Dynamic Collective Choice with Endogenous Status Quo[*]

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Abstract

This paper analyzes an ongoing bargaining situation in which preferences evolve over time and the previous agreement becomes the next status quo, determining the payoffs until a new agreement is reached. We show that the endogeneity of the status quo exacerbates the players' conflict of interest and decreases the responsiveness of the bargaining outcome to the environment. In some cases, it can lead the negotiations to complete gridlock. Compared to a bargaining protocol with an exogenous status quo, the status quo stays in place more often and equilibrium welfare is lower.

In a legislative setting, this model shows that the inertial effect of the endogenous status quo can be mitigated by concentrating decision power, and can be eliminated by sunset provisions.

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1 Introduction

This paper analyzes an ongoing collective decision problem in which i) there are shocks to the environment that affect individual preferences, and hence call for renegotiation of the past agreement; and ii) agreements are determined using an endogenous status quo protocol: the previous agreement stays in place and determines the payoffs until a new agreement is reached.

A prominent example of negotiations in a changing environment with an endogenous status quo is legislative bargaining. For instance, legislators' preferences over fiscal policies reflect heterogeneous ideologies and constituencies, but are also affected by shocks such as business cycles, changes in the country's credit rating, or the vagaries of public opinion. At the same time, in most democracies, a vast array of fiscal policies are set using an endogenous status quo: once enacted, the law or program continues in effect until further legislative action is taken. For example, about two-thirds of the U.S. federal budget—called mandatory spending—continues year after year by default. Outside of the fiscal sphere, many ideologically charged issues such as immigration, financial regulation, minimum wage, civil liberties, and national security are also affected by shocks (e.g., demographic transitions, financial innovation, national security threats) and are typically regulated by permanent legislation.[1]

The starting point of our analysis is the observation that the endogenous status quo creates a *dynamic linkage* between bargaining periods. In a changing environment, this linkage presents the negotiating parties with a trade-off between responding to the current shock and securing a favorable position for future bargaining. To illustrate this trade-off, consider the case of legislators in the U.S. Congress negotiating the size of mandatory spending. During a recession, generous deficit spending may be favored by all parties to stimulate short-term economic growth. During a boom, all parties may agree to use the extra tax revenues to bring the public debt under control. In normal times, however, legislators may genuinely disagree on the optimal level of public spending. Anticipating this disagreement, fiscal conservatives may be reluctant to increase public spending

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[1] Temporary legislation and sunset provisions—provisions attached to legislation that set its expiration date—are the exception rather than the norm in most democracies. Even in the U.S., which is probably one of the countries whose local and federal legislatures rely most on temporary legislation (Gersen 2007), the most important laws are typically permanent. For instance, in the fiscal sphere, the Social Security Act of 1935 and its later expansion by the Johnson administration's Great Society programs were permanent provisions. Likewise, the U.S. Earned Income Tax Credit and its subsequent expansions in 1986, 1990, 1993, and 2001 did not have a sunset provision. See Section 6 for a discussion of temporary legislation.

during a recession, out of fear that their liberal counterparts will veto a return to fiscal discipline when the economy improves. Similarly, liberals may refuse to lower spending in times of economic prosperity, out of fear that conservatives will oppose a fiscal expansion when the boom is over.

In this paper, we analyze the trade-off created by the combination of a changing environment and the endogenous status quo protocol. We show that it results in large and detrimental distortions in players' behavior, and we study ways of mitigating these distortions.

In the basic model, two players engage in an infinite sequence of collective choices over two policies, called left and right. Players' preferences are unambiguously ordered along the ideological line: one player has a stronger preference for left than her opponent. Both players, however, can prefer either alternative with positive probability. In each period, the state of the economy changes and affects players' preferences. At the beginning of each period one policy, called the current status quo, is in place. If both players agree to move away from the status quo, the new policy is implemented. Otherwise, the status quo stays in place. In both cases, the implemented policy determines the players' payoffs in this period and becomes the new status quo. We are looking for the stationary equilibria of this game.

We show that consistent with the motivating example the endogeneity of the status quo distorts players' behavior. Each player is willing to sacrifice her current payoff to secure a favorable status quo for the next period: in any period, she votes for her preferred status quo unless the relative payoff benefit from the other policy exceeds a certain positive cutoff. In other words, each player's vote is strictly biased in favor of one alternative.

The equilibrium analysis reveals that a player's preferred status quo—and hence her voting bias—is determined not by her expected preferences, but by her expected preferences *conditional on disagreement*. For this reason, even if both players on average prefer right, in equilibrium the more leftist player biases her vote in favor of left. In other words, players' voting biases depend not on their absolute but on their relative ideology.

This leads us to the central finding of the paper: the endogenous status quo exacerbates the ideological differences between players. To make this statement more formal, consider the alternative bargaining protocol in which the status quo is exogenously set in each period. With an exogenous status quo, today's policy has no impact on tomorrow's status quo, so players vote for their most preferred policy in every period. Therefore, they disagree only when their preferences disagree. Since under the endogenous status quo, the more leftist player biases her vote in favor of left and the more rightist player biases her vote in favor of right, the endogenous status quo protocol induces players to disagree more often than they would under the exogenous status quo protocol. As a result, the status quo stays in place more often, and the bargaining outcome is less responsive to the environment.

We show that the polarizing effect of the endogenous status quo can be quite dramatic. Arbitrarily similar players may become very biased for opposite alternatives and behave as if their interests were highly discordant. Moreover, if players are patient enough, the negotiations may end up in a gridlock in which players vote solely along ideological lines. Despite the fact that players' preferences agree with positive probability in every period, the bargaining outcome is completely unresponsive to the preference realization. It is worth noting that this result is not a direct consequence of players' patience, but stems instead from the fact that players' biases reinforce each other. More patient players care more about tomorrow's status quo, which increases players' voting biases and leads to more disagreement. A greater probability of disagreement further increases the importance of the status quo, which in turn further increases the voting biases.

In legislative bargaining, the behavior described above reminds us of what is commonly referred to as *partisanship:* each legislator votes for one particular alternative more often than is favored by her current preferences, and this bias results in more polarization. Although partisanship is often defined as a blind allegiance to a party or ideology, this paper shows that when the status quo is endogenous, a similar behavior can be generated by strategic considerations.

To assess the welfare effect of the endogeneity of the status quo, we compare the equilibrium welfare under the endogenous and the exogenous status quo protocols. Our analysis shows that with an exogenous status quo, players do not display any partisanship, the policy is more reactive to shocks, and welfare is higher.

