to Lemma 2, any investor with $\varepsilon < \varepsilon_t^*$ who has a trading opportunity in the OTC market, sells all his equity. Hence in a stationary equilibrium, the quantity of assets sold by investors to dealers in the OTC market is $Q^- = \delta G(\varepsilon^*) A_I^s$. From Lemma 2 we also know that an investor with $\varepsilon > \varepsilon_t^*$ who contacts a dealer will buy equity, and that the quantity he buys depends on whether the investor or the dealer has the bargaining

power. If the investor has the bargaining power then he purchases A_t^m/p_t equity shares. Thus the volume of assets traded by such investors is $Q^{+*} = \delta\theta[1 - G(\varepsilon^*)]A_t^m/p_t$. If instead the dealer has the bargaining power, then the investor purchases $A_t^m/p_t^o(\varepsilon)$ equity shares. Therefore the volume of assets traded by such investors is $Q^+ = \delta(1 - \theta) \int_{\varepsilon^*}^{\varepsilon^H} [A_t^m/p_t^o(\varepsilon)] dG(\varepsilon)$. The total quantity of equity shares traded in the OTC market is $\mathcal{V} = Q^- + Q^{+*} + Q^+$, or equivalently¹³

$$\mathcal{V} = \pi\tilde{A}_D^s + 2\delta G(\varepsilon^*)(A^s - \pi\tilde{A}_D^s). \quad (37)$$

Trade volume \mathcal{V} depends on γ and θ only indirectly, through ε^* . A decrease in γ or an increase in θ , increases the expected return to holding money, which makes more investors willing to sell equity for money in the OTC market, i.e., ε^* increases and so does trade volume. The indirect (through ε^*) positive effect on \mathcal{V} of an increase in the investors' trade probability δ is similar to an increase in θ , but in addition, δ directly increases trade volume since with a higher δ more investors are able to trade in the OTC market. These results are summarized in the following proposition.

Proposition 10 *Consider the formulation with $\alpha = 0$. In the stationary monetary equilibrium: (i) $\partial\mathcal{V}/\partial\gamma < 0$, (ii) $\partial\mathcal{V}/\partial\theta > 0$ and $\partial\mathcal{V}/\partial\delta > 0$.*

Next consider the formulation with $\delta = 0$. According to Lemma 3, the quantity traded in a meeting between two investors depends on whether the buyer or the seller of equity has the bargaining power. Suppose that investor i has preference type ε_i and investor j has preference type $\varepsilon_j < \varepsilon_i$. If investor i (in this case the buyer) makes the offer, then he purchases $\min\{A_t^m/p_t^o(\varepsilon_j), A^s\} = \mathbb{I}_{\{\varepsilon_j \leq \varepsilon^c\}}A^s + \mathbb{I}_{\{\varepsilon^c < \varepsilon_j\}} \frac{Z}{\varepsilon_j y + \phi^s}$ equity shares. Conversely, if investor j has the bargaining power, then investor i purchases $\min\{A_t^m/p_t^o(\varepsilon_i), A^s\} = \mathbb{I}_{\{\varepsilon_i \leq \varepsilon^c\}}A^s + \mathbb{I}_{\{\varepsilon^c < \varepsilon_i\}} \frac{Z}{\varepsilon_i y + \phi^s}$ equity shares. Hence the total quantity of equity shares traded in the OTC market is

$$\begin{aligned} \tilde{\mathcal{V}} = \alpha & \left\{ \eta \left[\int_{\varepsilon_L}^{\varepsilon^c} [1 - G(\varepsilon_j)] dG(\varepsilon_j) + \int_{\varepsilon^c}^{\varepsilon^H} \int_{\varepsilon_j}^{\varepsilon^H} \frac{\varepsilon^c y + \phi^s}{\varepsilon_j y + \phi^s} dG(\varepsilon_i) dG(\varepsilon_j) \right] \right. \\ & \left. + (1 - \eta) \left[\int_{\varepsilon_L}^{\varepsilon^c} G(\varepsilon_i) dG(\varepsilon_i) + \int_{\varepsilon^c}^{\varepsilon^H} \int_{\varepsilon_L}^{\varepsilon_i} \frac{\varepsilon^c y + \phi^s}{\varepsilon_i y + \phi^s} dG(\varepsilon_j) dG(\varepsilon_i) \right] \right\} A^s. \end{aligned}$$

¹³To obtain (37) we used the clearing condition for the interdealer market, $A_D^s + Q^- = Q^+ + Q^{+*}$ which implies $\mathcal{V} = Q^- + Q^{+*} + Q^+ = A_D^s + 2Q^-$. Also, note that \mathcal{V} is trade volume in the OTC market, but since every equity share traded in the first subperiod gets retraded in the second subperiod, total trade volume in the whole time period equals $2\mathcal{V}$.

In trades where the investor with no bargaining power has preference type $\varepsilon < \varepsilon^c$, all the equity holdings of the seller, A^s , are traded. In meetings where the investor with no bargaining power has preference type $\varepsilon > \varepsilon^c$, the cash constraint of the buyer binds, and only $\frac{Z}{\varepsilon_j y + \phi^s} = \frac{(\varepsilon^c y + \phi^s) A^s}{\varepsilon_j y + \phi^s}$ equity shares are traded. Notice that inflation only affects $\tilde{\mathcal{V}}$ indirectly, through its effect on ε^c (or equivalently, real balances, Z). Higher inflation reduces the value of real balances and this implies that the cash constraint will bind in more trades, causing trade volume to decline along the intensive margin (i.e., by reducing the quantity of equity traded in trades in which the agent with no bargaining power has relatively high valuation for the dividend). An increase in the contact probability α increases $\tilde{\mathcal{V}}$ along the extensive margin (more meetings among investors naturally result in larger trade volume), but an increase in α also increases real balances and therefore induces an increase in trade volume along the intensive margin. This intuition is formalized in the following proposition.

Proposition 11 *Consider the formulation with $\delta = 0$. In the stationary monetary equilibrium: (i) $\partial \tilde{\mathcal{V}} / \partial \gamma < 0$, and (ii) $\partial \tilde{\mathcal{V}} / \partial \alpha > 0$.*

6.3 Spreads

Focus on the formulation with $\alpha = 0$. Corollary 1 shows that when dealers with bargaining power execute trades on behalf of their investors, they charge an ask price $p_t^o(\varepsilon) > p_t$ to investors with $\varepsilon > \varepsilon^*$ who wish to buy equity, and pay a bid price $p_t^o(\varepsilon) < p_t$ to investors with $\varepsilon < \varepsilon^*$ who wish to sell equity. Thus in any transaction with an investor with preference type ε , the dealer earns a nominal spread $\mathcal{S}_t^m(\varepsilon) = |p_t^o(\varepsilon) - p_t|$. Define the real spread $\mathcal{S}(\varepsilon) = \mathcal{S}_t^m(\varepsilon) / p_t$, i.e.,

$$\mathcal{S}(\varepsilon) = \frac{|\varepsilon - \varepsilon^*| y}{\varepsilon^* y + \phi^s}.$$

The average real spread is $\bar{\mathcal{S}} = \int \mathcal{S}(\varepsilon) dG(\varepsilon)$, i.e.,

$$\bar{\mathcal{S}} = \left[\int_{\varepsilon_L}^{\varepsilon^*} (\varepsilon^* - \varepsilon) dG(\varepsilon) + \int_{\varepsilon^*}^{\varepsilon_H} (\varepsilon - \varepsilon^*) dG(\varepsilon) \right] \frac{y}{\varepsilon^* y + \phi^s}.$$

The change in the average spread in response to changes in γ , δ or θ , are ambiguous in general. The reason is that $\mathcal{S}(\varepsilon)$ is decreasing in ε^* for buyers with $\varepsilon > \varepsilon^*$, but may be increasing in ε^* for investors who wish to sell (i.e., those with $\varepsilon < \varepsilon^*$).

6.4 Speculation

According to Proposition 3 and Proposition 4, in a monetary equilibrium the equity price, ϕ^s , is larger than the expected present discounted value that any agent assigns to the dividend stream, i.e., $\phi_N^s \equiv [\beta\pi/(1-\beta\pi)]\bar{\varepsilon}y$. It is commonplace to define the *fundamental value* of the asset to be the expected present discounted value of the dividend stream, and to call any transaction value in excess of this fundamental value, a *bubble*.¹⁴ One could argue, of course, that the relevant notion of “fundamental value” should be calculated through market aggregation of diverse investor valuations, and taking into account the monetary policy stance as well as all the details of the market structure in which the asset is traded (such as the frequency of trading opportunities and the degree of market power of financial intermediaries) which ultimately also factor into the asset price in equilibrium. In any case, to avoid semantic controversies, we follow Harrison and Kreps (1978) and call the value of the asset in excess of the expected present discounted value of the dividend, the “speculative premium” which investors are willing to pay in anticipation of the capital gains they will reap when reselling the asset to investors with higher valuations in the future. So like Harrison and Kreps (1978), we say that investors exhibit *speculative behavior* if the right to resell a stock makes them willing to pay more for it than they would pay if obliged to hold it forever. Investors exhibit speculative behavior in the sense that they buy with the expectation to resell, and naturally the asset price incorporates the value of this option to resell: investors are willing to pay more for the asset than they would pay if obliged to hold it forever.

Consider the case pure-dealer case (i.e., $\alpha = 0$). According to Proposition 3, in a monetary equilibrium the speculative premium is

$$\mathcal{P} = \begin{cases} \frac{\beta\pi}{1-\beta\pi} (\varepsilon^* - \bar{\varepsilon}) y & \text{if } \beta < \gamma \leq \hat{\gamma} \\ \frac{\beta\pi}{1-\beta\pi} \delta\theta y \int_{\varepsilon_L}^{\varepsilon^*} G(\varepsilon) d\varepsilon & \text{if } \hat{\gamma} < \gamma < \bar{\gamma}. \end{cases}$$

Since $\partial\varepsilon^*/\partial\gamma < 0$ (Corollary 2), it is immediate that the speculative premium is decreasing in the rate of inflation. Intuitively, anticipated inflation reduces the real money balances used to finance asset trading, which limits the ability of high-valuation traders to purchase the asset from low-valuation traders. As a result, the speculative premium is decreasing in γ . Since $\partial\varepsilon^*/\partial(\delta\theta) > 0$ (see the proof of Proposition 7), the speculative premium is increasing in δ and θ . Intuitively, the speculative premium is the value of the option to resell the equity to

¹⁴See, e.g., Barlevy (2007), Scheinkman and Xiong (2003), and Xiong (2013) who discusses Harrison and Kreps’ paper in the context of what he calls the “resale option theory of bubbles.”

a higher valuation investor in the future, and the value of this resale option to the investor increases with the probability δ that the investor gets a trading opportunity in an OTC trading round and with the probability θ that he can capture the gains from trade in those trades. Interestingly, notice that the signs of these effects are exactly the opposite than they are for trade volume. So in low-inflation regimes, the model predicts large trade volume and a large speculative premium. The following proposition summarizes these results.

Proposition 12 *Consider the formulation with $\alpha = 0$. In the stationary monetary equilibrium: (i) $\partial\mathcal{P}/\partial\gamma < 0$, and (ii) $\partial\mathcal{P}/\partial(\delta\theta) > 0$.*

Consider a non-intermediated OTC market (i.e., $\delta = 0$). According to Proposition 4, in a monetary equilibrium the speculative premium is

$$\tilde{\mathcal{P}} = \frac{\beta\pi}{1 - \beta\pi} \alpha (1 - \eta) y\varphi(\varepsilon^c).$$

Higher inflation reduces real balances and therefore ε^c , which reduces the expected resale value of equity in the OTC market, so $\tilde{\mathcal{P}}$ decreases with inflation. An increase in the trade probability α has a positive direct effect on $\tilde{\mathcal{P}}$ (increase in the meeting probability) and also an indirect positive effect (α increases ε^c which in turn increases $\tilde{\mathcal{P}}$). These effects are summarized below.

Proposition 13 *Consider the formulation with $\delta = 0$. In the stationary monetary equilibrium: (i) $\partial\tilde{\mathcal{P}}/\partial\gamma < 0$, and (ii) $\partial\tilde{\mathcal{P}}/\partial\alpha > 0$.*

7 Endogenous trading delays

In this section we endogenize the supply of intermediation services and the length of the trading delays by allow for free entry of dealers. This formalizes the notion that a dealer's profit depends on the competition for order flow that he faces from other dealers.

In this section we use δ to denote a continuously differentiable function of the measure of dealers in the market, v , i.e., $\delta : \mathbb{R}_+ \rightarrow [0, 1]$, and let $\kappa(v) \equiv \delta(v)/v$. We assume $\delta'(v) > 0$, $\kappa'(v) < 0$, and $\delta''(v) < 0$. We also specify $\delta(0) = \lim_{v \rightarrow \infty} \kappa(v) = 0$, and $\lim_{v \rightarrow \infty} \delta(v) = \kappa(0) = 1$. These assumptions capture the notion that if the measure of dealers, v , is larger, then each investor contacts dealers faster, while the order flow for each individual dealer decreases.¹⁵ There is a large measure of dealers who can choose to participate in the market. Dealers who

¹⁵The matching function $\delta(v) = 1 - e^{-v}$ is an example that satisfies the maintained assumptions.

wish to provide intermediation services in the OTC market of period $t + 1$ must incur a real resource cost $k > 0$ in the second subperiod of period t , i.e., right before they can participate in the OTC round of trade. The cost k is in terms of the homogeneous good, and represents the ongoing expenses of running the dealership business.

7.1 Efficiency

With the notation introduced in Section 3, the planner's problem for the economy with free entry (and $\alpha = 0$) consists of choosing a nonnegative allocation

$$\left\{ v_t, \tilde{a}_{tD}, a'_{tD}, c_{tD}, h_{tD}, \tilde{a}_{tI}, a'_{tI}, [c_{tI}(\varepsilon_i), h_{tI}(\varepsilon_i)]_{\varepsilon \in [\varepsilon_L, \varepsilon_H]} \right\}_{t=0}^{\infty},$$

to maximize

$$\sum_{t=0}^{\infty} \beta^t \left\{ \delta(v_t) \int_{[\varepsilon_L, \varepsilon_H]} \varepsilon y a'_{tI}(d\varepsilon) + [1 - \delta(v_t)] \bar{\varepsilon} y a_{tI} + \int_{\varepsilon_L}^{\varepsilon_H} [c_{tI}(\varepsilon) - h_{tI}(\varepsilon)] dG(\varepsilon) + (c_{tD} - h_{tD}) v_t \right\}$$

subject to

$$v_t \tilde{a}_{tD} + \tilde{a}_{tI} \leq A^s \tag{38}$$

$$v_t a'_{tD} + \delta(v_t) \int_{[\varepsilon_L, \varepsilon_H]} a'_{tI}(d\varepsilon) \leq v_t a_{tD} + \delta(v_t) a_{tI} \tag{39}$$

$$\int_{\varepsilon_L}^{\varepsilon_H} c_{tI}(\varepsilon) dG(\varepsilon) + c_{tD} v_t + k v_{t+1} \leq \int_{\varepsilon_L}^{\varepsilon_H} h_{tI}(\varepsilon) dG(\varepsilon) + v_t h_{tD}, \tag{40}$$

(6) and (7). The following proposition characterizes the solution to the planner's problem.

Proposition 14 *The efficient allocation for the model with free entry of dealers has*

$$\beta \delta'(v_t) (\varepsilon_H - \bar{\varepsilon}) y (1 - \pi) A^s - k \leq 0, \text{ " = " if } v_t > 0, \tag{41}$$

$\tilde{a}_{tD} = (A^s - \tilde{a}_{tI})/v_t = A^s/v_t$, and $a'_{ti}(E) = \mathbb{I}_{\{\varepsilon_H \in E\}} [\pi/\delta(v_t) + (1 - \pi)] A^s$, where $\mathbb{I}_{\{\varepsilon_H \in E\}}$ is an indicator function that takes the value 1 if $\varepsilon_H \in E$, and 0 otherwise, for any $E \in \mathcal{F}([\varepsilon_L, \varepsilon_H])$.

7.2 Equilibrium

An equilibrium with free entry is characterized by the same equations that characterize an equilibrium in the baseline model of Section 2 (replacing δ with $\delta(v_t)$ and κ with $\kappa(v_t)$),

plus the following condition, which must hold in an equilibrium in which dealers are free to participate in the OTC of any period t

$$W_t^D(\mathbf{0}) - k \leq 0, \text{ with “} = \text{” if } v_{t+1} > 0. \quad (42)$$

For each t , the free-entry condition (42) can be used to determine the additional unknown v_{t+1} . Lemma 8 in the appendix shows that (42) can be written as

$$\Phi_{t+1} - k \leq 0, \text{ with “} = \text{” if } v_{t+1} > 0 \quad (43)$$

for all t , where

$$\begin{aligned} \Phi_{t+1} \equiv & \beta\kappa(v_{t+1})(1-\theta)\bar{\phi}_{t+1} \left[\int_{\varepsilon_L}^{\varepsilon_{t+1}^*} \frac{(\varepsilon_{t+1}^* - \varepsilon)y}{\varepsilon_{t+1}^* y + \phi_{t+1}^s} p_{t+1} A_{It+1}^s dG(\varepsilon) \right. \\ & \left. + \int_{\varepsilon_{t+1}^*}^{\varepsilon_H} \frac{(\varepsilon - \varepsilon_{t+1}^*)y}{\varepsilon y + \phi_{t+1}^s} A_{It+1}^m dG(\varepsilon) \right] \end{aligned} \quad (44)$$

is a dealer's discounted expected income from intermediation in the OTC market of period $t+1$.