In the legislative bargaining context, the last result provides a rationale for sunset provisions. A sunset provision is a clause that repeals a law, a tax change, or a program after a specific date, unless further legislative action is taken. Hence, an automatic sunset provision is strategically equivalent to an exogenous status quo. Sunset provisions have usually been advocated to improve parliamentary control of executive agencies, or to evaluate the efficiency of new laws. The rationale advanced by this paper has a more strategic flavor: sunset provisions sever the link between today's agreement and tomorrow's status quo, which mitigates conflicts of interest among legislators, and makes policies more responsive to the environment.

Our results extend to an $N-$player game with an arbitrary voting rule. Within this framework, we show that the inertial effect of the endogenous status quo depends on the dispersion of power implied by the voting rule. If a voting rule requires the approval of a larger set of players, the probability of disagreement increases; and it does so for two reasons. As in a static game, disagreement becomes more likely because more players have to agree. However, since disagreement becomes more likely, players become more partisan, which further increases the probability of disagreement. When the preference distribution is not too skewed, we show that increasing the dispersion of power is socially detrimental.

In principle, the decision process in a democracy is based on the simple majority rule, which, according to our definition, has a high concentration of power. However, in most modern democracies, legislative proposals have to pass several additional institutional hurdles to be enacted. These hurdles can take many forms, such as presidential vetoes, supermajority requirements, bicameralism, or judicial review by a constitutional court. Any such checks and balances increase the set of players whose agreement is necessary to change the policy, and thus increase partisanship. Our analysis therefore implies that the endogenous status quo exacerbates the inertial effect of checks and balances.

The conclusions of our analysis are presented in the legislative bargaining context, but we want to stress that they apply to other environments such as renegotiation of labor or financial contracts, trade agreements, and international treaties (e.g., for the World Trade Organization or the European Union). In particular, they have implications for monetary policy institutions. In some countries, monetary policy is set by a committee with heterogeneous preferences and beliefs, and the interest rate stays the same until the committee agrees to change it according to its internal voting rule.[2] Our results show that the endogeneity of the status quo, and the voting rule used in monetary policy making, can greatly affect the ability of the committee to respond to economic shocks and to smooth out the business cycle.

Despite its pervasiveness, the impact of the endogenous status quo in a changing environment has received little attention in the literature. This is likely due to the complexity of the strategic

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[2] See Riboni and Ruge-Murcia (2008) for more on the role of the status quo in monetary policy institutions.

interactions that it generates. This paper simplifies the analysis by restricting the choice set in each period to two alternatives, which eliminates the need to specify the details of the stage game, such as determining the proposer or the sequence of offers. However, the central result of this paper does not rely on this restriction. As long as in the stage game the more rightist player benefits from a more rightist status quo, under the endogenous status quo protocol this player will favor agreements that are to the right of what her current preferences suggest. This simple intuition suggests that the polarizing effect of the endogenous status quo should hold in richer environments.

The paper is organized as follows. Section 2 discusses the related literature. Section 3 describes the basic model. In Section 4, we solve a simple example that illustrates the main findings of the model. Section 5 formalizes these findings. Section 6 compares the equilibrium welfare under the endogenous and the exogenous status quo protocols. Section 7 extends the model to N players. Section 8 discusses how the results extend to more general preference distributions. Section 9 concludes. All proofs are in the appendix.

2 Related literature

The formal literature on dynamic bargaining with an endogenous status quo started with the seminal paper of Baron (1996).[3] His model has been extended in various settings by Baron and Herron (2003), Kalandrakis (2004, 2007), Cho (2005), Fong (2006), Bernheim et al. (2006), Battaglini and Palfrey (2007), Anesi (2010), Diermeier and Fong (2011), Baron et al. (2011), and Zapal (2011a).[4] These models, however, consider static environments: policies evolve over time not because preferences change, but because the set of actions available to each player varies across voting stages. Most of these papers focus on the impact of the bargaining protocol on the proposer power. We abstract away from the distributional issue of proposal power and focus instead on the efficiency and responsiveness of the policy-making process to preference shocks.[5]

It is, however, interesting to note that in our model the endogenous status quo exacerbates the

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[3] Epple and Riordan (1987) study a similar model but consider nonstationary equilibria. The principle of an evolving status quo was first introduced in the cooperative bargaining literature by Kalai (1977).

[4] The models of Bernheim et al. (2006) and Diermeier and Fong (2011) are cast in a single policy period, but they can be extended or interpreted as dynamic legislative bargaining games.

[5] Because most of these models consider the division of a pie of exogenous size or single-peaked preferences, equilibrium outcomes are always efficient in a static sense and can be inefficient in a dynamic sense only when citizens are risk-averse. In contrast, in our model preferences vary, and as a result, equilibrium outcomes may be Pareto inefficient even in a static sense, independent of risk aversion.

conflict between players, while in some of the aforementioned models, the endogenous status quo has a moderating effect: the endogenous status quo forces the proposer to get closer to the ideal point of the median voter (Baron 1996, Baron and Herron 2003, Zapal 2011a) and reduces the incentives of voters to expropriate each other (Diermeier and Fong 2011). The following observation explains the apparent contradiction between these results and ours. In these papers, players' behavior is driven by the fear of having their payoffs expropriated by the winning coalitions in the next period. With an endogenous status quo, players can minimize the cost of being excluded from the winning coalition tomorrow by implementing a moderate policy today. In our model, players' behavior is driven by the fear that the future status quo will not be in line with their preferences. With an endogenous status quo, players can minimize the probability of an unfavorable status quo tomorrow by voting today for a policy that is more in line with their own ideology.

Even though dynamic bargaining with an endogenous status quo in a stochastic environment is at the center of many economically relevant situations, the existing literature on this topic is scarce. This may be a consequence of the relative intractability of these games. As Romer and Rosenthal (1978) showed in a static setup with single-peaked preferences, the induced preferences over the status quo are typically not convex, which makes the multi-period extension technically hard to analyze. With a continuum of alternatives and an infinite horizon, the existence of the stationary equilibrium is not guaranteed even under standard preference specifications.[6]

To the best of our knowledge, only Diermeier and Fong (2008), Riboni and Ruge-Murcia (2008), Duggan and Kalandrakis (2009), Bowen et al. (2012), and Zapal (2011b) make progress on this front. Adding noise to the status quo, Duggan and Kalandrakis (2009) establish the existence of an equilibrium. The generality of their model does not allow an analytical characterization of the equilibrium, however, so they resort instead to numerical methods. Riboni and Ruge-Murcia (2008) analyze a game with quadratic utility functions and a finite state space. They analytically solve a two-period, two-state example, but use numerical solutions for the general model. Diermeier and Fong (2008) analyze a two-period three-state model with a richer institutional framework. Bowen et al. (2012) analyze a special case and focus on the interplay of the endogenous and the exogenous status quo in the U.S. budgetary process. Finally, Zapal (2011b) characterizes the infinite-horizon

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[6] See, e.g., Kalandrakis (2004, 2007) or Duggan and Kalandrakis (2009) for more on this issue.

equilibrium in a symmetric two-state case.[7] Our paper differs from these contributions in that we simplify the space of alternatives, but fully characterize the policy dynamics with an infinite bargaining horizon for any preference distributions. Moreover, our institutionally sparse model allows us to isolate the effect of the endogeneity of the status quo in a transparent way.