In order to interpret Φ_{t+1} , it is useful to define $Q_{t+1}(\varepsilon) \equiv \frac{1}{p_{t+1}^o(\varepsilon)} A_{It+1}^m$,

$$\begin{aligned} \mathcal{S}_{t+1}^b & \equiv \int_{\varepsilon_L}^{\varepsilon_{t+1}^*} [p_{t+1} - p_{t+1}^o(\varepsilon)] A_{It+1}^I \frac{dG(\varepsilon)}{G(\varepsilon_{t+1}^*)} \\ \mathcal{S}_{t+1}^a & \equiv \int_{\varepsilon_{t+1}^*}^{\varepsilon_H} [p_{t+1}^o(\varepsilon) - p_{t+1}] Q_{t+1}(\varepsilon) \frac{dG(\varepsilon)}{1 - G(\varepsilon_{t+1}^*)}, \end{aligned}$$

where $p_{t+1}^o(\varepsilon)$ (defined in (16)) is the nominal equity price that an investor faces when trading with a dealer with bargaining power. (So $p_{t+1}^o(\varepsilon)$ is a nominal bid price for investors with $\varepsilon < \varepsilon^*$, and the nominal ask price for investors with $\varepsilon > \varepsilon^*$.) With this notation, (44) can be written as

$$\Phi_{t+1} = \beta\kappa(v_{t+1})(1-\theta) \left\{ G(\varepsilon_{t+1}^*) \mathcal{S}_{t+1}^b + [1 - G(\varepsilon_{t+1}^*)] \mathcal{S}_{t+1}^a \right\} \bar{\phi}_{t+1}.$$

Consider a dealer who has the bargaining power in a meeting with a random investor with preference type $\varepsilon < \varepsilon_{t+1}^*$. In equilibrium, this investor wishes to sell the A_{It+1}^s equity shares that he is holding to the dealer, who pays the investor $p_{t+1}^o(\varepsilon)$ dollars per share, therefore earning $p_{t+1} - p_{t+1}^o(\varepsilon)$ dollars per share, for a total intermediation profit equal to $[p_{t+1} - p_{t+1}^o(\varepsilon)] A_{It+1}^s$ dollars. Hence \mathcal{S}_{t+1}^b is the expected value of the dealer's nominal profit from intermediation, conditional on having contacted an investor who wants to sell equity. Next consider a dealer who

has the bargaining power in a meeting with a random investor with preference type $\varepsilon > \varepsilon_{t+1}^*$. In equilibrium, this investor wishes to use spend all his money holdings, A_{It+1}^m , to purchase shares from the dealer. The dealer charges the investor $p_{t+1}^o(\varepsilon)$ dollars per share, so at this price the investor wishes to buy $Q_{t+1}(\varepsilon)$ shares from the dealer. The dealer earns $p_{t+1}^o(\varepsilon) - p_{t+1}$ dollars per share in this transaction, for a total intermediation profit equal to $[p_{t+1}^o(\varepsilon) - p_{t+1}] Q_{t+1}(\varepsilon)$ dollars. Hence \mathcal{S}_{t+1}^a is the expected value of the dealer's nominal profit from intermediation, conditional on having contacted an investor who wants to buy equity. Since the dealer can rebalance his portfolio of equity and cash freely in the interdealer market, the expected real value of his profit from intermediation is $\mathcal{S}_{t+1}^a \bar{\phi}_{t+1}$, conditional on having contacted a buyer, and $\mathcal{S}_{t+1}^b \bar{\phi}_{t+1}$ conditional on having met a seller. Hence $\beta \{G(\varepsilon_{t+1}^*) \mathcal{S}_{t+1}^b + [1 - G(\varepsilon_{t+1}^*)] \mathcal{S}_{t+1}^a\} \bar{\phi}_{t+1}$ is the discounted expected real income from intermediation to a dealer in period $t+1$, conditional on his contacting an investor in the OTC market (with probability $\kappa(v_{t+1})$), and conditional on the dealer having the bargaining power in the bilateral negotiation with the investor (with probability $1 - \theta$).

To simplify the exposition hereafter we specialize the analysis to the pure-dealer model with $\alpha = 0$. In a stationary equilibrium, $A_{Dt}^s = A_D^s$, $A_{It}^s = A_I^s$, $v_t = v$, $\phi_t^s = \phi^s$, $p_t \phi_t^m \equiv \bar{\phi}^s$, $\varepsilon_t^* = (\bar{\phi}^s - \phi^s)/y \equiv \varepsilon^*$, $\phi_t^m A_{It}^m \equiv Z$, $\phi_t^m A_{Dt}^m \equiv Z_D$, and $\phi_t^m / \phi_{t+1}^m = p_{t+1}/p_t = A_{t+1}^m / A_t^m = \gamma$, for all t , so $\Phi_t = \Phi$ for all t . A stationary equilibrium with entry is summarized by a sequence of nominal prices $\{\phi_t^m, p_t\}$, a vector of real asset prices, $(\phi^s, \bar{\phi}^s)$, a sequence of money holdings $\{A_{Dt}^m, A_{It}^m\}$, a vector of equity holdings (A_{Dt}^s, A_{It}^s) , real balances, Z , and a threshold ε^* that satisfy the conditions reported in Proposition 3 (with δ replaced by $\delta(v)$), together with a number of active dealers, v , that satisfies (45)

$$\Phi - k \leq 0, \text{ with “} = \text{” if } v > 0, \quad (45)$$

where

$$\Phi \equiv \beta \kappa(v) (1 - \theta) \left[A_I^s \int_{\varepsilon_L}^{\varepsilon^*} (\varepsilon^* - \varepsilon) dG(\varepsilon) + Z \int_{\varepsilon^*}^{\varepsilon_H} \frac{\varepsilon - \varepsilon^*}{\varepsilon y + \phi^s} dG(\varepsilon) \right] y. \quad (46)$$

Proposition 15 *Assume $k < \beta(1 - \theta)(\varepsilon_H - \bar{\varepsilon})y(1 - \pi)A^s$. The allocation implemented by the stationary monetary equilibrium converges to the symmetric efficient allocation as $\gamma \rightarrow \beta$, provided the bargaining power of dealers satisfies $1 - \theta = 1 - \frac{-\kappa'(v)v}{\kappa(v)}$.*

Proposition 15 establishes that with dealer entry, the Friedman rule achieves efficiency

only if the Hosios (1990) condition is satisfied.¹⁶ Under the Hosios condition, the share of the gain from trade that is apportioned by dealers in bilateral meetings, $1 - \theta$, equals the elasticity of the aggregate number of meetings with respect to the number of participating dealers, $1 + \kappa'(v)v/\kappa(v)$ (i.e., the contribution that the marginal dealer makes to the matching process). Thus generically, deviations from the Friedman rule could be welfare enhancing in the absence of other policies designed to restore the efficiency of the dealers' entry decision.

8 Related literature

The model builds on two strands of literature: the Search Theory of Money, and search-based models of financial trade in OTC markets. Specifically, we embed an OTC financial trading arrangement similar to Duffie et al. (2005) into a Lagos and Wright (2005) economy.

In the standard formulations of the Lagos-Wright framework, money (and sometimes other assets) are used as payment instruments to purchase consumption goods in bilateral markets mediated by search. We instead posit that money is used as a medium of exchange in OTC markets for financial assets. In the standard monetary model, money and other liquid assets help to allocate goods from producers to consumers, while in our current formulation, money helps to allocate financial assets among traders with heterogeneous valuations. This shift in the nature of the gains from trade offers a different perspective that delivers novel insights on the interaction between monetary policy and financial markets.

As a model of financial trade, the main strength of Duffie et al. (2005) is perhaps its realistic OTC market structure consisting of an interdealer market and bilateral negotiated trades between investors, and between investors and dealers. In Duffie et al. (2005) agents who wish to buy assets pay sellers with linear-utility transfers. In addition, utility transfers from buyers to sellers are unconstrained, so buyers effectively face no budget constraints in the financial market. Our formulation keeps the appealing market structure of Duffie et al. (2005) but improves upon its stylized model of financial transactions by considering traders who face standard budget constraints and use fiat money to purchase assets. These modifications make the standard OTC formulation amenable to general equilibrium analysis, and deliver a natural transmission mechanism through which monetary policy influences financial markets.

Our work is related to previous studies, e.g., Geromichalos et al. (2007), Jacquet and Tan

¹⁶This is a standard result in the monetary search literature, see, e.g., Berentsen et al. (2007).

(2010), Lagos and Rocheteau (2008), Lagos (2010a, 2010b, 2011), Lester et al. (2012), Nosal and Rocheteau (2013), that introduce a real asset that can (at least to some degree) be used along with money as a medium of exchange for consumption goods in variants of Lagos and Wright (2005). These papers identify the liquidity value of the asset with its usefulness in exchange, and find that when the asset is valuable as a medium of exchange, this manifests itself as a “liquidity premium” that makes the real asset price higher than the expected present discounted value of its financial dividend. High anticipated inflation reduces real money balances; this tightens bilateral trading constraints, which in turn increases the liquidity value and the real price of the asset. In contrast, we find that real asset prices are decreasing in the rate of anticipated inflation. There are some models that also build on Lagos and Wright (2005) where agents can use a real asset as collateral to borrow money that they subsequently use to purchase consumption goods. In those models, anticipated inflation reduces the demand for real balances which can in turn reduce the real price of the collateral asset needed to borrow money (see, e.g., He et al., 2012, and Li and Li, 2012). The difference is that in our setup inflation reduces the real asset price by constraining the reallocation of the financial asset from investors with low valuations to investors with relatively high valuations.

We share with two recent papers, Geromichalos and Herrenbrueck (2012) and Trejos and Wright (2012), the interest in bringing models of OTC trade in financial markets within the realm of modern monetary general equilibrium theory. Trejos and Wright (2012) offer an in-depth analysis of a model that nests Duffie et al. (2005) and the prototypical “second generation” monetary search model with divisible goods, indivisible money and unit upper bound on individual money holdings (e.g., Shi, 1995 or Trejos and Wright, 1995). Trejos and Wright emphasize the different nature of the gains from trade in both classes of models. In monetary models agents value consumption goods differently and use assets to buy goods, while in Duffie et al. (2005) agents trade because they value assets differently, and goods which are valued the same by all investors are used to pay for asset purchases. In our formulation there are gains from trading assets, as in Duffie et al. (2005), but agents pay with money, as in standard monetary models. Another difference with Trejos and Wright (2012) is that rather than assuming indivisible assets and unit upper bound on individual asset holdings as in Shi (1995), Trejos and Wright (1995) and Duffie et al. (2005), we work with divisible assets and unrestricted portfolios, as in Lagos and Wright (2005) and Lagos and Rocheteau (2009).

Geromichalos and Herrenbrueck (2012) is methodologically closer to our work. They extend

Lagos and Wright (2005) by adding a real asset that by assumption cannot be used to purchase goods in the decentralized market (as usual, at the end of every period agents choose next-period money and asset portfolios in a centralized market). The twist is that at the very beginning of every period, agents learn whether they will want to buy or sell in the subsequent decentralized market for consumption goods, and at that point they have access to a bilateral search market where they can retrade money and assets. This market allows agents to rebalance their positions depending on their need for money, e.g., those who will be buyers seek to buy money and sell assets. So although assets cannot be directly used to purchase consumption goods as in Geromichalos et al. (2007) or Lagos and Rocheteau (2008), assets can buy goods indirectly, i.e., by exchanging them for cash in the additional bilateral trading round at the beginning of the period. Geromichalos and Herrenbrueck use the model to revisit the link between asset prices and inflation. Their core results (they have several others) are similar to those obtained in models where the asset can be used directly as a medium of exchange for consumption goods, i.e., the asset carries a liquidity premium and higher inflation increases the real asset price in the centralized market. There are relevant differences between our work and Geromichalos and Herrenbrueck (2012). In our setup money allows agents to exploit gains from trading assets (as in Duffie et al.) rather than consumption goods (as in the money literature), which is why we find that inflation reduces asset prices. Also, we consider an OTC market with dealers who act as intermediaries, which allows us to study the effect of monetary policy on bid-ask spreads and dealers' incentives to supply liquidity services—the dimensions of financial liquidity that search based theories of OTC markets seek to explain.

The fact that the equilibrium asset price is larger than the expected present discounted value that any agent assigns to the dividend stream is reminiscent of the literature on speculative trading that can be traced back to Harrison and Kreps (1978). As in Harrison and Kreps, in our model speculation arises because traders have heterogeneous asset valuations that change over time: investors are willing to pay for the asset more than the present discounted value that they assign to the dividend stream, in anticipation of the capital gain they expect to obtain when reselling the asset to higher-valuation investors in the future. In terms of differences, in Harrison and Kreps traders have heterogeneous stubborn beliefs about the stochastic dividend process, and their motive for trading is that they all believe (at least some of them mistakenly) that by trading the asset they can profit at the expense of others. In our formulation traders simply have stochastic heterogeneous valuations for the dividend, as in Duffie et al. (2005). Our

model offers a new angle on the speculative premium embedded in the asset price, by showing how it depends on the underlying financial market structure and the prevailing monetary policy that jointly determine the likelihood and profitability of future resale opportunities.

9 Conclusion

We have developed a model in which money is used as a medium of exchange in financial transactions that take place in over-the-counter markets. We have used the theory to study the role that money and monetary policy play in shaping asset prices and the performance of OTC markets. In a monetary equilibrium, the asset price contains a speculative premium that makes the price larger than the expected present discounted value that any agent assigns to the dividend stream. We have also used the model to study the monetary and microstructure determinants of standard measures of financial liquidity of OTC markets, such as the size of bid-ask spreads, the volume of trade, and the incentives of dealers to supply immediacy, both by choosing to participate in the market-making activity, as well as by holding asset inventories on their own account. We have shown that there exist multiple equilibria as well as dynamic equilibria that resemble expectation driven “market crashes” or “liquidity crises” in which market liquidity suddenly dries up: dealers drastically reduce their market-making activity, trade volume drops, bid-ask spreads widen, and asset prices fall abruptly. We have also shown that high-inflation regimes are more prone to crises, and that when in a liquidity crisis, a low inflation (low nominal interest) policy can help markets recover.

A Proofs

Proof of Proposition 1. The choice variable a'_{tD} does not appear in the Planner's objective function, so $a'_{tD} = 0$ at an optimum. Also, (5) must bind for every t at an optimum, so the planner's problem is equivalent to

$$\begin{aligned} & \max_{\{\tilde{a}_{tD}, \tilde{a}_{tI}, \underline{a}_{tib(i)}, a'_{tI}\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t \left[\delta \int_{[\varepsilon_L, \varepsilon_H]} \varepsilon a'_{tI}(d\varepsilon) + (1 - \alpha - \delta) \bar{\varepsilon} a_{tI} \right. \\ & \left. + \int_{\mathcal{B}_t} \int \int \mathbb{I}_{\{i \leq b(i)\}} \left[\varepsilon_i \underline{a}_{tib(i)}(\varepsilon_i, \varepsilon_{b(i)}) + \varepsilon_{b(i)} \underline{a}_{tb(i)i}(\varepsilon_{b(i)}, \varepsilon_i) \right] dG(\varepsilon_i) dG(\varepsilon_{b(i)}) di \right] y \\ & \text{s.t. (2), (3), (6), (7) and } \delta \int_{[\varepsilon_L, \varepsilon_H]} a'_{tI}(d\varepsilon) \leq v a_{tD} + \delta a_{tI}. \end{aligned}$$

Let W^* denote the maximum value of this problem. Then clearly, $W^* \leq \bar{W}^*$, where

$$\begin{aligned} \bar{W}^* &= \max_{\{\tilde{a}_{tD}, \tilde{a}_{tI}\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t \left[\int \int \int_{\mathcal{B}_t} \mathbb{I}_{\{i \leq b(i)\}} \max(\varepsilon, \varepsilon') 2\tilde{a}_{tI} dG(\varepsilon) dG(\varepsilon') di \right. \\ & \left. + \varepsilon_H (v \tilde{a}_{tD} + \delta \tilde{a}_{tI}) + (1 - \alpha - \delta) \bar{\varepsilon} \tilde{a}_{tI} \right] \pi y + w, \end{aligned}$$

s.t. (3), where $w \equiv \frac{1}{1-\beta} [\alpha \varepsilon_B + \delta \varepsilon_H + (1 - \alpha - \delta) \bar{\varepsilon}] (1 - \pi) A^s y$ and

$$\varepsilon_B \equiv \int \int \max(\varepsilon, \varepsilon') dG(\varepsilon) dG(\varepsilon').$$

Rearrange this expression and substitute (3) (at equality) to obtain

$$\begin{aligned} \bar{W}^* &= \max_{\{\tilde{a}_{tI}\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t \{ \varepsilon_H A^s + [\alpha \varepsilon_B + \delta \varepsilon_H + (1 - \alpha - \delta) \bar{\varepsilon} - \varepsilon_H] \tilde{a}_{tI} \} \pi y + w \\ &= \frac{1}{1-\beta} \{ \pi \varepsilon_H + (1 - \pi) [\alpha \varepsilon_B + \delta \varepsilon_H + (1 - \alpha - \delta) \bar{\varepsilon}] \} y A^s. \end{aligned}$$

The allocation $\tilde{a}_{tD} = A^s/v$, $\tilde{a}_{tI} = 0$, and $\underline{a}_{tib(i)}(\varepsilon_i, \varepsilon_{b(i)}) = \mathbb{I}_{\{\varepsilon_{b(i)} < \varepsilon_i\}} 2a_{tI} + \mathbb{I}_{\{\varepsilon_{b(i)} = \varepsilon_i\}} a^o$, where $a^o \in [0, 2a_{tI}]$, together with the Dirac measure defined in the statement of the proposition, achieve \bar{W}^* and therefore solve the Planner's problem. ■

Proof of Lemma 1. Notice that (8) can be written as

$$W_t^D(\mathbf{a}_t) = \phi_t \mathbf{a}_t + W_t^D(\mathbf{0}) \tag{47}$$

with $W_t^D(\mathbf{0})$ given by (14). With (47), (9) is equivalent to

$$\hat{W}_t^D(\mathbf{a}_t) = \max_{\hat{a}_t^m, \hat{a}_t^s} [\phi_t^m \hat{a}_t^m + \phi_t^s \hat{a}_t^s + \xi(a_t^m + p_t a_t^s - \hat{a}_t^m - p_t \hat{a}_t^s) + \mu_m \hat{a}_t^m + \mu_s \hat{a}_t^s] + W_t^D(\mathbf{0})$$

where ξ is a Lagrange multiplier on the budget constraint $\hat{a}_t^m + p_t \hat{a}_t^s \leq a_t^m + p_t a_t^s$, and μ_m and μ_s are the multipliers on the nonnegativity constraints $\hat{a}_t^m \geq 0$ and $\hat{a}_t^s \geq 0$. The corresponding first-order necessary and sufficient conditions for \hat{a}_t^m and \hat{a}_t^s are

$$-\xi + \phi_t^m + \mu_m = 0 \quad (48)$$

$$-\xi p_t + \phi_t^s + \mu_s = 0 \quad (49)$$

$$\xi(a_t^m + p_t a_t^s - \hat{a}_t^m - p_t \hat{a}_t^s) = 0. \quad (50)$$

Clearly $\hat{a}_t^m = \hat{a}_t^s = 0$ is the solution if and only if $a_t^m = a_t^s = 0$, but more generally the solution could take one of three forms: (i) $\mu_s = 0 < \mu_m$, (ii) $\mu_s = \mu_m = 0$, or (iii) $\mu_m = 0 < \mu_s$. In case (i), (48)-(50) imply $\hat{a}_t^m = 0$, $\hat{a}_t^s = a_t^s + \frac{1}{p_t} a_t^m$, and $p_t \phi_t^m < \phi_t^s$. In case (ii), (48)-(50) imply $\hat{a}_t^m \in [0, a_t^m + p_t a_t^s]$, $\hat{a}_t^s = a_t^s + \frac{1}{p_t} (a_t^m - \hat{a}_t^m)$, and $\phi_t^s = p_t \phi_t^m$. In case (iii), (48)-(50) imply $\hat{a}_t^s = 0$, $\hat{a}_t^m = a_t^m + p_t a_t^s$, and $\phi_t^s < p_t \phi_t^m$. The expressions for \hat{a}_{td}^m and \hat{a}_{td}^s in Lemma 1 follow from these three cases. The value function (13) is obtained by substituting the optimal portfolio $(\hat{a}_{td}^m, \hat{a}_{td}^s)$ into (9). ■

Proof of Lemma 2. (i) Notice that (11) can be written as

$$W_t^I(\mathbf{a}_t) = \phi_t \mathbf{a}_t + W_t^I(\mathbf{0}) \quad (51)$$

where

$$W_t^I(\mathbf{0}) = T_t + \max_{\tilde{\mathbf{a}}_{t+1} \in \mathbb{R}_+^2} \left[\beta \int V_{t+1}^I(\mathbf{a}_{t+1}, \varepsilon) dG(\varepsilon) - \phi_t \tilde{\mathbf{a}}_{t+1} \right]$$

s.t. $\mathbf{a}_{t+1} = (\tilde{a}_{t+1}^m, \pi \tilde{a}_{t+1}^s + (1 - \pi) A^s)$.

With (13) and (51) the problem of the investor when he makes the ultimatum offer becomes

$$\max_{\bar{a}_{ti}^m, \bar{a}_{ti}^s, \bar{a}_{td}^m, \bar{a}_{td}^s} [\varepsilon y \bar{a}_{ti}^s + \phi_t^m \bar{a}_{ti}^m + \phi_t^s \bar{a}_{ti}^s]$$

s.t. $\bar{a}_{ti}^m + \bar{a}_{td}^m + p_t(\bar{a}_{ti}^s + \bar{a}_{td}^s) \leq a_{ti}^m + a_{td}^m + p_t(a_{ti}^s + a_{td}^s)$

$$\bar{a}_{td}^m + p_t \bar{a}_{td}^s \geq a_{td}^m + p_t a_{td}^s$$

$$\bar{a}_{ti}^m, \bar{a}_{ti}^s, \bar{a}_{td}^m, \bar{a}_{td}^s \in \mathbb{R}_+.$$

The corresponding Lagrangian is

$$\begin{aligned}\mathcal{L} = & (\phi_t^m + \mu_i^m - \xi) \bar{a}_{ti^*}^m + (\varepsilon y + \phi_t^s + \mu_i^s - \xi p_t) \bar{a}_{ti^*}^s \\ & + (\rho + \mu_d^m - \xi) \bar{a}_{td}^m + (\rho p_t + \mu_d^s - \xi p_t) \bar{a}_{td}^s + K,\end{aligned}$$

where $K \equiv \xi [a_{ti}^m + a_{td}^m + p_t(a_{ti}^s + a_{td}^s)] - \rho (a_{td}^m + p_t a_{td}^s)$, $\xi \in \mathbb{R}_+$ is the Lagrange multiplier associated with the budget constraint, $\rho \in \mathbb{R}_+$ is the multiplier on the dealer's individual rationality constraint, and $\mu_i^m, \mu_i^s, \mu_d^m, \mu_d^s \in \mathbb{R}_+$ are the multipliers for the nonnegativity constraints on $\bar{a}_{ti^*}^m, \bar{a}_{td}^m, \bar{a}_{td}^s$, respectively. The first-order necessary and sufficient conditions are

$$\phi_t^m + \mu_i^m - \xi = 0 \quad (52)$$

$$\varepsilon y + \phi_t^s + \mu_i^s - \xi p_t = 0 \quad (53)$$

$$\rho + \mu_d^m - \xi = 0 \quad (54)$$

$$\rho p_t + \mu_d^s - \xi p_t = 0 \quad (55)$$

and the complementary slackness conditions

$$\xi \{a_{ti}^m + a_{td}^m + p_t(a_{ti}^s + a_{td}^s) - [\bar{a}_{ti^*}^m + \bar{a}_{td}^m + p_t(\bar{a}_{ti^*}^s + \bar{a}_{td}^s)]\} = 0 \quad (56)$$

$$\rho [\bar{a}_{td}^m + p_t \bar{a}_{td}^s - (a_{td}^m + p_t a_{td}^s)] = 0 \quad (57)$$

$$\mu_i^m \bar{a}_{ti^*}^m = 0 \quad (58)$$

$$\mu_i^s \bar{a}_{ti^*}^s = 0 \quad (59)$$

$$\mu_d^m \bar{a}_{td}^m = 0 \quad (60)$$

$$\mu_d^s \bar{a}_{td}^s = 0. \quad (61)$$

First, notice that $\xi > 0$ at an optimum. To see this, assume the contrary, i.e., $\xi = 0$. Then (53) implies $\varepsilon y + \phi_t^s = -\mu_i^s \leq 0$ which is a contradiction since $\varepsilon y + \phi_t^s > 0$. If $\rho > 0$, then (57) implies

$$\bar{a}_{td}^m + p_t \bar{a}_{td}^s = a_{td}^m + p_t a_{td}^s. \quad (62)$$

If instead $\rho = 0$, then (54) and (55) imply $\mu_d^m = \xi > 0$ and $\mu_d^s = \xi p_t > 0$, which (using (60) and (61)) in turn imply $\bar{a}_{td}^m = \bar{a}_{td}^s = 0$. This can only be a solution if $a_{td}^m + p_t a_{td}^s = 0$ (since $\bar{a}_{td}^m + p_t \bar{a}_{td}^s \geq a_{td}^m + p_t a_{td}^s$ must hold at an optimum) in which case (62) also holds. Thus, we conclude that (62) must always hold at an optimum (and with $\rho > 0$ unless $a_{td}^m + p_t a_{td}^s = 0$). Since $\xi > 0$, (56) and (62) imply

$$\bar{a}_{ti^*}^m + p_t \bar{a}_{ti^*}^s = a_{ti}^m + p_t a_{ti}^s. \quad (63)$$

From (62) it is immediate that if $a_{td}^m + p_t a_{td}^s = 0$, then $\bar{a}_{td}^m = \bar{a}_{td}^s = 0$. So suppose $a_{td}^m + p_t a_{td}^s > 0$. In this case μ_d^m and μ_d^s cannot both be strictly positive. (To see this, assume the contrary, i.e., that $\mu_d^m > 0$ and $\mu_d^s > 0$. Then (60) and (61) imply $\bar{a}_{td}^m = \bar{a}_{td}^s = 0$, and (62) implies $a_{td}^m + p_t a_{td}^s = 0$, a contradiction.) Moreover, conditions (54) and (55) imply $\mu_d^s = \mu_d^m p_t$, so $\mu_d^s = \mu_d^m = 0$ must hold at an optimum. Hence when making the ultimatum offer, the investor is indifferent between offering the dealer any nonnegative pair $(\bar{a}_{td}^m, \bar{a}_{td}^s)$ that satisfies (62).

From (63) it is immediate that $\bar{a}_{ti}^m = \bar{a}_{ti}^s = 0$ if $a_{ti}^m + p_t a_{ti}^s = 0$. So suppose $a_{ti}^m + p_t a_{ti}^s > 0$. In this case μ_i^m and μ_i^s cannot both be strictly positive (if they were, then (58) and (59) would imply $\bar{a}_{ti}^m = \bar{a}_{ti}^s = 0$, and in turn (63) would imply $a_{ti}^m + p_t a_{ti}^s = 0$, a contradiction). There are three possible cases: (a) $\mu_i^s = 0 < \mu_i^m$, (b) $\mu_i^s = \mu_i^m = 0$, or (c) $\mu_i^m = 0 < \mu_i^s$. In every case, (52) and (53) imply

$$\varepsilon y + \phi_t^s + \mu_i^s = p_t \phi_t^m + p_t \mu_i^m. \quad (64)$$

In case (a), (58) implies $\bar{a}_{ti}^m = 0$, (63) implies $\bar{a}_{ti}^s = a_{ti}^m/p_t + a_{ti}^s$, and (64) implies that ε must satisfy $\varepsilon > \varepsilon_t^*$, where ε_t^* is as defined in (15). In case (b), (64) implies that ε must satisfy $\varepsilon = \varepsilon_t^*$ and the investor is indifferent between making any offer that leaves him with a nonnegative post-trade portfolio $(\bar{a}_{ti}^m, \bar{a}_{ti}^s)$ that satisfies (63). In case (c), (59) implies $\bar{a}_{ti}^s = 0$, (63) implies $\bar{a}_{ti}^m = a_{ti}^m + p_t a_{ti}^s$, and (64) implies that ε must satisfy $\varepsilon < \varepsilon_t^*$. The first, second, and third lines on the right side of the expressions for \bar{a}_{ti}^m , \bar{a}_{ti}^s , \bar{a}_{td}^m , and \bar{a}_{td}^s in part (i) of the statement of the lemma correspond cases (a), (b), and (c), respectively.

(ii) With (13) and (51) the problem of the dealer when it is his turn to make the ultimatum offer is equivalent to

$$\begin{aligned} & \max_{\bar{a}_{ti}^m, \bar{a}_{ti}^s, \bar{a}_{td}^m, \bar{a}_{td}^s} \bar{\phi}_t [\bar{a}_{td}^m + p_t \bar{a}_{td}^s] \\ \text{s.t. } & \bar{a}_{ti}^m + \bar{a}_{td}^m + p_t (\bar{a}_{ti}^s + \bar{a}_{td}^s) \leq a_{ti}^m + a_{td}^m + p_t (a_{ti}^s + a_{td}^s) \end{aligned} \quad (65)$$

$$\phi_t^m \bar{a}_{ti}^m + (\varepsilon y + \phi_t^s) \bar{a}_{ti}^s \geq \phi_t^m a_{ti}^m + (\varepsilon y + \phi_t^s) a_{ti}^s \quad (66)$$

$$\bar{a}_{ti}^m, \bar{a}_{ti}^s, \bar{a}_{td}^m, \bar{a}_{td}^s \in \mathbb{R}_+.$$

The corresponding Lagrangian is

$$\begin{aligned} \mathcal{L}' = & (\bar{\phi}_t + \mu_d^m - \xi) \bar{a}_{td}^m + (\bar{\phi}_t p_t + \mu_d^s - \xi p_t) \bar{a}_{td}^s \\ & + (\rho \phi_t^m + \mu_i^m - \xi) \bar{a}_{ti}^m + [\rho (\varepsilon y + \phi_t^s) + \mu_i^s - \xi p_t] \bar{a}_{ti}^s + K', \end{aligned}$$

where $K' \equiv \xi [a_{ti}^m + a_{td}^m + p_t (a_{ti}^s + a_{td}^s)] - \rho [\phi_t^m a_{ti}^m + (\varepsilon y + \phi_t^s) a_{ti}^s]$, $\xi \in \mathbb{R}_+$ is the Lagrange multiplier associated with the budget constraint, $\rho \in \mathbb{R}_+$ is the multiplier on the investor's

individual rationality constraint, and $\mu_i^m, \mu_i^s, \mu_d^m, \mu_d^s \in \mathbb{R}_+$ are the multipliers for the nonnegativity constraints on $\bar{a}_{ti}^m, \bar{a}_{ti}^s, \bar{a}_{td}^m, \bar{a}_{td}^s$, respectively. The first-order necessary and sufficient conditions are

$$\bar{\phi}_t + \mu_d^m - \xi = 0 \quad (67)$$

$$\bar{\phi}_t p_t + \mu_d^s - \xi p_t = 0 \quad (68)$$

$$\rho \phi_t^m + \mu_i^m - \xi = 0 \quad (69)$$

$$\rho(\varepsilon y + \phi_t^s) + \mu_i^s - \xi p_t = 0 \quad (70)$$

and the complementary slackness conditions

$$\xi \{a_{ti}^m + a_{td}^m + p_t(a_{ti}^s + a_{td}^s) - [\bar{a}_{ti}^m + \bar{a}_{td}^m + p_t(\bar{a}_{ti}^s + \bar{a}_{td}^s)]\} = 0 \quad (71)$$

$$\rho \{\phi_t^m \bar{a}_{ti}^m + (\varepsilon y + \phi_t^s) \bar{a}_{ti}^s - [\phi_t^m a_{ti}^m + (\varepsilon y + \phi_t^s) a_{ti}^s]\} = 0 \quad (72)$$

$$\mu_i^m \bar{a}_{ti}^m = 0 \quad (73)$$

$$\mu_i^s \bar{a}_{ti}^s = 0 \quad (74)$$

$$\mu_d^m \bar{a}_{td}^m = 0 \quad (75)$$

$$\mu_d^s \bar{a}_{td}^s = 0. \quad (76)$$

First, notice that $\xi > 0$ at an optimum. To see this, note that if $\xi = 0$ then (67) implies $\bar{\phi}_t + \mu_d^m = 0$ which is a contradiction since the left side is strictly positive ($\bar{\phi}_t > 0$ and $\mu_d^m \geq 0$ in a monetary equilibrium). Hence, at an optimum,

$$\bar{a}_{ti}^m + \bar{a}_{td}^m + p_t(\bar{a}_{ti}^s + \bar{a}_{td}^s) = a_{ti}^m + a_{td}^m + p_t(a_{ti}^s + a_{td}^s). \quad (77)$$

Second, observe that conditions (67) and (68), imply $p_t \mu_d^m = \mu_d^s$, so μ_d^m and μ_d^s have the same sign, i.e., either both are positive or both are zero.

If $\rho = 0$, then (69) and (70) imply $\mu_i^m = \xi > 0$ and $\mu_i^s = \xi p_t > 0$, which (using (73) and (74)) in turn imply $\bar{a}_{ti}^m = \bar{a}_{ti}^s = 0$. From the buyer's individual rationality constraint (66) it follows that this can be a solution only if $\phi_t^m a_{ti}^m + (\varepsilon y + \phi_t^s) a_{ti}^s = 0$, or equivalently only if $a_{ti}^m = a_{ti}^s = 0$. To obtain $(\bar{a}_{td}^m, \bar{a}_{td}^s)$, consider two cases: (a) $\mu_d^m = \mu_d^s = 0$, in which case $(\bar{a}_{td}^m, \bar{a}_{td}^s)$ need only satisfy $\bar{a}_{td}^m + p_t \bar{a}_{td}^s = a_{td}^m + p_t a_{td}^s$, or (b) $\mu_d^m > 0$ and $\mu_d^s > 0$, in which case $\bar{a}_{td}^m = \bar{a}_{td}^s = 0$, which according to (65), is only possible if $a_{td}^m = a_{td}^s = 0$. It is easy to see that the solution for case (a) can be obtained from the expressions for $\bar{a}_{ti}^m, \bar{a}_{ti}^s, \bar{a}_{td}^m$, and \bar{a}_{td}^s in part (ii) of the

statement of the lemma simply by setting $a_{ti}^m = a_{ti}^s = 0$, and the solution for case (b) can be obtained similarly, by setting $a_{ti}^m = a_{ti}^s = a_{td}^m = a_{td}^s = 0$.

If $\rho > 0$, then (72) implies

$$\phi_t^m \bar{a}_{ti}^m + (\varepsilon y + \phi_t^s) \bar{a}_{ti}^s = \phi_t^m a_{ti}^m + (\varepsilon y + \phi_t^s) a_{ti}^s. \quad (78)$$

There are eight possible configurations of to be considered: [Configuration 1] $\mu_i^s = \mu_d^m = \mu_d^s = 0 < \mu_i^m$. In this case (73) implies $\bar{a}_{ti}^m = 0$. Conditions (67)-(70) imply $\mu_i^m = (\varepsilon - \varepsilon_t^*) \bar{\phi}_t y / (\varepsilon y + \phi_t^s)$, and therefore $\varepsilon_t^* < \varepsilon$. Then from (77) and (78) it follows that

$$\bar{a}_{ti}^s = a_{ti}^s + \left(\frac{\varepsilon_t^* y + \phi_t^s}{\varepsilon y + \phi_t^s} \right) \frac{1}{p_t} a_{ti}^m$$

and $(\bar{a}_{td}^m, \bar{a}_{td}^s)$ is any nonnegative pair that satisfies

$$\bar{a}_{td}^m + p_t \bar{a}_{td}^s = a_{td}^m + p_t a_{td}^s + \frac{(\varepsilon - \varepsilon_t^*) y}{\varepsilon y + \phi_t^s} a_{ti}^m.$$

[Configuration 2] $\mu_i^m = \mu_i^s = \mu_d^m = \mu_d^s = 0$. In this case conditions (67)-(70) imply $\varepsilon = \varepsilon_t^*$, and (77) and (78) yield

$$\bar{a}_{ti}^m + p_t \bar{a}_{ti}^s = a_{ti}^m + p_t a_{ti}^s \quad (79)$$

$$\bar{a}_{td}^m + p_t \bar{a}_{td}^s = a_{td}^m + p_t a_{td}^s. \quad (80)$$

Hence the dealer is indifferent between making any offer $(\bar{a}_{ti}^m, \bar{a}_{ti}^s, \bar{a}_{td}^m, \bar{a}_{td}^s)$ such that $(\bar{a}_{ti}^m, \bar{a}_{ti}^s) \in \mathbb{R}_+$ satisfies (79), and $(\bar{a}_{td}^m, \bar{a}_{td}^s) \in \mathbb{R}_+$ satisfies (80). [Configuration 3] $\mu_i^m = \mu_d^m = \mu_d^s = 0 < \mu_i^s$. In this case condition (74) implies $\bar{a}_{ti}^s = 0$. Conditions (69) and (70) imply $\mu_i^s = (\varepsilon_t^* - \varepsilon) y \rho$, and therefore $\varepsilon < \varepsilon_t^*$. Then from (77) and (78) it follows that

$$\bar{a}_{ti}^m = a_{ti}^m + \frac{\varepsilon y + \phi_t^s}{\varepsilon_t^* y + \phi_t^s} p_t a_{ti}^s$$

and $(\bar{a}_{td}^m, \bar{a}_{td}^s)$ is any nonnegative pair that satisfies

$$\bar{a}_{td}^m + p_t \bar{a}_{td}^s = a_{td}^m + p_t a_{td}^s + \frac{(\varepsilon_t^* - \varepsilon) y}{\varepsilon_t^* y + \phi_t^s} p_t a_{ti}^s.$$