Montagnes (2010) looks at a two-period financial contracting environment in which the current contract serves as the default option in future negotiations. He shows that both contracting parties may prefer to commit ex ante to ceding a future decision power. Such a commitment breaks the dynamic linkage and avoids ex-ante inefficiencies.

Fernandez and Rodrik (1991) and Alesina and Drazen (1991) have emphasized that the distributional uncertainty of policy reforms can lead to status quo inertia. In our model, it is not the uncertainty but the evolution of preferences over time that drives the result.

Our results on policy responsiveness to shocks are related to the political economy literature on growth and the dynamics of welfare policies.[8] In this literature, the current policy affects future preferences (via private or public investment decisions). This dynamic linkage can generate policy persistence. In contrast, in our paper the implemented policy does not affect future preferences, but inertia emerges because today's policy affects players' positions for future bargaining.

Our results on the effect of the concentration of voting power contrast with the literature on distributive politics. As Buchanan and Tullock (1962) and Riker (1962) first argued, majoritarian decision making allows the concentration of benefits and the collectivization of costs, and thus leads to the adoption of inefficient pork-barrel programs.[9] Contrary to our model, the concentration of power exacerbates these perverse incentives, and efficiency is restored only when unanimity rule is used. Battaglini and Coate (2007, 2008) extend this framework to a dynamic legislative model of public finance. The availability of pork-barrel programs leads the minimal winning coalition to pass inefficient budgets and be present-biased; and the inefficiency is higher, the lower the supermajority requirement. In our model, the enduring nature of policies leads voters to be future-biased; and

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[7] In line with our results, Zapal (2011b) shows that under the endogenous status quo protocol, the policy may remain constant even though preferences evolve over time. However, contrary to our setup, for the particular reference distribution he considers, a constant policy is socially optimal.

[8] See, among others, Glomm and Ravikumar (1995), Krusell and Rios-Rull (1996, 1999), Saint Paul and Verdier (1997), Coate and Morris (1999), Benabou (2000), Saint Paul (2001), Hassler et al. (2003, 2005), Battaglini and Coate (2007, 2008), and Prato (2011).

[9] Ferejohn et al. (1987), Baron and Ferejohn (1989), and Baron (1991, 1993) first formalized this prediction in models of legislative bargaining.

they are more future biased the larger, the supermajority requirement.

Finally, Casella (2005) shows that linking voting decisions across time allows voters to express their preference intensity, which can be socially beneficial. Our results suggests that the endogenous status quo protocol, despite the pervasiveness of this institution, is not an efficient way to elicit preference intensity. Barbera and Jackson (2010) let ex ante identical voters choose the group decision rule after having learned their first-period preferences. Similarly to our paper, in their framework bundling the current and future decision rules generates inefficiencies. But since the dynamic linkage is only between the first period and the subsequent ones, sufficiently patient players always select the optimal voting rule.

3 The model

Two players, i and j, are in a relationship that lasts for infinitely many periods. In each period t, players adopt one of two alternatives, $y^t \in \{L, R\}$. The utility of player $k \in \{i, j\}$ in period t depends on the alternative adopted in period t and is given by

$$u\left(\theta_k^t, y^t\right) = \begin{cases} \theta_k^t \text{ if } y^t = R \\ -\theta_k^t \text{ if } y^t = L \end{cases}. \tag{1}$$

Hence, if θ_k^t is positive (negative), player k prefers alternatives R (L) to be implemented in period t. The realization of $\left(\theta_i^t, \theta_j^t\right)$ summarizes players' preferences over the current policy, so we refer to θ_k^t as player k's *current preference* in period t.

In each period, a state s is drawn from a finite set S. The process $\left(s^t\right)_{t\geq 1}$ is Markovian with a stationary and irreducible transition matrix and an initial distribution $\pi^0 \in \Delta(S)$.[10] The probability of moving from state $s \in S$ to state $s' \in S$ is denoted by $\pi(s, s')$. In each period, if the state is s, the preference parameters $\left(\theta_i^t, \theta_j^t\right)$ are drawn from a joint distribution with an integrable density function denoted by f^s. We assume that for all $k \in \{i, j\}$ and in each state, the marginal distribution of θ_k^t has full support.

Throughout the paper, subscripts refer to the individuals while superscripts refer either to the

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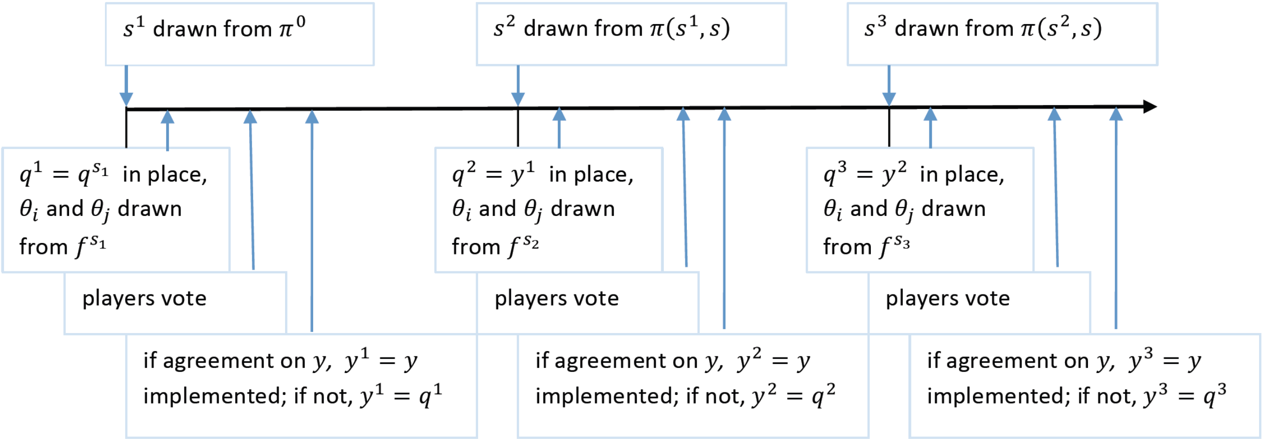
[10] A Markov process is irreducible if the probability of going from any state to any other state in a finite number of periods is strictly positive. It is stationary if the transition probabilities do not depend on time.

time period or to the state. For a generic parameter p, the bold symbol $\boldsymbol{p}$ refers to the vector (p_i, p_j), and p^S refers to a vector of state-dependent parameters $(p^s)_{s\in S}$. In particular, if the parameter p is real, $\boldsymbol{p}^S$ is an element of $\mathbb{R}^{2S}$.