[Configuration 4] $\mu_d^m = \mu_d^s = 0$, $0 < \mu_i^m$ and $0 < \mu_i^s$. In this case conditions (73) and (74) imply $\bar{a}_{ti}^m = \bar{a}_{ti}^s = 0$, which according to (78), is only possible if $a_{ti}^m = a_{ti}^s = 0$. Then $(\bar{a}_{td}^m, \bar{a}_{td}^s)$ is any nonnegative pair that satisfies (80). [Configuration 5] $\mu_i^s = 0 < \mu_i^m$, $0 < \mu_d^m$ and $0 < \mu_d^s$. In

this case conditions (73), (75) and (76) imply $\bar{a}_{ti}^m = \bar{a}_{td^*}^m = \bar{a}_{td^*}^s = 0$. Conditions (69) and (70) imply $\varepsilon_t^* < \varepsilon$. Then from (77) and (78) it follows that the following condition must hold:

$$a_{td}^s + \frac{1}{p_t} a_{td}^m = - \left[\frac{(\varepsilon - \varepsilon_t^*) y}{(\varepsilon - \varepsilon_t^*) y + p_t \phi_t^m} \right] \frac{1}{p_t} a_{ti}^m.$$

The term on the left side of the equality is nonnegative and the term on the right side of the equality is nonpositive (since $\varepsilon_t^* < \varepsilon$), so this condition can hold only if $a_{ti}^m = a_{td}^m = a_{td}^s = 0$. Therefore (77) implies $\bar{a}_{ti}^s = a_{ti}^s$. [Configuration 6] $\mu_i^m = \mu_i^s = 0$, $0 < \mu_d^m$ and $0 < \mu_d^s$. In this case conditions (75) and (76) imply $\bar{a}_{td^*}^m = \bar{a}_{td^*}^s = 0$. Conditions (69) and (70) imply $\varepsilon = \varepsilon_t^*$, and in turn conditions (77) and (78) imply $a_{td}^m + p_t a_{td}^s = 0$, or equivalently, $a_{td}^m = a_{td}^s = 0$ must hold, and $(\bar{a}_{ti}^m, \bar{a}_{ti}^s)$ is any nonnegative pair that satisfies (79). [Configuration 7] $\mu_i^m = 0 < \mu_i^s$, $0 < \mu_d^s$ and $0 < \mu_d^s$. In this case conditions (74)-(76) imply $\bar{a}_{ti}^s = \bar{a}_{td^*}^m = \bar{a}_{td^*}^s = 0$. Conditions (69) and (70) imply $\varepsilon < \varepsilon_t^*$. Then from (77) and (78) it follows that the following condition must hold:

$$\phi_t^m (a_{td}^m + p_t a_{td}^s) = - (\varepsilon_t^* - \varepsilon) y a_{ti}^s.$$

The term on the left side of the equality is nonnegative and the term on the right side of the equality is nonpositive (since $\varepsilon < \varepsilon_t^*$), so this condition can hold only if $\phi_t^m (a_{td}^m + p_t a_{td}^s) = a_{ti}^s = 0$. Therefore (78) implies $\bar{a}_{ti}^m = a_{ti}^m$. [Configuration 8] $0 < \mu_i^m$, $0 < \mu_i^s$, $0 < \mu_d^s$ and $0 < \mu_d^s$. In this case conditions (73)-(76) imply $\bar{a}_{ti}^m = \bar{a}_{ti}^s = \bar{a}_{td^*}^m = \bar{a}_{td^*}^s = 0$, which according to (77) is only possible, and the only possible solution if $a_{ti}^m = a_{ti}^s = a_{td}^m = a_{td}^s = 0$. To conclude, notice that the solutions for Configurations 1, 2, and 3, correspond to the first, second, and third lines of the expressions for \bar{a}_{ti}^m , \bar{a}_{ti}^s , $\bar{a}_{td^*}^m$, and $\bar{a}_{td^*}^s$ in part (ii) of the statement of the lemma. Similarly, the solution for Configuration 5 corresponds to the first line of the expressions for \bar{a}_{ti}^m , \bar{a}_{ti}^s , $\bar{a}_{td^*}^m$, and $\bar{a}_{td^*}^s$ in part (ii) of the statement of the lemma, with $a_{ti}^m = a_{td}^m = a_{td}^s = 0$. The solution for Configuration 6 corresponds to the second line of the expressions for \bar{a}_{ti}^m , \bar{a}_{ti}^s , $\bar{a}_{td^*}^m$, and $\bar{a}_{td^*}^s$ in part (ii) of the statement of the lemma, with $a_{td}^m = a_{td}^s = 0$. The solution for Configuration 7 corresponds to the third line of the expressions for \bar{a}_{ti}^m , \bar{a}_{ti}^s , $\bar{a}_{td^*}^m$, and $\bar{a}_{td^*}^s$ in part (ii) of the statement of the lemma, with $\phi_t^m (a_{td}^m + p_t a_{td}^s) = a_{ti}^s = 0$. Finally, it is easy to see that the solution for Configuration 4 can be obtained from the expressions for \bar{a}_{ti}^m , \bar{a}_{ti}^s , $\bar{a}_{td^*}^m$, and $\bar{a}_{td^*}^s$ in part (ii) of the statement of the lemma simply by setting $a_{ti}^m = a_{ti}^s = 0$, and the solution for case Configuration 8 can be obtained similarly, by setting $a_{ti}^m = a_{ti}^s = a_{td}^m = a_{td}^s = 0$. ■

Proof of Lemma 3. With (51) investor i 's problem when choosing his take-it-or-leave it offer

to investor j reduces to

$$\begin{aligned}
& \max_{\underline{a}_{ti}^m, \underline{a}_{ti}^s, \underline{a}_{tj}^m, \underline{a}_{tj}^s} [(\varepsilon_i y + \phi_t^s) \underline{a}_{ti}^s + \phi_t^m \underline{a}_{ti}^m] \\
& \text{s.t. } \underline{a}_{ti}^m + \underline{a}_{tj}^m \leq a_{ti}^m + a_{tj}^m \\
& \quad \underline{a}_{ti}^s + \underline{a}_{tj}^s \leq a_{ti}^s + a_{tj}^s \\
& \varepsilon_j y \underline{a}_{tj}^s + \phi_t^m \underline{a}_{tj}^m + \phi_t^s \underline{a}_{tj}^s \geq \varepsilon_j y a_{tj}^s + \phi_t^m a_{tj}^m + \phi_t^s a_{tj}^s \\
& \underline{a}_{ti}^m, \underline{a}_{ti}^s, \underline{a}_{tj}^m, \underline{a}_{tj}^s \in \mathbb{R}_+.
\end{aligned}$$

If $\phi_t^m = 0$, then $\underline{a}_{ti}^s = a_{ti}^s$ and $\underline{a}_{tj}^s = a_{tj}^s$ (the bargaining outcome is no trade between investors i and j) so suppose $\phi_t^m > 0$ for the rest of the proof. The Lagrangian corresponding to investor i 's problem is

$$\begin{aligned}
\mathcal{L} = & (\phi_t^m + \mu_i^m - \xi^m) \underline{a}_{ti}^m + (\varepsilon_i y + \phi_t^s + \mu_i^s - \xi^s) \underline{a}_{ti}^s \\
& + (\rho \phi_t^m + \mu_j^m - \xi^m) \underline{a}_{tj}^m + [\rho(\varepsilon_j y + \phi_t^s) + \mu_j^s - \xi^s] \underline{a}_{tj}^s + K'',
\end{aligned}$$

where $K'' \equiv \xi^m(a_{ti}^m + a_{tj}^m) + \xi^s(a_{ti}^s + a_{tj}^s) - \rho(\varepsilon_j y a_{tj}^s + \phi_t^m a_{tj}^m + \phi_t^s a_{tj}^s)$, $\xi^m \in \mathbb{R}_+$ is the multiplier associated with the bilateral constraint on money holdings, $\xi^s \in \mathbb{R}_+$ is the multiplier associated with the bilateral constraint on equity holdings, $\rho \in \mathbb{R}_+$ is the multiplier on investor j 's individual rationality constraint, and $\mu_i^m, \mu_i^s, \mu_j^m, \mu_j^s \in \mathbb{R}_+$ are the multipliers for the nonnegativity constraints on $\bar{a}_{ti}^m, \bar{a}_{ti}^s, \bar{a}_{tj}^m, \bar{a}_{tj}^s$, respectively. The first-order necessary and sufficient conditions are

$$\phi_t^m + \mu_i^m - \xi^m = 0 \tag{81}$$

$$\varepsilon_i y + \phi_t^s + \mu_i^s - \xi^s = 0 \tag{82}$$

$$\rho \phi_t^m + \mu_j^m - \xi^m = 0 \tag{83}$$

$$\rho(\varepsilon_j y + \phi_t^s) + \mu_j^s - \xi^s = 0 \tag{84}$$

and the complementary slackness conditions

$$\xi^m(a_{ti}^m + a_{tj}^m - \underline{a}_{ti^*}^m - \underline{a}_{tj}^m) = 0 \quad (85)$$

$$\xi^s(a_{ti}^s + a_{tj}^s - \underline{a}_{ti^*}^s - \underline{a}_{tj}^s) = 0 \quad (86)$$

$$\rho(\varepsilon_j y \underline{a}_{tj}^s + \phi_t^m \underline{a}_{tj}^m + \phi_t^s \underline{a}_{tj}^s - \varepsilon_j y a_{tj}^s - \phi_t^m a_{tj}^m - \phi_t^s a_{tj}^s) = 0 \quad (87)$$

$$\mu_i^m \underline{a}_{ti^*}^m = 0 \quad (88)$$

$$\mu_i^s \underline{a}_{ti^*}^s = 0 \quad (89)$$

$$\mu_j^m \underline{a}_{tj}^m = 0 \quad (90)$$

$$\mu_j^s \underline{a}_{tj}^s = 0. \quad (91)$$

If $\xi^m = 0$, (81) implies $0 < \phi_t^m = -\mu_i^m \leq 0$, a contradiction. If $\xi^s = 0$, (82) implies $0 < \varepsilon_j y + \phi_t^s = -\mu_i^s \leq 0$, another contradiction. Hence $\xi^m > 0$ and $\xi^s > 0$, so (85) and (86) imply

$$\underline{a}_{ti^*}^m + \underline{a}_{tj}^m = a_{ti}^m + a_{tj}^m \quad (92)$$

$$\underline{a}_{ti^*}^s + \underline{a}_{tj}^s = a_{ti}^s + a_{tj}^s. \quad (93)$$

If $\rho = 0$, (83) and (84) imply $\mu_j^m = \xi^m > 0$ and $\mu_j^s = \xi^s > 0$, and (90) and (91) imply $\underline{a}_{tj}^m = \underline{a}_{tj}^s = 0$. From investor's j individual rationality constraint, this can only be a solution if $a_{tj}^m = a_{tj}^s = 0$, and if this is the case (85) and (86) imply $(\underline{a}_{ti^*}^m, \underline{a}_{ti^*}^s) = (a_{ti}^m, a_{ti}^s)$. Hereafter suppose $\rho > 0$ which using (87) implies

$$\phi_t^m \underline{a}_{tj}^m + (\varepsilon_j y + \phi_t^s) \underline{a}_{tj}^s = \phi_t^m a_{tj}^m + (\varepsilon_j y + \phi_t^s) a_{tj}^s. \quad (94)$$

If $\mu_i^m > 0$ and $\mu_j^m > 0$, (88) and (90) imply $\underline{a}_{ti^*}^m = \underline{a}_{tj}^m = 0$ which by (92), is only possible if $a_{ti}^m = a_{tj}^m = 0$. But then (94) implies $\underline{a}_{tj}^s = a_{tj}^s$, and (93) implies $\underline{a}_{ti^*}^s = a_{ti}^s$. Similarly, if $\mu_i^s > 0$ and $\mu_j^s > 0$, (89) and (91) imply $\underline{a}_{ti^*}^s = \underline{a}_{tj}^s = 0$ which by (93), is only possible if $a_{ti}^s = a_{tj}^s = 0$. But then (94) implies $\underline{a}_{tj}^m = a_{tj}^m$, and (92) implies $\underline{a}_{ti^*}^m = a_{ti}^m$. If $\mu_i^m > 0$ and $\mu_i^s > 0$, then (88) and (89) imply $\underline{a}_{ti^*}^m = \underline{a}_{ti^*}^s = 0$, and according to (92), (93) and (94), this is only possible if $a_{ti}^m = a_{ti}^s = 0$. Conditions (92) and (93) in turn imply $(\underline{a}_{tj}^m, \underline{a}_{tj}^s) = (a_{tj}^m, a_{tj}^s)$. Similarly, if $\mu_j^m > 0$ and $\mu_j^s > 0$, then (90) and (91) imply $\underline{a}_{tj}^m = \underline{a}_{tj}^s = 0$, and according to (94) this is only possible if $a_{tj}^m = a_{tj}^s = 0$. Conditions (92) and (93) in turn imply $(\underline{a}_{ti^*}^m, \underline{a}_{ti^*}^s) = (a_{ti}^m, a_{ti}^s)$. So far we have simply verified that there is no trade between investors i and j , i.e., $(\underline{a}_{ti^*}^m, \underline{a}_{ti^*}^s) = (a_{ti}^m, a_{ti}^s)$ and

$(\underline{a}_{tj}^m, \underline{a}_{tj}^s) = (a_{tj}^m, a_{tj}^s)$, if $a_{ti}^m = a_{tj}^m = 0$, or $a_{ti}^s = a_{tj}^s = 0$, or $a_{ti}^m = a_{ti}^s = 0$, or $a_{tj}^m = a_{tj}^s = 0$. Thus there are seven binding patterns for $(\mu_i^m, \mu_i^s, \mu_j^m, \mu_j^s)$ that remain to be considered.

(i) $\mu_i^m = \mu_i^s = \mu_j^m = \mu_j^s = 0$. Conditions (81)-(84) imply that this case is only possible if $\varepsilon_i = \varepsilon_j$, and conditions (92), (93) and (94), imply that the solution consists of any pair of post trade portfolios $(\underline{a}_{ti^*}^m, \underline{a}_{ti^*}^s)$ and $(\underline{a}_{tj}^m, \underline{a}_{tj}^s)$ that satisfy

$$\begin{aligned}\underline{a}_{tj}^m &= a_{tj}^m - \frac{\varepsilon_j y + \phi_t^s}{\phi_t^m} (a_{ti}^s - \underline{a}_{ti^*}^s) \\ \underline{a}_{ti^*}^m &= a_{ti}^m + \frac{\varepsilon_j y + \phi_t^s}{\phi_t^m} (a_{ti}^s - \underline{a}_{ti^*}^s) \\ \underline{a}_{tj}^s &= a_{ti}^s + a_{tj}^s - \underline{a}_{ti^*}^s \\ \underline{a}_{ti^*}^s &\in \left[a_{ti}^s - \min \left(\frac{\phi_t^m}{\varepsilon_j y + \phi_t^s} a_{tj}^m, a_{ti}^s \right), a_{ti}^s + \min \left(\frac{\phi_t^m}{\varepsilon_j y + \phi_t^s} a_{ti}^m, a_{tj}^s \right) \right].\end{aligned}$$

(ii) $\mu_i^s = \mu_j^m = \mu_j^s = 0 < \mu_i^m$. Condition (88) implies $\underline{a}_{ti^*}^m = 0$, and from (92) we obtain $\underline{a}_{tj}^m = a_{ti}^m + a_{tj}^m$. Then condition (94) yields

$$\underline{a}_{tj}^s = a_{tj}^s - \frac{\phi_t^m}{\varepsilon_j y + \phi_t^s} a_{ti}^m$$

and condition (93) implies

$$\underline{a}_{ti^*}^s = a_{ti}^s + \frac{\phi_t^m}{\varepsilon_j y + \phi_t^s} a_{ti}^m.$$

Notice that $\mu_j^s = 0$ requires $\underline{a}_{tj}^s \geq 0$ which is equivalent to

$$\phi_t^m a_{ti}^m \leq (\varepsilon_j y + \phi_t^s) a_{tj}^s.$$

Conditions (81)-(84) imply $\mu_i^m = (\varepsilon_i - \varepsilon_j) y \frac{\phi_t^m}{\varepsilon_j y + \phi_t^s}$, so $\mu_i^m > 0$ requires $\varepsilon_j < \varepsilon_i$.

(iii) $\mu_i^m = \mu_j^m = \mu_j^s = 0 < \mu_i^s$. Condition (89) implies $\underline{a}_{ti^*}^s = 0$, and from (93) we obtain $\underline{a}_{tj}^s = a_{ti}^s + a_{tj}^s$. Then condition (94) yields

$$\underline{a}_{tj}^m = a_{tj}^m - \frac{\varepsilon_j y + \phi_t^s}{\phi_t^m} a_{ti}^s$$

and condition (92) implies

$$\underline{a}_{ti^*}^m = a_{ti}^m + \frac{\varepsilon_j y + \phi_t^s}{\phi_t^m} a_{ti}^s.$$

Notice that $\mu_j^m = 0$ requires $\underline{a}_{tj}^m \geq 0$ which is equivalent to

$$(\varepsilon_j y + \phi_t^s) a_{ti}^s \leq \phi_t^m a_{tj}^m.$$

Conditions (81)-(84) imply $\mu_i^s = (\varepsilon_j - \varepsilon_i)y$, so $\mu_i^s > 0$ requires $\varepsilon_i < \varepsilon_j$.

(iv) $\mu_i^m = \mu_i^s = \mu_j^s = 0 < \mu_j^m$. Condition (90) implies $\underline{a}_{tj}^m = 0$, and from (92) we obtain $\underline{a}_{ti}^m = a_{ti}^m + a_{tj}^m$. Then (93) and (94) imply

$$\begin{aligned}\underline{a}_{tj}^s &= a_{tj}^s + \frac{\phi_t^m}{\varepsilon_j y + \phi_t^s} a_{tj}^m \\ \underline{a}_{ti}^s &= a_{ti}^s - \frac{\phi_t^m}{\varepsilon_j y + \phi_t^s} a_{tj}^m.\end{aligned}$$

Notice that $\mu_i^s = 0$ requires $\underline{a}_{ti}^s \geq 0$ which is equivalent to

$$\phi_t^m a_{tj}^m \leq (\varepsilon_j y + \phi_t^s) a_{ti}^s.$$

Conditions (81)-(84) imply $\mu_j^m = (\varepsilon_j - \varepsilon_i)y \frac{\phi_t^m}{\varepsilon_j y + \phi_t^s}$, so $\mu_j^m > 0$ requires $\varepsilon_i < \varepsilon_j$.