The game proceeds as follows. Each period starts with one alternative in place. We call this alternative *the status quo* in period t and denote it by q^t. At the beginning of each period, the state s^t and subsequently the preference profile $\boldsymbol{\theta}^t$ are drawn. After players observe q^t, s^t, and $\boldsymbol{\theta}^t$, they vote on which alternative to adopt in period t. If both players vote for the same alternative, this alternative is implemented. If they disagree, the status quo q^t stays in place. The implemented alternative y^t, be it the new agreement or the status quo q^t, determines the payoffs in period t and becomes the status quo for the next period q^{t+1}. Players discount future payoffs with the same factor $\delta \in (0,1)$. The initial status quo is allowed to depend on the initial state and is denoted $q^S \in \{R, L\}^S$. We denote the corresponding game by $\Gamma^{en}_{q^S,\pi^0}$. The following diagram summarizes this game.



In this game, the alternative implemented in some period t has no effect on the state and the preferences in future periods, so each period is an independent social choice problem. The dynamic linkage between periods comes solely from the endogeneity of the status quo. In the sequel, to isolate the effect of the endogeneity of the status quo on equilibrium behavior and welfare, we shall compare $\Gamma^{en}_{q^S,\pi^0}$ to the game $\Gamma^{ex}_{q^S,\pi^0}$, which differs from $\Gamma^{en}_{q^S,\pi^0}$ only in that the status quo is exogenously fixed at q^S in every period. That is, in each period t, if the state is s, the status quo is q^s independent of previous policies and players' actions.

As is customary in dynamic voting games with an infinite horizon, we look for stationary equilibria in stage-undominated strategies (henceforth, equilibria) as defined in Baron and Kalai (1993).[11] As shown in Baron and Kalai (1993), these equilibria have a focal point property that derives from their simplicity. In the legislative sphere, the stationarity assumption can be justified on the grounds that the game is played by a sequence of legislators who are never certain to be reelected. In such cases, the institutional memory required for more sophisticated nonstationary equilibria involving infinitely nested punishment strategies may be inappropriate. Stage-undomination is a standard equilibrium refinement in voting games, which basically amounts to assuming that in every period, players cast their votes as if they were pivotal. This refinement rules out pathological equilibria such as both players always voting for the status quo.[12]

A few comments on the modeling assumptions are in order. First, our setup allows the preferences to be correlated across players and across time. The stationarity of the preference distribution is a simplifying assumption, which is consistent with the recurring nature of the shocks that affect issues such as taxation, public spending, immigration, or civil liberties (e.g., economic cycles, demographic transitions, public opinion swings, or national security threats). Second, we analyze a two-player game which requires unanimity for changing the status quo, but in section 7 we show that our results extend to an $N-$player game with a large class of voting rules. Third, restricting attention to two alternatives allows us to abstract away from the details of the stage game and the issue of proposal power.[13] It thereby allows us to isolate in a transparent way the effect of the endogeneity of the status quo on the equilibrium outcomes. Fourth, what players know about each other's current preferences is immaterial. Finally, we assume that today's action has no impact on tomorrow's preferences (that is, π does not depend on the status quo q) because this dynamic linkage has already received some attention in the dynamic political economy literature (as noted

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[11] Stage-undominated stationary equilibria, or variants thereof, are used in almost all of the infinite-horizon models cited in this paper. The only exceptions that we are aware of are Epple and Riordan (1987), and Baron and Ferejohn (1989). Both papers prove results that have the flavor of the folk theorem in repeated games.

[12] Moreover, the equilibria eliminated by this refinement hinge on details of the bargaining protocols which are difficult to map to reality. For instance, they would disappear if we assumed instead that players vote sequentially. See, e.g., Acemoglu et al. (2009).

[13] With two alternatives, many static bargaining protocols are equivalent. In particular, using standard equilibrium concepts, equilibrium outcomes are the same when players vote simultaneously or sequentially, when they make take-it-or-leave-it offers, or when we allow for n rounds of bargaining within each period with either a random or an alternating proposer.

in the literature review in section 2). Ruling this linkage out allows us to isolate the effect of the endogenous status quo.

4 An example

We start by solving a simple example that illustrates the workings of the model. The results derived for this example are formalized and generalized in the next sections.

Assume that $|S| = 1$, $\theta_i^t = \bar{\theta}_i + \varepsilon^t$, and $\theta_j^t = \bar{\theta}_j + \varepsilon^t$, where $\bar{\theta}_i,\bar{\theta}_j \in \mathbb{R}$, $\left(\varepsilon^t\right)_{t\geq 1}$ is i.i.d. over time, and for each t, $\varepsilon^t \sim N(0,1)$. Hence, players' preferences are perfectly correlated, ε^t is the common shock, and $\bar{\theta}_i$ and $\bar{\theta}_j$ are the expected preferences of players i and j, respectively. Since $|S| = 1$, we will denote the game with an endogenous status quo by Γ_q^{en} and the game with an exogenous status quo by Γ_q^{ex}.

Let us first derive the equilibrium characterization of Γ_q^{en} with a simple heuristic reasoning. For any player $k \in \{i,j\}$, the policy implemented in period t impacts player k's payoff via two channels. First, it affects her current payoff θ_k^t. Second, it determines the future status quo. Let $V_k(q)$ be the continuation value for player $k \in \{i,j\}$ when the status quo is q. Since player k votes as if she were pivotal, in period t she votes for R if

$$\theta_k^t + \delta V_k(R) > -\theta_k^t + \delta V_k(L),$$

and for L if the reverse inequality holds. Therefore, she uses a cutoff strategy with the following cutoff:

$$c_k = \frac{\delta}{2}\left(V_k(L) - V_k(R)\right). \tag{2}$$

Observe that future payoffs depend on the current status quo only if players disagree in the next period. Such disagreement occurs when players' preferences θ_i^t and θ_j^t are on opposite sides of their respective cutoffs c_i and c_j. Hence, we can rewrite (2) as follows:

$$\begin{aligned} c_k = \frac{\delta}{2} \quad & \left(\int_{-\infty}^{c_j}\int_{c_i}^{\infty}\left(-\theta_k + \delta V_k(L) - \left(\theta_k + \delta V_k(R)\right)\right) f(\boldsymbol{\theta})\, d\theta_i d\theta_j\right. \\ & \left. + \int_{c_j}^{\infty}\int_{-\infty}^{c_i}\left(-\theta_k + \delta V_k(L) - \left(\theta_k + \delta V_k(R)\right)\right) f(\boldsymbol{\theta})\, d\theta_i d\theta_j\right). \end{aligned}$$

Substituting (2) inside the above integral, we determine that the equilibrium cutoffs solve the following fixed-point problem:

$$c_k = \delta \left(\int_{-\infty}^{c_j} \int_{c_i}^{\infty} (c_k - \theta_k) f(\boldsymbol{\theta})\, d\theta_i d\theta_j + \int_{c_j}^{\infty} \int_{-\infty}^{c_i} (c_k - \theta_k) f(\boldsymbol{\theta})\, d\theta_i d\theta_j \right). \tag{3}$$

One can interpret the equilibrium cutoffs in Γ_q^{en} as *voting biases*. With positive probability, a player with a positive (negative) cutoff votes for L (R) even though her current preferences favor R (L). Players' voting behavior is biased because they face a trade-off between implementing a policy that is optimal according to their current preferences and securing a status quo that is more in line with their expected preferences. From (2), we see that the sign of c_k determines whether player k prefers the status quo to be R (c_k negative) or L (c_k positive), and the absolute value of c_k measures the intensity of this preference. Hence, $\theta_k - c_k$ measures player k's intertemporal preferences over the current policy. Equation (3) shows that the voting bias of each player is given by her expected intertemporal preferences in the next period *conditional on disagreement*.

In contrast to Γ_q^{en}, in the game in which the status quo is exogenous in every period, current agreements have no effect on future periods. Therefore, in any equilibrium of Γ_q^{ex}, both players vote myopically for their most preferred policy in every period. In other words, in Γ_q^{ex} players also use cutoff strategies, but the cutoffs are zero. Comparing the equilibria of Γ_q^{en} and Γ_q^{ex} shows that the signs and magnitude of the equilibrium cutoffs in Γ_q^{en} completely capture the effect of the endogeneity of the status quo on players' voting behavior.

We solve Equation (3) numerically for $\delta = 0.9$ and $\bar{\theta}_i = 0.5$, while varying $\bar{\theta}_j$. When $\bar{\theta}_j = -0.5$, then the resulting cutoffs are $c_i \approx -4.49$ and $c_j \approx 4.49$. When $\bar{\theta}_j = 0.1$, they are $c_i \approx -0.04$ and $c_j \approx 0.038$.

We would like to point out a few features of this example. First, in both numerical examples, $c_i < 0 < c_j$. Hence, player i is biased for R while player j is biased for L. Notice that this happens even though in the case in which $(\bar{\theta}_i, \bar{\theta}_j) = (0.5, 0.1)$, both players prefer R on average. The reason for this finding is that players' voting biases reflect their preferences over the next period's status quo, and the status quo matters only in case of disagreement. In both cases, $\bar{\theta}_i > \bar{\theta}_j$, so when players disagree, i prefers R while j prefers L. Hence, the direction of players' voting biases is determined not by their absolute but by their relative ideological position.

Second, $c_i < 0 < c_j$ means that player i behaves as if she were more rightist than she is, while player j behaves as if she were more leftist than she is. Hence, the voting cutoffs in Γ_q^{en} act as a polarization-magnifying preference shift: players disagree more often than their actual preferences do. More precisely, players' current preferences disagree—and hence, the players vote for opposite alternatives in Γ_q^{ex}—when $\varepsilon^t \in \left(-\bar{\theta}_i, -\bar{\theta}_j\right)$. In the equilibrium of Γ_q^{en}, players vote for opposite alternatives when $\varepsilon^t \in \left(-\bar{\theta}_i + c_i, -\bar{\theta}_j + c_j\right)$. Comparing these two disagreement regions shows that the endogeneity of the status quo increases the probability that players disagree.

Third, note that this polarizing effect can be large: for example, when $\left(\bar{\theta}_i, \bar{\theta}_j\right) = (0.5, 0.1)$, in any period the probability that players' preferences disagree is 0.15, while the probability that players vote for opposite alternatives in Γ_q^{en} is 0.3. When $\left(\bar{\theta}_i, \bar{\theta}_j\right) = (0.5, -0.5)$, these probabilities are 0.38 and close to 1, respectively.

Finally, by comparing the equilibrium of Γ_q^{en} for the two preference distributions, we see that the voting biases increase with the preference polarization: as we make the leftist player even more leftist (as $\bar{\theta}_j$ decreases), i's bias in favor of right $(-c_i)$ and j's bias in favor of left (c_j) increase. Hence, more polarized players disagree more often not only because their preferences are farther apart, but also because their preference polarization is magnified by their voting behavior. In fact, as δ tends to 1, for $\left(\bar{\theta}_i, \bar{\theta}_j\right) = (0.5, -0.5)$, the voting cutoffs tend to $c_i = -\infty$ and $c_j = \infty$. That is, as players become very patient, the negotiations reach complete gridlock: players always vote for opposite alternatives even though their preferences agree with positive probability in every period.

The following observation is key to understanding the magnitude of the equilibrium cutoffs in Γ_q^{en}. Even if players expect that their opponent will use a cutoff equal to zero, they still expect some disagreement. Hence, player j prefers to use a positive cutoff to defend L as a status quo, and player i prefers to use a negative cutoff to defend R as a status quo. But if players use nonzero cutoffs, the probability of disagreement, and hence the probability that the status quo stays in place, increases. This makes defending the status quo even more important. Realizing that, each player has an incentive to become even more biased, which further increases the probability of disagreement. In other words, players' voting biases reinforce each other.

Whenever the voting cutoffs are nonzero, players implement Pareto-dominated alternatives with positive probability. For example, when $q^t = R$, $\theta_j^t < 0$, and $c_i < \theta_i^t < 0$, player i vetoes the Pareto-optimal alternative L. This suggests that the endogeneity of the status quo is socially detrimental.