(v) $\mu_i^m = \mu_i^s = \mu_j^m = 0 < \mu_j^s$. Condition (91) implies $\underline{a}_{tj}^s = 0$, and from (93) we obtain $\underline{a}_{ti}^s = a_{ti}^s + a_{tj}^s$. Then (92) and (94) imply

$$\begin{aligned}\underline{a}_{tj}^m &= a_{tj}^m + \frac{\varepsilon_j y + \phi_t^s}{\phi_t^m} a_{tj}^s \\ \underline{a}_{ti}^m &= a_{ti}^m - \frac{\varepsilon_j y + \phi_t^s}{\phi_t^m} a_{tj}^s.\end{aligned}$$

Notice that $\mu_i^m = 0$ requires $\underline{a}_{ti}^m \geq 0$ which is equivalent to

$$(\varepsilon_j y + \phi_t^s) a_{tj}^s \leq \phi_t^m a_{ti}^m.$$

Conditions (81)-(84) imply $\mu_j^s = (\varepsilon_i - \varepsilon_j)y$, so $\mu_j^s > 0$ requires $\varepsilon_j < \varepsilon_i$.

(vi) $\mu_i^m, \mu_j^s \in \mathbb{R}_{++}$ and $\mu_i^s = \mu_j^m = 0$. In this case, conditions (88) and (91) give $\underline{a}_{ti}^m = \underline{a}_{tj}^s = 0$, and (92) and (93) imply $\underline{a}_{tj}^m = a_{ti}^m + a_{tj}^m$ and $\underline{a}_{ti}^s = a_{ti}^s + a_{tj}^s$. Condition (94) implies the following restriction must be satisfied

$$\phi_t^m a_{ti}^m = (\varepsilon_j y + \phi_t^s) a_{tj}^s.$$

Conditions (81)-(84) imply $\mu_i^m = (\rho - 1)\phi_t^m$ and $\mu_j^s = (\varepsilon_i - \varepsilon_j)y - (\rho - 1)(\varepsilon_j y + \phi_t^s)$, so $\mu_i^m > 0$ requires $\rho > 1$, and μ_j^s requires $\varepsilon_j < \varepsilon_i$.

(vii) $\mu_i^m = \mu_j^s = 0$ and $\mu_i^s, \mu_j^m \in \mathbb{R}_{++}$. In this case, conditions (89) and (90) give $\underline{a}_{ti}^s = \underline{a}_{tj}^m = 0$, and (92) and (93) imply $\underline{a}_{ti}^m = a_{ti}^m + a_{tj}^m$ and $\underline{a}_{tj}^s = a_{ti}^s + a_{tj}^s$. Condition (94) implies the following restriction must be satisfied

$$\phi_t^m a_{tj}^m = (\varepsilon_j y + \phi_t^s) a_{ti}^s.$$

Conditions (81)-(84) imply $\mu_j^m = (1 - \rho) \phi_t^m$ and $\mu_i^s = (\varepsilon_j - \varepsilon_i) y - (1 - \rho) (\varepsilon_j y + \phi_t^s)$, so $\mu_j^m > 0$ requires $\rho \in (0, 1)$, and $\mu_i^s > 0$ requires $\varepsilon_i < \varepsilon_j$. ■

Proof of Lemma 4. (i) With Lemma 1, (10) becomes

$$\begin{aligned} V_t^D(a_{td}^m, a_{td}^s) &= \kappa \theta \int \bar{\phi}_t [\bar{a}_{td}^m + p_t \bar{a}_{td}^s - (a_{td}^m + p_t a_{td}^s)] dH_t(\mathbf{a}_{ti}, \varepsilon) \\ &\quad + \kappa (1 - \theta) \int \bar{\phi}_t [\bar{a}_{td}^m + p_t \bar{a}_{td}^s - (a_{td}^m + p_t a_{td}^s)] dH_t(\mathbf{a}_{ti}, \varepsilon) \\ &\quad + \bar{\phi}_t (a_{td}^m + p_t a_{td}^s) + W_t^D(\mathbf{0}) \end{aligned}$$

where we have used the more compact notation introduced in Lemma 2, i.e., $\bar{a}_{ti}^k \equiv \bar{a}_{i^*}^k(\mathbf{a}_{ti}, \mathbf{a}_{td}, \varepsilon; \boldsymbol{\psi}_t)$, $\bar{a}_{td}^k \equiv \bar{a}_d^k(\mathbf{a}_{ti}, \mathbf{a}_{td}, \varepsilon; \boldsymbol{\psi}_t)$, $\bar{a}_{ti}^k \equiv \bar{a}_i^k(\mathbf{a}_{ti}, \mathbf{a}_{td}, \varepsilon; \boldsymbol{\psi}_t)$, and $\bar{a}_{td}^k \equiv \bar{a}_{d^*}^k(\mathbf{a}_{ti}, \mathbf{a}_{td}, \varepsilon; \boldsymbol{\psi}_t)$, for $k = m, s$. Use Corollary 1 to arrive at

$$\begin{aligned} V_t^D(a_{td}^m, a_{td}^s) &= \kappa (1 - \theta) \int \bar{\phi}_t \left[\mathbb{I}_{\{\varepsilon < \varepsilon_t^*\}} \frac{(\varepsilon_t^* - \varepsilon) y}{\varepsilon_t^* y + \phi_t^s} p_t a_{ti}^s + \mathbb{I}_{\{\varepsilon_t^* \leq \varepsilon\}} \frac{(\varepsilon - \varepsilon_t^*) y}{\varepsilon y + \phi_t^s} a_{ti}^m \right] dH_t(\mathbf{a}_{ti}, \varepsilon) \\ &\quad + \bar{\phi}_t (a_{td}^m + p_t a_{td}^s) + W_t^D(\mathbf{0}) \end{aligned}$$

where $\mathbb{I}_{\{\varepsilon < \varepsilon_t^*\}}$ is an indicator function that takes the value 1 if $\varepsilon < \varepsilon_t^*$, and 0 otherwise. To obtain (17), use the fact that $dH_t(\mathbf{a}_{ti}, \varepsilon) = dF_t^I(\mathbf{a}_{ti}) dG(\varepsilon)$.

(ii) With (51) and the notation introduced in Lemma 2 and Lemma 3, (12) becomes

$$\begin{aligned} V_t^I(a_{ti}^m, a_{ti}^s, \varepsilon_i) &= \delta \theta \int [\phi_t^m (\bar{a}_{ti}^m - a_{ti}^m) + (\varepsilon_i y + \phi_t^s) (\bar{a}_{ti}^s - a_{ti}^s)] dF_t^D(\mathbf{a}_{td}) \\ &\quad + \delta (1 - \theta) \int [\phi_t^m (\bar{a}_{ti}^m - \bar{a}_{ti}^m) + (\varepsilon_i y + \phi_t^s) (\bar{a}_{ti}^s - a_{ti}^s)] dF_t^D(\mathbf{a}_{td}) \\ &\quad + \alpha \int \tilde{\eta}(\varepsilon_i, \varepsilon_j) [\phi_t^m (\underline{a}_{ti}^m - a_{ti}^m) + (\varepsilon_i y + \phi_t^s) (\underline{a}_{ti}^s - a_{ti}^s)] dH_t(\mathbf{a}_{tj}, \varepsilon_j) \\ &\quad + \alpha \int [1 - \tilde{\eta}(\varepsilon_i, \varepsilon_j)] [\phi_t^m (\underline{a}_{ti}^m - a_{ti}^m) + (\varepsilon_i y + \phi_t^s) (\underline{a}_{ti}^s - a_{ti}^s)] dH_t(\mathbf{a}_{tj}, \varepsilon_j) \\ &\quad + \phi_t^m a_{ti}^m + (\varepsilon_i y + \phi_t^s) a_{ti}^s + W_t^I(\mathbf{0}). \end{aligned}$$

Use $\tilde{\eta}(\varepsilon_i, \varepsilon_j) \equiv \eta \mathbb{I}_{\{\varepsilon_j < \varepsilon_i\}} + (1 - \eta) \mathbb{I}_{\{\varepsilon_i < \varepsilon_j\}} + (1/2) \mathbb{I}_{\{\varepsilon_i = \varepsilon_j\}}$ and substitute the bargaining outcomes

reported in Lemma 2 and Lemma 3 to obtain

$$\begin{aligned}
V_t^I(a_{ti}^m, a_{ti}^s, \varepsilon_i) &= \delta\theta \mathbb{I}_{\{\varepsilon_t^* \leq \varepsilon_i\}} \frac{(\varepsilon_i - \varepsilon_t^*)y}{\varepsilon_t^*y + \phi_t^s} \phi_t^m a_{ti}^m + \delta\theta \mathbb{I}_{\{\varepsilon_i < \varepsilon_t^*\}} (\varepsilon_t^* - \varepsilon_i) y a_{ti}^s \\
&+ \alpha\eta \int \int \mathbb{I}_{\{\varepsilon_j \leq \varepsilon_i\}} \left[-\phi_t^m \min\{p_t^o(\varepsilon_j) a_{tj}^s, a_{ti}^m\} \right. \\
&+ (\varepsilon_i y + \phi_t^s) \min\left\{\frac{a_{ti}^m}{p_t^o(\varepsilon_j)}, a_{tj}^s\right\} \left. \right] dF_t^I(\mathbf{a}_{tj}) dG(\varepsilon_j) \\
&+ \alpha(1-\eta) \int \int \mathbb{I}_{\{\varepsilon_i < \varepsilon_j\}} \left[\phi_t^m \min\{p_t^o(\varepsilon_j) a_{ti}^s, a_{tj}^m\} \right. \\
&- (\varepsilon_i y + \phi_t^s) \min\left\{\frac{a_{tj}^m}{p_t^o(\varepsilon_j)}, a_{ti}^s\right\} \left. \right] dF_t^I(\mathbf{a}_{tj}) dG(\varepsilon_j) \\
&+ \phi_t^m a_{ti}^m + (\varepsilon_i y + \phi_t^s) a_{ti}^s + W_t^I(\mathbf{0}). \tag{95}
\end{aligned}$$

From (11), we anticipate that as in Lagos and Wright (2005), the beginning-of-period distribution of assets across investors will be degenerate, i.e., $(a_{t+1j}^m, a_{t+1j}^s) = (A_{It+1}^m, A_{It+1}^s)$ for all $j \in \mathcal{I}$, so (95) can be written as (18). ■

Proof of Lemma 5. With Lemma 4, the dealer's problem in the second subperiod of period t , (14), becomes

$$W_t^D(\mathbf{0}) = \max_{\tilde{\mathbf{a}}_{t+1} \in \mathbb{R}_+^2} \{(\beta \bar{\phi}_{t+1} - \phi_t^m) \tilde{a}_{t+1}^m + [\beta \pi \bar{\phi}_{t+1} p_{t+1} - \phi_t^s] \tilde{a}_{t+1}^s\} + \beta V_{t+1}^D(\mathbf{0}). \tag{96}$$

From (18),

$$\begin{aligned}
\int V_{t+1}^I(a_{t+1}^m, a_{t+1}^s, \varepsilon_i) dG(\varepsilon_i) &= \phi_{t+1}^m a_{t+1}^m + \int (\varepsilon_i y + \phi_{t+1}^s) a_{t+1}^s dG(\varepsilon_i) + W_{t+1}^I(\mathbf{0}) \\
&+ \delta\theta \int \mathbb{I}_{\{\varepsilon_{t+1}^* \leq \varepsilon_i\}} \frac{(\varepsilon_i - \varepsilon_{t+1}^*)y}{\varepsilon_{t+1}^*y + \phi_{t+1}^s} \phi_{t+1}^m a_{t+1}^m dG(\varepsilon_i) \\
&+ \delta\theta \int \mathbb{I}_{\{\varepsilon_i < \varepsilon_{t+1}^*\}} (\varepsilon_{t+1}^* - \varepsilon_i) y a_{t+1}^s dG(\varepsilon_i) \\
&+ \alpha\eta \int \left[\frac{\phi_{t+1}^m a_{t+1}^m}{A_{It+1}^s} - \phi_{t+1}^s \right] \frac{1}{y} \int_{\varepsilon_j} \frac{(\varepsilon_i - \varepsilon_j)y}{\varepsilon_j y + \phi_{t+1}^s} \phi_{t+1}^m a_{t+1}^m dG(\varepsilon_i) dG(\varepsilon_j) \\
&+ \alpha\eta \int \left[\frac{\phi_{t+1}^m a_{t+1}^m}{A_{It+1}^s} - \phi_{t+1}^s \right] \frac{1}{y} \int_{\varepsilon_j} (\varepsilon_i - \varepsilon_j) y A_{It+1}^s dG(\varepsilon_i) dG(\varepsilon_j) \\
&+ \alpha(1-\eta) \int \left[\frac{\phi_{t+1}^m A_{It+1}^m}{a_{t+1}^s} - \phi_{t+1}^s \right] \frac{1}{y} \int^{\varepsilon_j} \frac{(\varepsilon_j - \varepsilon_i)y}{\varepsilon_j y + \phi_{t+1}^s} \phi_{t+1}^m A_{It+1}^m dG(\varepsilon_i) dG(\varepsilon_j) \\
&+ \alpha(1-\eta) \int \left[\frac{\phi_{t+1}^m A_{It+1}^m}{a_{t+1}^s} - \phi_{t+1}^s \right] \frac{1}{y} \int^{\varepsilon_j} (\varepsilon_j - \varepsilon_i) y a_{t+1}^s dG(\varepsilon_i) dG(\varepsilon_j)
\end{aligned}$$

so the investor's problem (11) can be written as in (51), with

$$\begin{aligned}
W_t^I(\mathbf{0}) = & \max_{\tilde{a}_{t+1}^m \in \mathbb{R}_+} \left\{ -\phi_t^m \tilde{a}_{t+1}^m + \beta \left[\left(1 + \delta \theta \int \mathbb{I}_{\{\varepsilon_{t+1}^* \leq \varepsilon_i\}} \frac{(\varepsilon_i - \varepsilon_{t+1}^*) y}{\varepsilon_{t+1}^* y + \phi_{t+1}^s} dG(\varepsilon_i) \right. \right. \\
& + \alpha \eta \int \left[\frac{\phi_{t+1}^m a_{t+1}^m}{A_{t+1}^s} - \phi_{t+1}^s \right] \frac{1}{y} \int_{\varepsilon_j} \frac{(\varepsilon_i - \varepsilon_j) y}{\varepsilon_j y + \phi_{t+1}^s} dG(\varepsilon_i) dG(\varepsilon_j) \Big) \phi_{t+1}^m \tilde{a}_{t+1}^m \\
& + \left. \left. \alpha \eta \int \left[\frac{\phi_{t+1}^m \tilde{a}_{t+1}^m}{A_{t+1}^s} - \phi_{t+1}^s \right] \frac{1}{y} \int_{\varepsilon_j} (\varepsilon_i - \varepsilon_j) y dG(\varepsilon_i) dG(\varepsilon_j) A_{t+1}^s \right] \right\} \\
& + \max_{\tilde{a}_{t+1}^s \in \mathbb{R}_+} \left\{ -\phi_t^s \tilde{a}_{t+1}^s + \beta \left[\left(\int (\varepsilon_i y + \phi_{t+1}^s) dG(\varepsilon_i) \right. \right. \right. \\
& + \delta \theta \int \mathbb{I}_{\{\varepsilon_i < \varepsilon_{t+1}^*\}} (\varepsilon_{t+1}^* - \varepsilon_i) y dG(\varepsilon_i) \\
& + \alpha (1 - \eta) \int \left[\frac{\phi_{t+1}^m A_{t+1}^m}{a_{t+1}^s} - \phi_{t+1}^s \right] \frac{1}{y} \int^{\varepsilon_j} (\varepsilon_j - \varepsilon_i) y dG(\varepsilon_i) dG(\varepsilon_j) \Big) a_{t+1}^s \\
& + \left. \left. \alpha (1 - \eta) \int \left[\frac{\phi_{t+1}^m A_{t+1}^m}{a_{t+1}^s} - \phi_{t+1}^s \right] \frac{1}{y} \int^{\varepsilon_j} \frac{(\varepsilon_j - \varepsilon_i) y}{\varepsilon_j y + \phi_{t+1}^s} dG(\varepsilon_i) dG(\varepsilon_j) \phi_{t+1}^m A_{t+1}^m \right] \right\} \\
& + T_t + \beta W_{t+1}^I(\mathbf{0}), \tag{97}
\end{aligned}$$

where $a_{t+1}^s = \pi \tilde{a}_{t+1}^s + (1 - \pi) A^s$. The first-order necessary and sufficient conditions for optimization of (96) are (19) and (20). The first-order necessary and sufficient conditions for optimization of (97) are (21) and (22). ■

Proof of Proposition 2. In a stationary equilibrium, the dealer's Euler equations in Lemma 5 become

$$\begin{aligned}
\gamma & \geq \beta, \text{ " = " if } \tilde{a}_{t+1d}^m > 0 \\
\phi^s & \geq \frac{\beta \pi}{1 - \beta \pi} \varepsilon^* y, \text{ " = " if } \tilde{a}_{t+1d}^s > 0.
\end{aligned}$$

The maintained assumption $\gamma > \beta$ implies $\tilde{a}_{t+1d}^m = 0$. Similarly, in a stationary monetary equilibrium the investor's Euler equations in Lemma 5 become

$$\begin{aligned}
\gamma = & \beta \left[1 + \delta \theta \int_{\varepsilon^*}^{\varepsilon^H} \frac{(\varepsilon_i - \varepsilon^*) y}{\varepsilon^* y + \phi^s} dG(\varepsilon_i) \right. \\
& + \left. \alpha \eta \int_{\varepsilon^c}^{\varepsilon^H} \int_{\varepsilon_j}^{\varepsilon^H} \frac{(\varepsilon_i - \varepsilon_j) y}{\varepsilon_j y + \phi^s} dG(\varepsilon_i) dG(\varepsilon_j) \right] \tag{98}
\end{aligned}$$

$$\phi^s \geq \frac{\beta\pi}{1-\beta\pi} \left[\bar{\varepsilon} + \delta\theta \int_{\varepsilon_L}^{\varepsilon^*} (\varepsilon^* - \varepsilon_i) dG(\varepsilon_i) + \alpha(1-\eta) \varphi(\varepsilon^c) \right] y$$

where $\varepsilon^c \equiv [Z/A_I^s - \phi^s]/y$, $\varphi(\varepsilon) \equiv \int_{\varepsilon_L}^{\varepsilon} \int_{\varepsilon_L}^{\varepsilon_j} (\varepsilon_j - \varepsilon_i) dG(\varepsilon_i) dG(\varepsilon_j)$, and the second condition holds with “=” if $\tilde{a}_{t+1}^s > 0$. Together, the dealer’s and the investor’s Euler equations for equity imply

$$\phi^s = \frac{\beta\pi}{1-\beta\pi} \max \left\{ \varepsilon^*, \bar{\varepsilon} + \delta\theta \int_{\varepsilon_L}^{\varepsilon^*} (\varepsilon^* - \varepsilon_i) dG(\varepsilon_i) + \alpha(1-\eta) \varphi(\varepsilon^c) \right\} y. \quad (99)$$

As $\gamma \rightarrow \beta$, (98) implies

$$\beta\delta\theta \int_{\varepsilon^*}^{\varepsilon_H} \frac{(\varepsilon_i - \varepsilon^*)y}{\varepsilon^*y + \phi^s} dG(\varepsilon_i) + \alpha\eta \int_{\varepsilon^c}^{\varepsilon_H} \int_{\varepsilon_j}^{\varepsilon_H} \frac{(\varepsilon_i - \varepsilon_j)y}{\varepsilon_jy + \phi^s} dG(\varepsilon_i) dG(\varepsilon_j) \rightarrow 0,$$

a condition that can only hold if $\varepsilon^* \rightarrow \varepsilon_H$ and $\varepsilon^c \rightarrow \varepsilon_H$. The fact that $\varepsilon^* \rightarrow \varepsilon_H$ means that among investors who contact dealers, only those with preference type ε_H purchase equity. The fact that $\varepsilon^c \rightarrow \varepsilon_H$ implies that in bilateral trades between investors, the investor with the higher valuation purchases all his counterparty’s equity holdings (the investor who wishes to buy is never constrained by his real money balances as $\gamma \rightarrow \beta$). Finally, as $\gamma \rightarrow \beta$,

$$\phi^s \rightarrow \frac{\beta\pi}{1-\beta\pi} \max \{ \varepsilon_H, \bar{\varepsilon} + \delta\theta(\varepsilon_H - \bar{\varepsilon}) + \alpha(1-\eta) \varphi(\varepsilon_H) \} y = \frac{\beta\pi}{1-\beta\pi} \varepsilon_H y$$

so $\tilde{a}_{t+1}^s = A^s$ for all t , i.e., only dealers hold equity overnight. ■

Lemma 6 Consider $\hat{\gamma}$ and $\bar{\gamma}$ as defined in (23). Then $\hat{\gamma} < \bar{\gamma}$.

Proof of Lemma 6. Define $\Gamma(\zeta) : \mathbb{R} \rightarrow \mathbb{R}$ by $\Gamma(\zeta) \equiv \beta[1 + \delta\theta(1 - \beta\pi)\zeta]$. Let $\hat{\zeta} \equiv \frac{(1-\delta\theta)(\bar{\varepsilon}-\bar{\varepsilon})}{\delta\theta\bar{\varepsilon}}$ and $\bar{\zeta} \equiv \frac{\bar{\varepsilon}-\varepsilon_L}{\beta\pi\bar{\varepsilon}+(1-\beta\pi)\varepsilon_L}$, so that $\hat{\gamma} = \Gamma(\hat{\zeta})$ and $\bar{\gamma} = \Gamma(\bar{\zeta})$. Since Γ is strictly increasing, $\hat{\gamma} < \bar{\gamma}$ if and only if $\hat{\zeta} < \bar{\zeta}$. With (24) and the fact that $\bar{\varepsilon} \equiv \int_{\varepsilon_L}^{\varepsilon_H} \varepsilon dG(\varepsilon) = \varepsilon_H - \int_{\varepsilon_L}^{\varepsilon_H} G(\varepsilon) d\varepsilon$,

$$\hat{\zeta} = \frac{\int_{\hat{\varepsilon}}^{\varepsilon_H} [1 - G(\varepsilon)] d\varepsilon}{\bar{\varepsilon} + \delta\theta \int_{\varepsilon_L}^{\hat{\varepsilon}} G(\varepsilon) d\varepsilon},$$

so clearly,

$$\hat{\zeta} < \frac{\int_{\varepsilon_L}^{\varepsilon_H} [1 - G(\varepsilon)] d\varepsilon}{\bar{\varepsilon}} = \frac{\bar{\varepsilon} - \varepsilon_L}{\bar{\varepsilon}} < \bar{\zeta}.$$

Hence $\hat{\gamma} < \bar{\gamma}$. ■

Lemma 7 *In a stationary equilibrium, the interdealer market clearing condition $\bar{A}_{Dt}^s + \bar{A}_{It}^s = A_{Dt}^s + \delta A_{It}^s$ is equivalent to*

$$\delta \theta [1 - G(\varepsilon^*)] \left(A_I^s + \frac{Z}{\varepsilon^* y + \phi^s} \right) + \delta (1 - \theta) \int_{\varepsilon^*}^{\varepsilon^H} \left[A_I^s + \frac{Z}{\varepsilon y + \phi^s} \right] dG(\varepsilon) = A_D^s + \delta A_{It}^s.$$

Proof of Lemma 7. Use $\delta = \kappa v$ in $\bar{A}_{Dt}^s + \bar{A}_{It}^s = A_{Dt}^s + \delta A_{It}^s$ to obtain

$$\begin{aligned} & \theta \int \{ \hat{a}_d^s [\bar{a}_d(\mathbf{a}_{ti}, \mathbf{a}_{td}, \varepsilon; \boldsymbol{\psi}_t); \boldsymbol{\psi}_t] + \bar{a}_{i^*}^s(\mathbf{a}_{ti}, \mathbf{a}_{td}, \varepsilon; \boldsymbol{\psi}_t) \} dF_t^D(\mathbf{a}_{td}) dF_t^I(\mathbf{a}_{ti}) dG(\varepsilon) \\ & + (1 - \theta) \int \{ \hat{a}_d^s [\bar{a}_{d^*}(\mathbf{a}_{ti}, \mathbf{a}_{td}, \varepsilon; \boldsymbol{\psi}_t); \boldsymbol{\psi}_t] + \bar{a}_i^s(\mathbf{a}_{ti}, \mathbf{a}_{td}, \varepsilon; \boldsymbol{\psi}_t) \} dF_t^D(\mathbf{a}_{td}) dF_t^I(\mathbf{a}_{ti}) dG(\varepsilon) \\ & = \int a_{td}^s dF_t^D(\mathbf{a}_{td}) + \int a_{ti}^s dF_t^I(\mathbf{a}_{ti}) + \frac{(1 - \kappa)v}{\delta} \int [a_{td}^s - \hat{a}_d^s(\mathbf{a}_{td}; \boldsymbol{\psi}_t)] dF_t^D(\mathbf{a}_{td}). \end{aligned} \quad (100)$$

Since $\phi_t^s = \phi^s < \varepsilon^* y + \phi^s = \bar{\phi}^s \equiv p_t \phi_t^m$ in a stationary equilibrium, Lemma 1 implies

$$\hat{a}_d^s [\bar{a}_d(\mathbf{a}_{ti}, \mathbf{a}_{td}, \varepsilon; \boldsymbol{\psi}_t); \boldsymbol{\psi}_t] = \hat{a}_d^s [\bar{a}_{d^*}(\mathbf{a}_{ti}, \mathbf{a}_{td}, \varepsilon; \boldsymbol{\psi}_t); \boldsymbol{\psi}_t] = \hat{a}_d^s(\mathbf{a}_{td}; \boldsymbol{\psi}_t) = 0. \quad (101)$$

With (101) and the fact that $\int a_{ti}^s dF_t^I(\mathbf{a}_{ti}) = A_I^s$ and $v \int a_{td}^s dF_t^D(\mathbf{a}_{td}) = A_D^s$, (100) becomes

$$\begin{aligned} A_I^s + \frac{1}{\delta} A_D^s &= \theta \int \bar{a}_{i^*}^s(\mathbf{a}_{ti}, \mathbf{a}_{td}, \varepsilon; \boldsymbol{\psi}_t) dF_t^D(\mathbf{a}_{td}) dF_t^I(\mathbf{a}_{ti}) dG(\varepsilon) \\ &+ (1 - \theta) \int \bar{a}_i^s(\mathbf{a}_{ti}, \mathbf{a}_{td}, \varepsilon; \boldsymbol{\psi}_t) dF_t^D(\mathbf{a}_{td}) dF_t^I(\mathbf{a}_{ti}) dG(\varepsilon). \end{aligned} \quad (102)$$

From Lemma 2,

$$\begin{aligned} \bar{a}_{i^*}^s(\mathbf{a}_{ti}, \mathbf{a}_{td}, \varepsilon; \boldsymbol{\psi}_t) &= \mathbb{I}_{\{\varepsilon^* < \varepsilon\}} \left(a_{ti}^s + \frac{1}{p_t} a_{ti}^m \right) + \mathbb{I}_{\{\varepsilon = \varepsilon^*\}} \bar{a}_{i^*}^s \\ \bar{a}_i^s(\mathbf{a}_{ti}, \mathbf{a}_{td}, \varepsilon; \boldsymbol{\psi}_t) &= \mathbb{I}_{\{\varepsilon^* < \varepsilon\}} \left[a_{ti}^s + \left(\frac{\varepsilon^* y + \phi^s}{\varepsilon y + \phi^s} \right) \frac{1}{p_t} a_{ti}^m \right] + \mathbb{I}_{\{\varepsilon = \varepsilon^*\}} \bar{a}_i^s \end{aligned}$$

where $\bar{a}_{i^*}^s, \bar{a}_i^s \in [0, a_{ti}^s + a_{ti}^m/p_t]$, so (102) becomes

$$A_I^s + \frac{1}{\delta} A_D^s = \theta [1 - G(\varepsilon^*)] \left(A_I^s + \frac{1}{p_t} A_{It}^m \right) + (1 - \theta) \int_{\varepsilon^*}^{\varepsilon^H} \left[A_I^s + \left(\frac{\varepsilon^* y + \phi^s}{\varepsilon y + \phi^s} \right) \frac{1}{p_t} A_{It}^m \right] dG(\varepsilon).$$

Finally, use $p_t \phi_t^m = \bar{\phi}^s = \varepsilon^* y + \phi^s$ to arrive at the expression in the statement of the lemma. ■

Proof of Proposition 3. In an equilibrium with no money (or no valued money), there is no trade in the OTC market. The first-order conditions for a dealer d and an investor i in the time- t Walrasian market are

$$\begin{aligned} \phi_t^s &\geq \beta \pi \phi_{t+1}^s, \text{ “} = \text{” if } \tilde{a}_{t+1d}^s > 0 \\ \phi_t^s &\geq \beta \pi (\bar{\varepsilon} y + \phi_{t+1}^s), \text{ “} = \text{” if } \tilde{a}_{t+1i}^s > 0. \end{aligned}$$

In a stationary equilibrium, $\phi_t^s/\phi_{t+1}^s = 1 > \beta$ so no dealer holds equity. The Walrasian market for equity can only clear if $\phi^s = \frac{\beta\pi}{1-\beta\pi}\bar{\varepsilon}y$. This establishes parts (i) and (iii) in the statement of the proposition.

Next, we turn to monetary equilibria. With $\alpha = 0$, in a stationary equilibrium (19)-(22) become

$$\gamma \geq \beta, \text{ " = " if } \tilde{a}_{t+1d}^m > 0 \quad (103)$$

$$\phi^s \geq \beta\pi\bar{\phi}^s, \text{ " = " if } \tilde{a}_{t+1d}^s > 0 \quad (104)$$

$$1 \geq \frac{\beta}{\gamma} \left[1 + \delta\theta y \frac{\int_{\varepsilon^*}^{\varepsilon_H} [1 - G(\varepsilon)] d\varepsilon}{\varepsilon^*y + \phi^s} \right], \text{ " = " if } \tilde{a}_{t+1i}^m > 0 \quad (105)$$

$$\phi^s \geq \frac{\beta\pi}{1-\beta\pi} \left[\bar{\varepsilon} + \delta\theta \int_{\varepsilon_L}^{\varepsilon^*} G(\varepsilon) d\varepsilon \right] y, \text{ " = " if } \tilde{a}_{t+1i}^s > 0. \quad (106)$$

(In (103) we have used the fact that $\bar{\phi}^s = \varepsilon^*y + \phi^s > \phi^s$.) Under our maintained assumption $\beta < \gamma$, (103) implies $\tilde{a}_{t+1d}^m = Z_D = 0$, so (105) must hold with equality for some investor in a monetary equilibrium. Thus in order to find a monetary equilibrium there are three possible equilibrium configurations to consider depending on the the various binding patterns of the complementary slackness conditions (104) and (106). The market-clearing condition, $\bar{A}_{Dt}^s + \bar{A}_{It}^s = A_{Dt}^s + \delta A_{It}^s$ will apply to all three configurations. Lemma 7 shows that this condition can be written as

$$\delta\theta [1 - G(\varepsilon^*)] \left(A_I^s + \frac{Z}{\varepsilon^*y + \phi^s} \right) + \delta(1-\theta) \int_{\varepsilon^*}^{\varepsilon_H} \left(A_I^s + \frac{Z}{\varepsilon y + \phi^s} \right) dG(\varepsilon) = A_D^s + \delta A_I^s, \quad (107)$$

and this condition can be rearranged to deliver (26). Hereafter the proof proceeds by construction, in three steps.

Step 1: Try to construct a stationary monetary equilibrium with $\tilde{a}_{t+1d}^s = 0$ for all $d \in \mathcal{D}$, and $\tilde{a}_{t+1i}^s > 0$ for some $i \in \mathcal{I}$. The equilibrium conditions for this case are (107) together with

$$\phi^s > \beta\pi\bar{\phi}^s \quad (108)$$

$$1 = \frac{\beta}{\gamma} \left[1 + \delta\theta y \frac{\int_{\varepsilon^*}^{\varepsilon_H} [1 - G(\varepsilon)] d\varepsilon}{\varepsilon^*y + \phi^s} \right] \quad (109)$$

$$\phi^s = \frac{\beta\pi}{1-\beta\pi} \left[\bar{\varepsilon} + \delta\theta \int_{\varepsilon_L}^{\varepsilon^*} G(\varepsilon) d\varepsilon \right] y \quad (110)$$

$$\tilde{a}_{t+1d}^m = 0 \text{ for all } d \in \mathcal{D} \quad (111)$$

$$\tilde{a}_{t+1i}^m \geq 0, \text{ with “} > \text{” for some } i \in \mathcal{I} \quad (112)$$

$$\tilde{a}_{t+1d}^s = 0 \text{ for all } d \in \mathcal{D} \quad (113)$$

$$\tilde{a}_{t+1i}^s \geq 0, \text{ with “} > \text{” for some } i \in \mathcal{I}. \quad (114)$$

Conditions (109) and (110), are to be solved for the two unknowns: ε^* and ϕ^s . Substitute (110) into (109) to obtain

$$1 = \frac{\beta}{\gamma} \left[1 + \delta\theta \frac{\int_{\varepsilon^*}^{\varepsilon_H} [1 - G(\varepsilon)] d\varepsilon}{\varepsilon^* + \frac{\beta\pi}{1-\beta\pi} \left[\bar{\varepsilon} + \delta\theta \int_{\varepsilon_L}^{\varepsilon^*} G(\varepsilon) d\varepsilon \right]} \right] \quad (115)$$

which is a single equation in ε^* . Define

$$T(x) \equiv \frac{\int_x^{\varepsilon_H} [1 - G(\varepsilon)] d\varepsilon}{\frac{1}{1-\beta\pi}x + \frac{\beta\pi}{1-\beta\pi}\hat{T}(x)} - \frac{\gamma - \beta}{\beta\delta\theta} \quad (116)$$

with

$$\hat{T}(x) \equiv \bar{\varepsilon} - x + \delta\theta \int_{\varepsilon_L}^x G(\varepsilon) d\varepsilon, \quad (117)$$

and notice that ε^* solves (115) if and only if it satisfies $T(\varepsilon^*) = 0$. T is a continuous real-valued function on $[\varepsilon_L, \varepsilon_H]$, with

$$T(\varepsilon_L) = \frac{\bar{\varepsilon} - \varepsilon_L}{\varepsilon_L + \frac{\beta\pi}{1-\beta\pi}\bar{\varepsilon}} - \frac{\gamma - \beta}{\beta\delta\theta},$$

$$T(\varepsilon_H) = -\frac{\gamma - \beta}{\beta\delta\theta} < 0,$$

and

$$T'(x) = -\frac{[1-G(x)]\left\{x + \frac{\beta\pi}{1-\beta\pi}\left[\bar{\varepsilon} + \delta\theta \int_{\varepsilon_L}^x G(\varepsilon)d\varepsilon\right]\right\} + \left[\int_x^{\varepsilon_H} [1-G(\varepsilon)]d\varepsilon\right]\left\{1 + \frac{\beta\pi}{1-\beta\pi}\delta\theta G(x)\right\}}{\left\{x + \frac{\beta\pi}{1-\beta\pi}\left[\bar{\varepsilon} + \delta\theta \int_{\varepsilon_L}^x G(\varepsilon)d\varepsilon\right]\right\}^2} < 0.$$

Hence if $T(\varepsilon_L) > 0$, or equivalently, if $\gamma < \bar{\gamma}$ (with $\bar{\gamma}$ is as defined in (23)) then there exists a unique $\varepsilon^* \in (\varepsilon_L, \varepsilon_H)$ that satisfies $T(\varepsilon^*) = 0$ (and $\varepsilon^* \downarrow \varepsilon_L$ as $\gamma \uparrow \bar{\gamma}$). Given ε^* , ϕ^s is given by (110). Given ε^* and ϕ^s , the values of Z , $\bar{\phi}^s$, ϕ_t^m and p_t are obtained using (26) (with $A_I^s = A^s$ and $A_D^s = 0$), (27), (28) and (29). To conclude this step, notice that for this case to be an equilibrium (108) must hold, or equivalently, using (??) and (110), it must be that $\hat{T}(\varepsilon^*) > 0$, where \hat{T} is the continuous function on $[\varepsilon_L, \varepsilon_H]$ defined in (117). Notice that $\hat{T}'(x) = -[1 - \delta\theta G(x)] < 0$, and $\hat{T}(\varepsilon_H) = -(1 - \delta\theta)(\varepsilon_H - \bar{\varepsilon}) < 0 < \bar{\varepsilon} - \varepsilon_L = \hat{T}(\varepsilon_L)$, so there exists a unique $\hat{\varepsilon} \in (\varepsilon_L, \varepsilon_H)$ such that $\hat{T}(\hat{\varepsilon}) = 0$. (Since $\hat{T}(\bar{\varepsilon}) > 0$, and $\hat{T}' < 0$, it follows that

$\bar{\varepsilon} < \hat{\varepsilon}$.) Then $\hat{T}'(x) < 0$ implies $\hat{T}(\varepsilon^*) \geq 0$ if and only if $\varepsilon^* \leq \hat{\varepsilon}$, with “=” for $\varepsilon^* = \hat{\varepsilon}$. With (116), we know that $\varepsilon^* < \hat{\varepsilon}$ if and only if $T(\hat{\varepsilon}) < 0 = T(\varepsilon^*)$, i.e., if and only if

$$\beta \left[1 + \frac{\delta\theta(1-\beta) \int_{\hat{\varepsilon}}^{\varepsilon_H} [1-G(\varepsilon)] d\varepsilon}{\beta\bar{\varepsilon} + (1-\beta)\hat{\varepsilon} + \beta\delta\theta \int_{\varepsilon_L}^{\hat{\varepsilon}} G(\varepsilon) d\varepsilon} \right] < \gamma.$$

Since $\hat{T}(\hat{\varepsilon}) = (1-\delta\theta)(\bar{\varepsilon}-\hat{\varepsilon}) + \delta\theta \int_{\hat{\varepsilon}}^{\varepsilon_H} [1-G(\varepsilon)] d\varepsilon = 0$, this last condition is equivalent to $\hat{\gamma} < \gamma$, where $\hat{\gamma}$ is as defined in (23). The allocations and asset prices described in this step correspond to those in the statement of the proposition for the case with $\gamma \in (\hat{\gamma}, \bar{\gamma})$.