When we compare the equilibrium welfare in Γ_q^{ex} and in Γ_q^{en}, we indeed find that fixing the status quo improves welfare in both of the considered cases. Perhaps more surprisingly, the identity of the exogenous status quo does not matter: even when $\left(\bar{\theta}_i, \bar{\theta}_j\right) = (0.5, 0.1)$, that is, when R is socially better when players disagree, the welfare is higher when the default is fixed at L than when it is endogenous. It is also worth noting that the loss of welfare in Γ_q^{en} is so large that when $\left(\bar{\theta}_i, \bar{\theta}_j\right) = (0.5, -0.5)$ and δ is sufficiently large, both players would prefer to cede their veto power and let their opponent be the dictator instead of playing Γ_q^{en}.[14]

5 The equilibrium

The following proposition characterizes the equilibria of the game Γ_{q^S,π^0}^{en} for general preference distributions.

Proposition 1 *In any equilibrium of* Γ_{q^S,π^0}^{en}*, the players use state-dependent, status-quo-independent cutoff strategies: there exists* $\boldsymbol{c}^S \in \mathbb{R}^{2S}$ *such that in state* $s \in S$*, player* $k \in \{i, j\}$ *votes for* R *if* $\theta_k > c_k^s$ *and for* L *if* $\theta_k < c_k^s$.
The equilibrium cutoffs are the fixed points of the mapping $\boldsymbol{H}^S$ *defined as follows: for all* $s \in S$ *and all* $\boldsymbol{c}^S \in \mathbb{R}^{2S}$,

$$\boldsymbol{H}^s\left(\boldsymbol{c}^S\right) = \delta \sum_{s' \in S} \pi\left(s, s'\right) \left(\int_{-\infty}^{c_j^{s'}} \int_{c_i^{s'}}^{\infty} \left(\boldsymbol{c}^{s'} - \boldsymbol{\theta}\right) f^{s'}\left(\boldsymbol{\theta}\right) d\theta_i d\theta_j + \int_{c_j^{s'}}^{\infty} \int_{-\infty}^{c_i^{s'}} \left(\boldsymbol{c}^{s'} - \boldsymbol{\theta}\right) f^{s'}\left(\boldsymbol{\theta}\right) d\theta_i d\theta_j \right). \tag{4}$$

The set of equilibrium cutoffs is a complete lattice for the partial order $(\leq, \geq)^S$ *defined as follows: for all* $\boldsymbol{c}^S, \boldsymbol{d}^S \in \mathbb{R}^{2S}$, $\boldsymbol{c}^S \,(\leq, \geq)^S\, \boldsymbol{d}^S$ *if for all* $s \in S$, $c_i^s \leq d_i^s$ *and* $c_j^s \geq d_j^s$.

Consistent with the example solved in Section 4, players use cutoff strategies. However, since the continuation of the game depends in general on the state, cutoffs are state-dependent. Equation (4) states that the cutoffs of player k are given by the expected intertemporal preferences of player k, $c_k^s - \theta_k$, conditional on players disagreeing in the next period, i.e., conditional on $c_i^s - \theta_i$ and $c_i^s - \theta_i$ being of opposite sign.

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[14] See example 5 in Dziuda and Loeper (2010) for a formal proof of this result.

The cutoff structure of the equilibria holds for any preference distribution. However, the properties of the voting cutoffs depend on that distribution. For clarity of exposition, we make the following assumption:

Assumption 1 *In all states $s \in S$, $\theta_i \geq \theta_j$ with probability 1, and in some state $s' \in S$, θ_i and θ_j are of opposite sign with positive probability.*

Assumption 1 has a natural interpretation in political economy or monetary policy applications: players can be unambiguously ranked on the ideological spectrum. Player i is always more rightist than player j (there is no preference reversal), and players disagree with positive probability.[15] Note, however, that this assumption imposes no restriction on the preference distribution of a single player nor on the severity of the conflict of interest between players: both players might prefer policy L arbitrarily often in some state s and policy R arbitrarily often in another state s'. Assumption 1 is relaxed in Section 8.

5.1 Voting biases and disagreement

The following proposition states that in $\Gamma^{en}_{q^S,\pi^0}$, players' voting behavior is more extreme than their actual preferences: the more leftist player is always biased in favor of L and the more rightist player is always biased in favor of R.

Proposition 2 *In all equilibria of $\Gamma^{en}_{q^S,\pi^0}$, for all $s \in S$, $c^s_i < 0$ and $c^s_j > 0$.*

To understand the consequences of the endogeneity of the status quo, we compare the players' behavior under the bargaining protocol $\Gamma^{en}_{q^S,\pi^0}$ with their behavior under the bargaining protocol $\Gamma^{ex}_{q^S,\pi^0}$. The exogenous status quo protocol is a natural alternative to the endogenous status quo protocol because the former is arguably the simplest protocol which severs the link between today's policy and tomorrow's status quo. The following remark states that in $\Gamma^{ex}_{q^S,\pi^0}$, the players consider each period in isolation and vote according to their current preferences.

Remark 1 *In the unique equilibrium of $\Gamma^{ex}_{q^S,\pi^0}$, the players use voting cutoffs $c^s_i = c^s_j = 0$ in all states and all periods.*

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[15] If players never disagree, the bargaining situation is trivial. Our results would still hold, but strict inequalities would have to be replaced by weak ones.

The following corollary compares $\Gamma^{en}_{q^S,\pi^0}$ with $\Gamma^{ex}_{q^S,\pi^0}$ and delivers the main qualitative insight of this paper: the endogenous status quo amplifies the ideological differences between players. They disagree more often, which increases the status quo inertia.

Corollary 1 *For all π^0, q^S, and q'^S, in any equilibrium the probability that players vote for opposite alternatives in any period t and state s is higher in $\Gamma^{en}_{q^S,\pi^0}$ than in $\Gamma^{ex}_{q'^S,\pi^0}$.*

To understand Corollary 1, note that Proposition 2 and Remark 1 imply the following: If the status quo is exogenous, in a given period t, players disagree when

$$\theta^t_j \leq 0 \leq \theta^t_i.$$

If instead the status quo is endogenous, players disagree when

$$\theta^t_j - c^s_j \leq 0 \leq \theta^t_i - c^s_i.$$

Since $c^s_i \leq 0$ and $c^s_j \geq 0$, the above inequalities imply that the set of preference realizations for which players disagree is greater under the endogenous status quo.

The equilibrium behavior of the players in $\Gamma^{en}_{q^S,\pi^0}$ reminds us of what is commonly referred to as *partisanship.* One dictionary definition for *partisanship* is "a prejudice in favor of a particular cause; a bias." In multiparty systems, this term carries a negative connotation: it refers to those who wholly support their party's policies and are reluctant to acknowledge any common ground with their political opponents. This definition resonates with the players' voting behavior in our model: each player favors a distinct alternative for which she votes more often than is justified by her current preferences, and this in turn leads to more disagreement. This model hence shows that when the status quo is endogenous, partisanship can be generated by strategic considerations.