Step 2: Try to construct a stationary monetary equilibrium with $a_{t+1d}^s > 0$ for some $d \in \mathcal{D}$, and $a_{t+1i}^s = 0$ for all $i \in \mathcal{I}$. The equilibrium conditions are (107), (109), (111), (112), together with

$$\phi^s = \beta\pi\bar{\phi}^s \tag{118}$$

$$\phi^s > \frac{\beta\pi}{1-\beta\pi} \left[\bar{\varepsilon} + \delta\theta \int_{\varepsilon_L}^{\varepsilon^*} G(\varepsilon) d\varepsilon \right] y, \text{ “=” if } \tilde{a}_{t+1i}^s > 0. \tag{119}$$

$$\tilde{a}_{t+1d}^s \geq 0, \text{ with “>” for some } d \in \mathcal{D} \tag{120}$$

$$\tilde{a}_{t+1i}^s = 0, \text{ for all } i \in \mathcal{I}. \tag{121}$$

The conditions (109) and (118) are to be solved for ε^* and ϕ^s . First use $\bar{\phi}^s = \varepsilon^*y + \phi^s$ in (118) to obtain

$$\phi^s = \frac{\beta\pi}{1-\beta\pi} \varepsilon^*y. \tag{122}$$

Substitute (122) in (109) to obtain

$$1 = \frac{\beta}{\gamma} \left[1 + \frac{\delta\theta(1-\beta\pi) \int_{\varepsilon^*}^{\varepsilon_H} [1-G(\varepsilon)] d\varepsilon}{\varepsilon^*} \right] \tag{123}$$

which is a single equation in ε^* . Define

$$R(x) \equiv \frac{(1-\beta\pi) \int_x^{\varepsilon_H} [1-G(\varepsilon)] d\varepsilon}{x} - \frac{\gamma-\beta}{\beta\delta\theta} \tag{124}$$

and notice that ε^* solves (123) if and only if it satisfies $R(\varepsilon^*) = 0$. R is a continuous real-valued function on $[\varepsilon_L, \varepsilon_H]$, with

$$R(\varepsilon_L) = \frac{(1-\beta\pi)(\bar{\varepsilon}-\varepsilon_L)}{\varepsilon_L} - \frac{\gamma-\beta}{\beta\delta\theta}$$

$$R(\varepsilon_H) = -\frac{\gamma-\beta}{\beta\delta\theta}$$

and

$$R'(x) = -\frac{[1 - G(x)]x + \int_x^{\varepsilon_H} [1 - G(\varepsilon)] d\varepsilon}{\frac{1}{1-\beta\pi}x^2} < 0.$$

Hence if $R(\varepsilon_L) > 0$, or equivalently, if

$$\gamma < \beta \left[1 + \frac{\delta\theta(1-\beta\pi)(\bar{\varepsilon} - \varepsilon_L)}{\varepsilon_L} \right] \equiv \gamma^o$$

then there exists a unique $\varepsilon^* \in (\varepsilon_L, \varepsilon_H)$ that satisfies $R(\varepsilon^*) = 0$ (and $\varepsilon^* \downarrow \varepsilon_L$ as $\gamma \uparrow \gamma^o$). Having solved for ε^* , ϕ^s is obtained from (122). Given ε^* and ϕ^s , the values of Z , $\bar{\phi}^s$, ϕ_t^m and p_t are obtained using (26) (with $A_D^s = A^s - A_I^s = \pi A^s$), (27), (28) and (29). Notice that for this case to be an equilibrium (119) must hold, or equivalently, using (122), it must be that $\hat{T}(\varepsilon^*) < 0$, which is in turn equivalent to $\hat{\varepsilon} < \varepsilon^*$. With (124), we know that $\hat{\varepsilon} < \varepsilon^*$ if and only if $R(\varepsilon^*) = 0 < R(\hat{\varepsilon})$, i.e., if and only if

$$\gamma < \beta \left[1 + \frac{\delta\theta(1-\beta\pi) \int_{\hat{\varepsilon}}^{\varepsilon_H} [1 - G(\varepsilon)] d\varepsilon}{\hat{\varepsilon}} \right],$$

which using the fact that $\hat{T}(\hat{\varepsilon}) = 0$, can be written as $\gamma < \hat{\gamma}$. To summarize, the prices and allocations constructed in this step constitute a stationary monetary equilibrium provided $\gamma \in (\beta, \min(\hat{\gamma}, \gamma^o))$. To conclude this step, we show that $\hat{\gamma} < \bar{\gamma} < \gamma^o$, which together with the previous step will mean that there is no stationary monetary equilibrium for $\gamma \geq \bar{\gamma}$ (thus establishing part (ii) in the statement of the proposition). It is clear that $\bar{\gamma} < \gamma^o$, and we know that $\hat{\gamma} < \bar{\gamma}$ from Lemma 6. Therefore the allocations and asset prices described in this step correspond to those in the statement of the proposition for the case with $\gamma \in (\beta, \min(\hat{\gamma}, \gamma^o)) = (\beta, \hat{\gamma})$.

Step 3: Try to construct a stationary monetary equilibrium with $\tilde{a}_{t+1d}^s > 0$ for some $d \in \mathcal{D}$, and $\tilde{a}_{t+1i}^s > 0$ for some $i \in \mathcal{I}$. The equilibrium conditions are (107), (109), (110), (111), (112), and (118) with

$$\tilde{a}_{t+1i}^s \geq 0 \text{ and } \tilde{a}_{t+1d}^s \geq 0, \text{ with " > " for some } i \in \mathcal{I} \text{ or some } d \in \mathcal{I}.$$

Notice that ε^* and ϕ^s are obtained as in Step 2. Now, however, (110) must also hold, which together with (122) implies that

$$0 = \bar{\varepsilon} - \varepsilon^* + \delta\theta \int_{\varepsilon_L}^{\varepsilon^*} G(\varepsilon) d\varepsilon$$

or equivalently, (since the right side is just $\hat{T}(\varepsilon^*)$), that $\varepsilon^* = \hat{\varepsilon}$. In other words, this condition requires $R(\hat{\varepsilon}) = \hat{T}(\hat{\varepsilon})$, or equivalently, we must have $\gamma = \hat{\gamma}$. As before, the market-clearing condition (26) is used to obtain Z , while (27), (28), and (29) imply $\bar{\phi}^s$, ϕ_t^m , and p_t , respectively. The allocations and asset prices described in this step correspond to those in the statement of the proposition for the case with $\gamma = \hat{\gamma}$.

Combined, Steps 1-3 prove part (iv) in the statement of the proposition. Part (v)(a) is immediate from (110) and (116), and part (v)(b) from (122) and (124). ■

Corollary 2 *The marginal type, ε^* , characterized in Proposition 3 is strictly decreasing in the rate of inflation, i.e., $\frac{\partial \varepsilon^*}{\partial \gamma} < 0$ both for $\gamma \in (\beta, \hat{\gamma})$, and for $\gamma \in (\hat{\gamma}, \bar{\gamma})$.*

Proof of Corollary 2. For $\gamma \in (\beta, \hat{\gamma})$, implicitly differentiate $R(\varepsilon^*) = 0$ (with R given by (124)), and for $\gamma \in (\hat{\gamma}, \bar{\gamma})$, implicitly differentiate $T(\varepsilon^*) = 0$ (with T given by (116)) to obtain

$$\frac{\partial \varepsilon^*}{\partial \gamma} = \begin{cases} -\frac{\varepsilon^*}{\beta \delta \theta (1 - \beta \pi) [1 - G(\varepsilon^*)] + \gamma - \beta} & \text{if } \beta < \gamma < \hat{\gamma} \\ -\frac{\beta \delta \theta \int_{\varepsilon^*}^H [1 - G(\varepsilon)] d\varepsilon}{\left\{ 1 + \beta \delta \theta \left[\frac{\pi G(\varepsilon^*)}{1 - \beta \pi} + \frac{1 - G(\varepsilon^*)}{\gamma - \beta} \right] \right\} (\gamma - \beta)^2} & \text{if } \hat{\gamma} < \gamma < \bar{\gamma}. \end{cases}$$

Clearly, $\partial \varepsilon^* / \partial \gamma < 0$ for $\gamma \in (\beta, \hat{\gamma})$, and for $\gamma \in (\hat{\gamma}, \bar{\gamma})$. ■

Proof of Proposition 4. With $\delta = 0$, in any stationary equilibrium the Euler equations for a dealer d obtained in Lemma 5 reduce to

$$\begin{aligned} \gamma &\geq \beta, \text{ with “} = \text{” if } \tilde{a}_{t+1d}^m > 0 \\ \phi^s &\geq \beta \pi \phi^s, \text{ with “} = \text{” if } \tilde{a}_{t+1d}^m > 0. \end{aligned}$$

The maintained assumption $\gamma > \beta$ and the fact that the equity will be valued in any equilibrium imply $\tilde{a}_{t+1d}^m = \tilde{a}_{t+1d}^m = 0$ for all $d \in \mathcal{D}$. Since dealers are inactive in any stationary equilibrium, we focus on investors for the remainder of the proof. In an equilibrium with no money (or no valued money), there is no trade in the OTC market. The first-order condition for an investor i in the time- t Walrasian market is

$$\phi_t^s \geq \beta \pi (\bar{\varepsilon} y + \phi_{t+1}^s), \text{ “} = \text{” if } \tilde{a}_{t+1i}^s > 0.$$

In a stationary equilibrium the Walrasian market for equity can only clear if $\phi^s = \frac{\beta \pi}{1 - \beta \pi} \bar{\varepsilon} y$. This establishes parts (i) and (iii) in the statement of the proposition. In a stationary monetary

equilibrium the Euler equations for an investor obtained in Lemma 5 reduce to

$$\gamma = \beta \left\{ 1 + \alpha \eta \int_{\varepsilon^c}^{\varepsilon_H} \int_{\varepsilon_j}^{\varepsilon_H} \frac{(\varepsilon_i - \varepsilon_j) y}{\varepsilon_j y + \phi^s} dG(\varepsilon_i) dG(\varepsilon_j) \right\} \quad (125)$$

$$\phi^s = \frac{\beta \pi}{1 - \beta \pi} \left[\bar{\varepsilon} + \alpha (1 - \eta) \int_{\varepsilon_L}^{\varepsilon^c} \int_{\varepsilon_L}^{\varepsilon_j} (\varepsilon_j - \varepsilon_i) dG(\varepsilon_i) dG(\varepsilon_j) \right] y \quad (126)$$

where

$$\varepsilon^c \equiv \left[\frac{Z}{A_I^s} - \phi^s \right] \frac{1}{y}. \quad (127)$$

Condition (125) can be substituted into (126) to obtain a single equation in the unknown ε^c , namely $\bar{T}(\varepsilon^c) = 0$, where $\bar{T} : [\varepsilon_L, \varepsilon_H] \rightarrow \mathbb{R}$ is defined by

$$\bar{T}(\varepsilon^c) \equiv \beta \alpha \eta \int_{\varepsilon^c}^{\varepsilon_H} \int_{\varepsilon_j}^{\varepsilon_H} \frac{\varepsilon_i - \varepsilon_j}{\varepsilon_j + \frac{\beta \pi}{1 - \beta \pi} \left[\bar{\varepsilon} + \alpha (1 - \eta) \int_{\varepsilon_L}^{\varepsilon^c} \int_{\varepsilon_L}^{\varepsilon_j} (\varepsilon_j - \varepsilon_i) dG(\varepsilon_i) dG(\varepsilon_j) \right]} dG(\varepsilon_i) dG(\varepsilon_j) + \beta - \gamma.$$

Notice that $\bar{T}(\varepsilon_H) = \beta - \gamma < 0$ and

$$\bar{T}(\varepsilon_L) = \beta \alpha \eta \int_{\varepsilon_L}^{\varepsilon_H} \int_{\varepsilon_j}^{\varepsilon_H} \frac{\varepsilon_i - \varepsilon_j}{\varepsilon_j + \frac{\beta \pi}{1 - \beta \pi} \bar{\varepsilon}} dG(\varepsilon_i) dG(\varepsilon_j) + \beta - \gamma,$$

so since \bar{T} is continuous, a stationary monetary equilibrium exists if $\gamma < \tilde{\gamma}$ with $\tilde{\gamma}$ defined as in (33). In addition,

$$\begin{aligned} \bar{T}'(\varepsilon^c) = & - \left[\beta \alpha \eta \int_{\varepsilon^c}^{\varepsilon_H} \frac{(\varepsilon_i - \varepsilon^c) y}{\varepsilon^c y + \phi^s} dG(\varepsilon_i) G'(\varepsilon^c) \right. \\ & \left. + \frac{(\beta \alpha)^2 \pi \eta (1 - \eta)}{1 - \beta \pi} \int_{\varepsilon^c}^{\varepsilon_H} \int_{\varepsilon_j}^{\varepsilon_H} \frac{(\varepsilon_i - \varepsilon_j) \int_{\varepsilon_L}^{\varepsilon^c} (\varepsilon^c - \varepsilon) dG(\varepsilon) G'(\varepsilon^c)}{(\varepsilon_j + \phi^s / y)^2} dG(\varepsilon_i) dG(\varepsilon_j) \right] \end{aligned}$$

is negative, so a stationary monetary equilibrium exists if and only if $\gamma < \tilde{\gamma}$, and there cannot be more than one stationary monetary equilibrium. Condition (34) is just (126), condition (36) is $\bar{T}(\varepsilon^c) = 0$, and (35) follows from (127). This establishes parts (ii) and (iv). Part (v) is immediate from (36). ■

Proof of Proposition 5. Recall that $\partial \varepsilon^* / \partial \gamma < 0$ (Corollary 2). (i) From (25),

$$\frac{\partial \phi^s}{\partial \gamma} = \frac{\beta \pi}{1 - \beta \pi} \left[\mathbb{I}_{\{\beta < \gamma \leq \hat{\gamma}\}} + \mathbb{I}_{\{\hat{\gamma} < \gamma < \bar{\gamma}\}} \delta \theta G(\varepsilon^*) \right] y \frac{\partial \varepsilon^*}{\partial \gamma} < 0.$$

(ii) Condition (27) implies $\partial \bar{\phi}^s / \partial \gamma = (\partial \varepsilon^* / \partial \gamma) y + \partial \phi^s / \partial \gamma < 0$. (iii) Differentiate (26) to obtain

$$\frac{\partial Z}{\partial \varepsilon^*} = \delta Z \frac{G'(\varepsilon^*) A_I^s + \left[G'(\varepsilon^*) (\varepsilon^* y + \phi^s) + \theta [1 - G(\varepsilon^*)] \left(y + \frac{\partial \phi^s}{\partial \varepsilon^*} \right) + (1 - \theta) \frac{\partial \phi^s}{\partial \varepsilon^*} \int_{\varepsilon^*}^{\varepsilon_H} \left(\frac{\varepsilon^* y + \phi^s}{\varepsilon y + \phi^s} \right)^2 dG(\varepsilon) \right]}{A_D^s + \delta G(\varepsilon^*) A_I^s} \frac{Z}{(\varepsilon^* y + \phi^s)^2} > 0. \quad (128)$$

Hence $\partial Z/\partial\gamma = (\partial Z/\partial\varepsilon^*)(\partial\varepsilon^*/\partial\gamma) < 0$. From (28), $\partial\phi_t^m/\partial\gamma = (1/A_t^m)\partial Z/\partial\gamma < 0$. ■

Proof of Proposition 6. First, notice that $\partial\varepsilon^c/\partial\gamma = 1/\bar{T}'(\varepsilon^c) < 0$, where $\bar{T}(\cdot)$ is the mapping defined in the proof of Proposition 4. (i) Differentiate (34) to obtain

$$\frac{\partial\phi^s}{\partial\gamma} = \frac{\beta\pi}{1-\beta\pi}\alpha(1-\eta)G'(\varepsilon^c)\int_{\varepsilon_L}^{\varepsilon^c}(\varepsilon^c-\varepsilon_i)y dG(\varepsilon_i)\frac{\partial\varepsilon^c}{\partial\gamma} < 0.$$

(ii) From (35), $\partial Z/\partial\gamma = [(\partial\varepsilon^c/\partial\gamma)y + \partial\phi^s/\partial\gamma]A^s < 0$, and since $Z = \phi_t^m A_t^m$, $\partial\phi_t^m/\partial\gamma = (\partial Z/\partial\gamma)(1/A_t^m) < 0$. ■

Proof of Proposition 7. From condition (30),

$$\frac{\partial\varepsilon^*}{\partial(\delta\theta)} = \frac{\frac{\gamma-\beta}{\delta\theta}[\varepsilon^* + \beta\pi(\bar{\varepsilon} - \varepsilon^*)\mathbb{I}_{\{\hat{\gamma} < \gamma\}}]}{\beta\delta\theta(1-\beta\pi)[1-G(\varepsilon^*)] + (\gamma-\beta)\{1 + \beta\pi[\delta\theta G(\varepsilon^*) - 1]\mathbb{I}_{\{\hat{\gamma} < \gamma\}}\}} > 0. \quad (129)$$