We use the following definition throughout the paper.

Definition 1 *The partisanship of player $k \in \{i,j\}$ in state $s \in S$ is $|c^s_k|$.*

5.2 The magnitude of partisanship

The game $\Gamma^{en}_{q^S,\pi^0}$ may have multiple equilibria. Multiplicity is driven by the fact that partisanship feeds on itself. If players expect their opponent to be more partisan, they expect to disagree more often. Therefore, defending the preferred status quo becomes more important, which means that players will vote in a more partisan way.

This strategic complementarity also implies that the set of equilibrium cutoffs has a lattice structure, as stated in Proposition 1. From Proposition 2, for each player, the cutoff sign is the same across equilibria. Therefore, there exist a least and a most partisan equilibria. The following proposition further shows that the partisanship ranking coincides with the Pareto order: more partisan equilibria are Pareto worse.

Proposition 3 *If* $\boldsymbol{c}^S$ *and* $\boldsymbol{d}^S$ *are two equilibrium cutoffs of* $\Gamma^{en}_{q^S,\pi^0}$ *such that* $\boldsymbol{d}^S\,(\leq,\geq)^S\,\boldsymbol{c}^S$, *then* $\boldsymbol{c}^S$ *Pareto dominates* $\boldsymbol{d}^S$. *In particular, the least and the most partisan equilibria, i.e., the least and the greatest equilibria for the order* $(\leq,\geq)^S$, *are the Pareto best and worst equilibria, respectively.*

When deriving comparative statics and determining the magnitude of partisanship, we use Pareto efficiency as a selection criterion and focus on the least partisan equilibrium.[16] However, the same comparative statics holds for the most partisan equilibrium.

The following definition will be helpful when deriving comparative statics with respect to the preference distribution f^S:

Definition 2 *Let* f^S *and* g^S *be two preference distributions. The distribution* f^S *is more polarized than* g^S *if there exists a random variable* $\boldsymbol{\varepsilon}^S$ *with support on* $(\mathbb{R}_+\times\mathbb{R}_-)^S$ *and a random variable* $\boldsymbol{\theta}^S$ *such that the p.d.f.s of* $\boldsymbol{\theta}^S$ *and* $\boldsymbol{\theta}^S+\boldsymbol{\varepsilon}^S$ *are* g^S *and* f^S, *respectively.*

We use the terminology "more polarized" because if g^S satisfies Assumption 1, then f^S can be obtained from g^S by shifting the preferences of the rightist player farther to the right and the preferences of the leftist player farther to the left.

Denote by $\boldsymbol{c}^S\left(\delta,f^S\right)$ the cutoffs in the least partisan equilibrium of $\Gamma^{en}_{q^S,\pi^0}$ with a discount factor δ and a preference distribution f^S. The next proposition shows how partisanship varies with the main preference parameters.

Proposition 4 *In the least partisan equilibrium,*

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[16] An additional support for this equilibrium selection can be found in the proof of Proposition 10, where it is shown that the least partisan equilibrium is the limit of the finite-horizon version of the game $\Gamma^{en}_{q^S,\pi^0}$ as the bargaining horizon goes to infinity.

a) partisanship increases with patience: $\boldsymbol{c}^S\left(\delta, f^S\right)$ *is increasing in* δ *in the order* $(\leq, \geq)^S$*;*

b) partisanship increases with the polarization of the preferences: if g^S *is more polarized than* f^S*, then* $\boldsymbol{c}^S\left(\delta, g^S\right)(\leq, \geq)^S\, \boldsymbol{c}^S\left(\delta, f^S\right)$*.*

The intuition for part (a) is that when players trade off the policy's adequacy for the current environment versus securing a favorable status quo for tomorrow, more patient players put more weight on the latter, and thus are more partisan. As for part (b), the preferences of more polarized players are more likely to disagree, which makes the status quo more important, and thus increases partisanship. This result reinforces the findings of Corollary 1 in that it shows that status quo endogeneity exacerbates the ideological differences between players; and the more polarized the players, the greater the exacerbation.

The next proposition delivers the main quantitative finding: the polarizing effect of the endogenous status quo can be dramatic.

Proposition 5 *There exists a preference distribution* g^S *such that for all* f^S *that are more polarized than* g^S*,* $\lim_{\delta\to 1} \boldsymbol{c}^s\left(\delta, f^S\right) = (-\infty, +\infty)$*, for all* $s \in S$*.*

Proposition 5 states that when players are sufficiently polarized and patient, their partisanship can lead to complete gridlock. Even though in all periods, their current preferences agree with positive probability, they always vote for opposite alternatives. As a result, the policy is totally unresponsive to the shocks to the environment.[17]

Observe that this result is not a mechanical consequence of increasing patience. The alternative adopted in period t impacts players' payoffs in some subsequent period t' only if players' preferences disagree for all periods between $t+1$ and t'. For any finite level of partisanship, such disagreement happens with probability smaller than 1. Hence, the difference in continuation value induced by different status quos stays finite even as $\delta \to 1$. For this reason, irrespective of the players' patience, the best response of a player to a finite level of partisanship of her opponent is also a finite level of partisanship. What drives the completely unresponsive behavior of patient players is the vicious cycle in which patience increases partisanship, partisanship then increases the life expectancy of the status quo, which in turn increases partisanship.

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[17] Assumption 1 is not needed for Proposition 5. More precisely, we show in the appendix that g^S can be obtained from any preference distribution by shifting players in opposite directions.

Proposition 5 does not state how polarized the preference distribution g^S must be. The following corollary shows that gridlock can arise with a modest degree of polarization.

Corollary 2 *Let* $|S| = 1$, $\theta_i = \bar{\theta} + \varepsilon$ *and* $\theta_j = -\bar{\theta} + \varepsilon$, *where* $\bar{\theta} \in \mathbb{R}^+$ *measures the players' ideological polarization and* ε *is a random shock with distribution symmetric around zero. If* $\bar{\theta} \geq E(|\varepsilon|)$, *then* $\lim_{\delta \to 1} \boldsymbol{c}\left(\delta, f^S\right) = (-\infty, +\infty)$.

The condition in Corollary 2 is only sufficient for gridlock. For the example solved in Section 4, which is a special case of the setup from Corollary 2 with $\varepsilon \sim N(0,1)$, numerical simulations show that gridlock occurs when $\bar{\theta} \geq 0.35$. At $\bar{\theta} = 0.35$, players' preferences agree with probability 0.73, but as they become very patient, they always vote for opposite alternatives.