(i) From (34),

$$\frac{\partial\phi^s}{\partial(\delta\theta)} = \begin{cases} \frac{\beta\pi}{1-\beta\pi}\frac{\partial\varepsilon^*}{\partial(\delta\theta)}y > 0 & \text{if } \beta < \gamma \leq \hat{\gamma} \\ \frac{\beta\pi}{1-\beta\pi}\left[\int_{\varepsilon_L}^{\varepsilon^*}G(\varepsilon)d\varepsilon + \delta\theta G(\varepsilon^*)\frac{\partial\varepsilon^*}{\partial(\delta\theta)}\right]y > 0 & \text{if } \hat{\gamma} < \gamma < \bar{\gamma}. \end{cases}$$

(ii) From (27), $\partial\bar{\phi}^s/\partial(\delta\theta) = [\partial\varepsilon^*/\partial(\delta\theta)]y + \partial\phi^s/\partial(\delta\theta) > 0$. (iii) For $\gamma \in (\hat{\gamma}, \bar{\gamma})$, (26) implies $\partial Z/\partial\delta = (\partial Z/\partial\varepsilon^*)(\partial\varepsilon^*/\partial\delta) > 0$ (the sign follows from (128) and (129)), and therefore $\partial\phi_t^m/\partial\delta = (\partial Z/\partial\delta)(1/A_t^m) > 0$. ■

Proof of Proposition 8. Implicit differentiation of $\bar{T}(\varepsilon^c) = 0$ implies

$$\frac{\partial\varepsilon^c}{\partial\alpha} = \frac{\int_{\varepsilon^c}^{\varepsilon^H}\int_{\varepsilon_j}^{\varepsilon^H}\frac{\eta(1-\beta\pi)(\varepsilon_i-\varepsilon_j)[(1-\beta\pi)\varepsilon_j+\beta\pi\bar{\varepsilon}]}{\{(1-\beta\pi)\varepsilon_j+\beta\pi[\bar{\varepsilon}+\alpha(1-\eta)\varphi(\varepsilon^c)]\}^2}dG(\varepsilon_i)dG(\varepsilon_j)}{\int_{\varepsilon^c}^{\varepsilon^H}\frac{\alpha\eta(1-\beta\pi)(\varepsilon_i-\varepsilon_j)}{(1-\beta\pi)\varepsilon^c+\beta\pi[\bar{\varepsilon}+\alpha(1-\eta)\varphi(\varepsilon^c)]}dG(\varepsilon_i)G'(\varepsilon^c)+\int_{\varepsilon^c}^{\varepsilon^H}\int_{\varepsilon_j}^{\varepsilon^H}\frac{\beta\pi\alpha^2\eta(1-\eta)(1-\beta\pi)(\varepsilon_i-\varepsilon_j)\varphi'(\varepsilon^c)}{\{(1-\beta\pi)\varepsilon_j+\beta\pi[\bar{\varepsilon}+\alpha(1-\eta)\varphi(\varepsilon^c)]\}^2}dG(\varepsilon_i)dG(\varepsilon_j)} > 0.$$

(i) Differentiate (34) to arrive at

$$\frac{\partial\phi^s}{\partial\alpha} = \frac{\beta\pi(1-\eta)}{1-\beta\pi}\left[\varphi(\varepsilon^c) + \alpha\int_{\varepsilon_L}^{\varepsilon^c}(\varepsilon^c-\varepsilon_i)dG(\varepsilon_i)dG(\varepsilon^c)\frac{\partial\varepsilon^c}{\partial\alpha}\right]y > 0.$$

(ii) From (35),

$$\frac{\partial Z}{\partial\alpha} = \left(\frac{\partial\varepsilon^c}{\partial\alpha}y + \frac{\partial\phi^s}{\partial\alpha}\right)A^s > 0,$$

and since $Z = \phi_t^m A_t^m$, it follows that $\partial\phi_t^m/\partial\alpha > 0$. ■

Proof of Proposition 9. (i) The result is immediate from the expression for A_D^s in Proposition 3. (ii) From (23) and (24),

$$\frac{\partial \hat{\gamma}}{\partial (\delta \theta)} = \beta (1 - \beta \pi) \left\{ \frac{(1 - \delta \theta) \bar{\varepsilon}}{[1 - \delta \theta G(\hat{\varepsilon})] \hat{\varepsilon}^2} \int_{\varepsilon_L}^{\hat{\varepsilon}} G(\varepsilon) d\varepsilon - \frac{\hat{\varepsilon} - \bar{\varepsilon}}{\hat{\varepsilon}} \right\}.$$

Notice that $\partial \hat{\gamma} / \partial (\delta \theta)$ approaches a positive value as $\delta \theta \rightarrow 0$, and a negative value as $\delta \theta \rightarrow 1$. Also, $\hat{\gamma} \rightarrow \beta$ both when $\delta \theta \rightarrow 0$, and when $\delta \theta \rightarrow 1$. Hence $\gamma > \beta = \lim_{\delta \theta \rightarrow 0} \hat{\gamma} = \lim_{\delta \theta \rightarrow 1} \hat{\gamma}$ for a range of values of $\delta \theta$ close to 0 and a range of values of $\delta \theta$ close to 1. For those ranges of values of $\delta \theta$, $A_D^s = 0$. In between those ranges there must exist values of $\delta \theta$ such that $\gamma < \hat{\gamma}$ which implies $A_D^s > 0$. ■

Proof of Proposition 10. (i) Differentiate (37) to get

$$\frac{\partial \mathcal{V}}{\partial \gamma} = 2\delta G'(\varepsilon^*) (A^s - \pi \tilde{A}_D^s) \frac{\partial \varepsilon^*}{\partial \gamma} < 0,$$

where the inequality follows from Corollary 2. (ii) From (37),

$$\begin{aligned} \frac{\partial \mathcal{V}}{\partial \theta} &= 2\delta G'(\varepsilon^*) (A^s - \pi \tilde{A}_D^s) \frac{\partial \varepsilon^*}{\partial \theta} \\ \frac{\partial \mathcal{V}}{\partial \delta} &= 2 \left[G(\varepsilon^*) + \delta G'(\varepsilon^*) \frac{\partial \varepsilon^*}{\partial \delta} \right] (A^s - \pi \tilde{A}_D^s) \end{aligned}$$

and both are positive since $\partial \varepsilon^* / \partial (\delta \theta) > 0$ (see (129)). ■

Proof of Proposition 10. Rewrite $\tilde{\mathcal{V}}$ as

$$\begin{aligned} \tilde{\mathcal{V}} &= \alpha A^s \int_{\varepsilon_L}^{\varepsilon^c} \{ \eta [1 - G(\varepsilon_i)] + (1 - \eta) G(\varepsilon_i) \} dG(\varepsilon_i) \\ &\quad + \alpha A^s \int_{\varepsilon^c}^{\varepsilon^H} \{ \eta [1 - G(\varepsilon_i)] + (1 - \eta) G(\varepsilon_i) \} \frac{\varepsilon^c y + \phi^s}{\varepsilon_i y + \phi^s} dG(\varepsilon_i). \end{aligned}$$

Differentiate to obtain

$$\frac{\partial \tilde{\mathcal{V}}}{\partial \varepsilon^c} = \alpha A^s \int_{\varepsilon^c}^{\varepsilon^H} \{ \eta [1 - G(\varepsilon_i)] + (1 - \eta) G(\varepsilon_i) \} \frac{\partial}{\partial \varepsilon^c} \left[\frac{\varepsilon^c y + \phi^s}{\varepsilon_i y + \phi^s} \right] dG(\varepsilon_i),$$

where

$$\frac{\partial}{\partial \varepsilon^c} \left[\frac{\varepsilon^c y + \phi^s}{\varepsilon_i y + \phi^s} \right] = \frac{\varepsilon_i y + \phi^s + (\varepsilon_i - \varepsilon^c) \frac{\partial \phi^s}{\partial \varepsilon^c}}{(\varepsilon_i y + \phi^s)^2} y A^s > 0 \text{ for } \varepsilon_i > \varepsilon^c.$$

Hence, $\partial \tilde{\mathcal{V}} / \partial \varepsilon^c > 0$. Thus $\partial \tilde{\mathcal{V}} / \partial \gamma = (\partial \tilde{\mathcal{V}} / \partial \varepsilon^c) (\partial \varepsilon^c / \partial \gamma) < 0$, since $\partial \varepsilon^c / \partial \gamma < 0$ (see proof of Proposition 6), which establishes (i). For part (ii), simply notice that $\partial \tilde{\mathcal{V}} / \partial \alpha = \tilde{\mathcal{V}} / \alpha + (\partial \tilde{\mathcal{V}} / \partial \varepsilon^c) (\partial \varepsilon^c / \partial \alpha) > 0$. ■

Proof of Proposition 12. (i) For $\beta < \gamma \leq \hat{\gamma}$, $\partial \mathcal{P} / \partial \gamma = [\beta \pi / (1 - \beta \pi)] y (\partial \varepsilon^* / \partial \gamma) < 0$, and for $\hat{\gamma} < \gamma < \bar{\gamma}$, $\partial \mathcal{P} / \partial \gamma = [\beta \pi / (1 - \beta \pi)] \delta \theta y G(\varepsilon^*) (\partial \varepsilon^* / \partial \gamma) < 0$. (ii) For $\beta < \gamma \leq \hat{\gamma}$, $\partial \mathcal{P} / \partial (\delta \theta) = [\beta \pi / (1 - \beta \pi)] y (\partial \varepsilon^* / \partial (\delta \theta)) > 0$, and for $\hat{\gamma} < \gamma < \bar{\gamma}$, $\partial \mathcal{P} / \partial \gamma = [\beta \pi / (1 - \beta \pi)] \{ \delta \theta G(\varepsilon^*) [\partial \varepsilon^* / \partial (\delta \theta)] + \int_{\varepsilon_L}^{\varepsilon^*} G(\varepsilon) d\varepsilon \} y > 0$. ■

Proof of Proposition 13. (i) $\partial \tilde{\mathcal{P}} / \partial \gamma = [\beta \pi / (1 - \beta \pi)] y \alpha (1 - \eta) \varphi'(\varepsilon^c) (\partial \varepsilon^c / \partial \gamma) < 0$. (ii) $\partial \tilde{\mathcal{P}} / \partial \alpha = [\beta \pi / (1 - \beta \pi)] (1 - \eta) y \{ \alpha \varphi'(\varepsilon^c) (\partial \varepsilon^c / \partial \alpha) + \varphi(\varepsilon^c) \} > 0$. ■

Lemma 8 *In any equilibrium, the free-entry condition (42) can be written as (43).*

Proof of Lemma 8. With (14), the left side of condition (42) can be written as

$$\max_{(a_{t+1}^m, a_{t+1}^s) \in \mathbb{R}_+^2} [\beta V_{t+1}^D(a_{t+1}^m, a_{t+1}^s) - (\phi_t^m a_{t+1}^m + \phi_t^s a_{t+1}^s)] - k.$$

And with (17), this last expression becomes

$$\max_{(a_{t+1}^m, a_{t+1}^s) \in \mathbb{R}_+^2} [(\beta \bar{\phi}_{t+1} - \phi_t^m) a_{t+1}^m + (\beta \bar{\phi}_{t+1} p_{t+1} - \phi_t^s) a_{t+1}^s] + \beta V_{t+1}^D(\mathbf{0}) - k, \quad (130)$$

where

$$V_{t+1}^D(\mathbf{0}) \equiv \kappa(v_{t+1}) (1 - \theta) \bar{\phi}_{t+1} \left[p_{t+1} A_{t+1}^I \int_{\varepsilon_L}^{\varepsilon_{t+1}^*} \frac{(\varepsilon_{t+1}^* - \varepsilon) y}{\varepsilon_{t+1}^* y + \phi_{t+1}^s} dG(\varepsilon) + M_{t+1}^I \int_{\varepsilon_{t+1}^*}^{\varepsilon_H} \frac{(\varepsilon - \varepsilon_{t+1}^*) y}{\varepsilon y + \phi_{t+1}^s} dG(\varepsilon) \right] + \max \{ W_{t+1}^D(\mathbf{0}) - k, 0 \},$$

is as in Lemma 4, except for the last term, which reflects the fact that the dealer has to bear cost k in order to participate in the OTC market of the following period. In equilibrium, the dealer optimization (conditions (19) and (20)) implies

$$\max_{(a_{t+1}^m, a_{t+1}^s) \in \mathbb{R}_+^2} [(\beta \bar{\phi}_{t+1} - \phi_t^m) a_{t+1}^m + (\beta \bar{\phi}_{t+1} p_{t+1} - \phi_t^s) a_{t+1}^s] = 0.$$

Also, (42) implies $\max \{ W_{t+1}^D(\mathbf{0}) - k, 0 \} = 0$. Hence (130) reduces to $\Phi_{t+1} - k$, with Φ_{t+1} as defined below (43). ■

Proof of Proposition 14. The choice variable a'_{tD} does not appear in the Planner's objective function, so $a'_{tD} = 0$ at an optimum. Since (40) must bind for every t at an optimum, the planner's problem is equivalent to

$$W^{**} = \max_{\{v_t, \bar{a}_{tD}, \bar{a}_{tI}, a'_{tI}\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t \left\{ \delta(v_t) \int_{[\varepsilon_L, \varepsilon_H]} \varepsilon y a'_{tI}(d\varepsilon) + [1 - \delta(v_t)] \bar{\varepsilon} y a_{tI} - k v_{t+1} \right\}$$

subject to (6), (7), (38) and (39). Clearly, $\int_{[\varepsilon_L, \varepsilon_H]} \varepsilon y a'_{tI} (d\varepsilon) \leq \varepsilon_H y$ and (39) must bind at an optimum, so $W^{**} \leq \bar{W}^{**}$, where

$$\begin{aligned}
\bar{W}^{**} &= \max_{\{v_t, \tilde{a}_{tD}, \tilde{a}_{tI}\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t \{ [v_t a_{tD} + \delta(v_t) a_{tI}] \varepsilon_H y + [1 - \delta(v_t)] \bar{\varepsilon} y a_{tI} - k v_{t+1} \} \\
&= \max_{\{v_t, \tilde{a}_{tI}\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t \{ [\pi \varepsilon_H + (1 - \pi) \{ \delta(v_t) \varepsilon_H + [1 - \delta(v_t)] \bar{\varepsilon} \}] y A^s \\
&\quad - [1 - \delta(v_t)] (\varepsilon_H - \bar{\varepsilon}) \pi y \tilde{a}_{tI} - k v_{t+1} \} \\
&= \max_{\{v_t\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t \{ [\pi \varepsilon_H + (1 - \pi) \{ \delta(v_t) \varepsilon_H + [1 - \delta(v_t)] \bar{\varepsilon} \}] y A^s - k v_{t+1} \} \\
&= \sum_{t=0}^{\infty} \beta^t \{ [\pi \varepsilon_H + (1 - \pi) \{ \delta(v_t^*) \varepsilon_H + [1 - \delta(v_t^*)] \bar{\varepsilon} \}] y A^s - k v_{t+1}^* \},
\end{aligned}$$

where the maximization in the first line is subject to (6), (7) and (38) (which must bind at an optimum), the second line has been obtained by substituting these constraints into the objective function, and $\{v_t^*\}$ in the last line denotes the sequence of v_t characterized by (41). The allocation in the statement of the proposition achieves \bar{W}^{**} and therefore solves the Planner's problem. ■

Proof of Proposition 15. Consider a stationary equilibrium with free entry (for the model with $\alpha = 0$). As $\gamma \rightarrow \beta$, (30) implies

$$\frac{\int_{\varepsilon^*}^{\varepsilon_H} [1 - G(\varepsilon)] d\varepsilon}{\varepsilon^* + \beta\pi \left[\bar{\varepsilon} - \varepsilon^* + \delta(v) \theta \int_{\varepsilon_L}^{\varepsilon^*} G(\varepsilon) d\varepsilon \right] \mathbb{I}_{\{\hat{\gamma} < \gamma\}}} \rightarrow 0$$

which in turn implies $\varepsilon^* \rightarrow \varepsilon_H$. The dealer's and the investor's Euler equations for equity in Lemma 5 imply

$$\phi^s = \frac{\beta\pi}{1 - \beta\pi} \max \left\{ \varepsilon^*, \bar{\varepsilon} + \delta(v) \theta \int_{\varepsilon_L}^{\varepsilon^*} (\varepsilon^* - \varepsilon_i) dG(\varepsilon_i) \right\} y,$$

and as $\varepsilon^* \rightarrow \varepsilon_H$, $\max \left\{ \varepsilon^*, \bar{\varepsilon} + \delta(v) \theta \int_{\varepsilon_L}^{\varepsilon^*} (\varepsilon^* - \varepsilon_i) dG(\varepsilon_i) \right\} \rightarrow \max \{ \varepsilon_H, \bar{\varepsilon} + \delta(v) \theta (\varepsilon_H - \bar{\varepsilon}) \} = \varepsilon_H$, so $\tilde{A}_D^s \rightarrow A^s$, i.e., only dealers hold equity overnight. Thus, from (46), $\Phi - k \rightarrow \Pi(v)$, where

$$\Pi(v) \equiv \beta\kappa(v) (1 - \theta) (\varepsilon_H - \bar{\varepsilon}) y (1 - \pi) A^s - k.$$

Notice that

$$\lim_{v \rightarrow \infty} \Pi(v) = -k < 0 < \beta(1 - \theta) (\varepsilon_H - \bar{\varepsilon}) y (1 - \pi) A^s - k = \Pi(0)$$

and $\Pi'(v) = \beta\kappa'(v)(1-\theta)(\varepsilon_H - \bar{\varepsilon})y(1-\pi)A^s < 0$, so there exists a unique $v \in (0, \infty)$ that satisfies $\Pi(v) = 0$. To conclude we only need to show that under the hypothesis of the proposition, $\Pi(v) = 0$ is equivalent to (41). Notice that $\delta''(v) < 0$ implies $\kappa(v) = \delta(v)/v \leq \delta'(0)$ for any $v \geq 0$. In particular, for $v = 0$ this implies $1 \leq \delta'(0)$. Hence

$$0 < \beta(1-\theta)(\varepsilon_H - \bar{\varepsilon})y(1-\pi)A^s - k \leq \beta\delta'(0)(\varepsilon_H - \bar{\varepsilon})y(1-\pi)A^s - k$$

which means that $v > 0$ in the Planner's solution. Then (41) must hold with equality and the optimal v satisfies

$$\beta\delta'(v)(\varepsilon_H - \bar{\varepsilon})y(1-\pi)A^s - k = 0. \tag{131}$$

Finally, notice that $\delta'(v) = \kappa(v) + \kappa'(v)v$, so if the Hosios condition holds, $1-\theta = 1 - \frac{-\kappa'(v)v}{\kappa(v)} = \frac{\delta'(v)}{\kappa(v)}$ and (131) is identical to $\Pi(v) = 0$. ■

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