6 Welfare Analysis

In this section, we compare the equilibrium level of the utilitarian welfare under the endogenous and exogenous status quo protocols, denoted by $W\left(\Gamma^{en}_{q^S,\pi^0}\right)$ and $W\left(\Gamma^{ex}_{q^S,\pi^0}\right)$, respectively. This comparison is relevant for two reasons. First, the exogenous status quo is the natural alternative to the endogenous status quo in that it is arguably the simplest protocol that breaks the linkage between today's policy and tomorrow's status quo.

Second, even though the protocol of the endogenous status quo is prevalent in many dynamic bargaining settings (e.g., legislative bargaining, trade agreements at the WTO, monetary policy in the U.S.), the exogenous status quo is the most common alternative.[18] For instance, in the U.S. budget process, federal spending is divided into two categories. One—called mandatory spending—continues year after year by default. The other one—called discretionary spending—requires annual appropriation bills, which means that the status quo is exogenously fixed at zero.[19] In the legislative sphere, the exogenous status quo is also implemented in the form of automatic sunset provisions.

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[18] For example, the permanent provisions of the Agricultural Adjustment Act of 1938 and the Agriculture Act of 1949 serve as a fixed status quo for U.S. farm bills (Kwan 2009). Also, bilateral international agreements implicitely have an exogenous status quo of no agreement because either country can unilaterally opt out. See Lowi (1969), Weaver (1985, 1988), Hird (1991), and Gersen (2007) for more detailed studies of the ongoing and temporary nature of the laws enacted by the U.S. congress.

[19] Mandatory spending, also called direct spending, consists almost entirely of entitlement programs such as Social Security benefits, Medicare and Medicaid. Discretionary spending includes the budgets of most federal agencies (e.g., defense, national parks) and pork barrel projects. Mandatory and discretionary spending currently represents about two thirds and one third of the federal budget, respectively.

A sunset provision is a clause that specifies a duration after which an act expires, unless further legislative action is taken. One example of automatic sunset provisions is the sunset legislation in twenty-four U.S. states that requires automatic termination of a state agency, board, commission, or committee.[20]

The utilitarian welfare under the endogenous and the exogenous status quo protocols differs for two reasons. First, the endogenous status quo creates partisanship, while the exogenous status quo does not (see Remark 1). Partisanship, in turn, is detrimental to welfare as Pareto-dominated alternatives are implemented with positive probability; for example, when $q^t = R$, $\theta_j^t < 0$, and $c_i^s < \theta_i^t < 0$, player i vetoes the Pareto-optimal alternative L. Second, these two protocols induce a different distribution over the status quo in each period. However, by appropriately choosing a state-dependent status quo in $\Gamma^{ex}_{q^S,\pi^0}$, one can induce a distribution which weakly outperforms the distribution induced by the endogenous status quo protocol. The next proposition formalizes this observation.

Proposition 6 *There exists* $q'^S \in \{R, L\}^S$ *such that for all* $\pi^0 \in \Delta(S)$, $q^S \in \{R, L\}^S$, *and* $\delta \in (0, 1)$, *we have* $W\left(\Gamma^{en}_{q^S,\pi^0}\right) < W\left(\Gamma^{en}_{q'^S,\pi^0}\right)$.

Proposition 6 provides an argument in favor of the exogenous status quo protocol and automatic sunset provisions. Though we are not the first ones to advocate the use of sunset provisions, the rationale behind our recommendation is novel. Sunset provisions have traditionally been advocated for two reasons: to improve legislative oversight of executive agencies and regulations through periodic reviews, and to ensure ex-post evaluation of policies with uncertain effects. The argument advanced by our model has instead a more strategic underpinning: by severing the link between today's decisions and tomorrow's status quo, sunset provisions decrease the partisanship of the supporters and opponents of a policy and make its enactment and repeal more responsive to the current needs of legislators.

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[20] See The Book of the States, 2011, Council of State Governments. See Kearney (1990) for more on the use of sunset provisions by U.S. state legislatures. In the U.S., automatic sunset clauses are less common at the federal level, although there have been attempts to introduce them systematically in Congress (the Federal Sunset Act). In the budget process, the Byrd rule is equivalent to imposing an automatic sunset clause on any provision that increases the deficit and that does not garner a filibuster-proof majority. In a similar spirit, in 2007, the Liberal Democratic Party in Australia proposed an automatic sunset provision in all legislation that does not get the support of a 75 percent parliamentary supermajority. In Canada, any law that overrides the Canadian Charter of Rights and Freedoms (section 33) has an automatic 5-year sunset. In Germany, all emergency legislation has an automatic sunset of six months.

In light of Proposition 6, it may seem surprising that the endogenous status quo is so commonly used. A natural explanation for this comes from political economy considerations. Proposition 6 provides a normative result but is mute on individual preferences over bargaining protocols. In fact, there might not exist an exogenous status that Pareto improves on the endogenous status quo. Moreover, even when a Pareto improving exogenous status quo exists, for many ideologically charged policies such as income taxation, immigration, or hand-weapon regulation, different political actors will favor different exogenous status quos. A careful analysis of the negotiations over bargaining protocols is left for future research, but it should be clear that players' disagreement over the identity of the exogenous status quo might prevent them from changing the protocol.

Finally, we should emphasize that Proposition 6 requires the exogenous status quo to be state-dependent. This means that in order to improve upon the endogenous status quo, the exogenous status quo may need to depend on the variables that affect players' preferences. For instance, in the case of monetary policy, an optimal status quo should depend on the unemployment rate and the inflation rate.[21] In the case of welfare policies, an optimal exogenous status quo should be tied to the level of fiscal revenues, the number of recipients, the cost of living, and to business cycle indicators (e.g., the growth rate and the capacity utilization rate). Since the aforementioned variables are verifiable, and many countries already use them explicitly in the policy making process, the requirement that the status quo be state-dependent is not unrealistic in these cases.[22]

However, for some policies, the relevant state variables are not verifiable. For instance, in the case of national security, the need to restrict civil liberties depends on the likelihood of future threats, but the latter can hardly be measured in an impartial way. When the exogenous status quo cannot be state-dependent, the welfare comparison becomes less clear. The reason is that in different states, different status quos may be socially optimal. Under the endogenous status quo, players' voting behavior differs across states, and hence the equilibrium status quo distribution is state-dependent. Under the state-independent status quo, this is obviously not the case. Example

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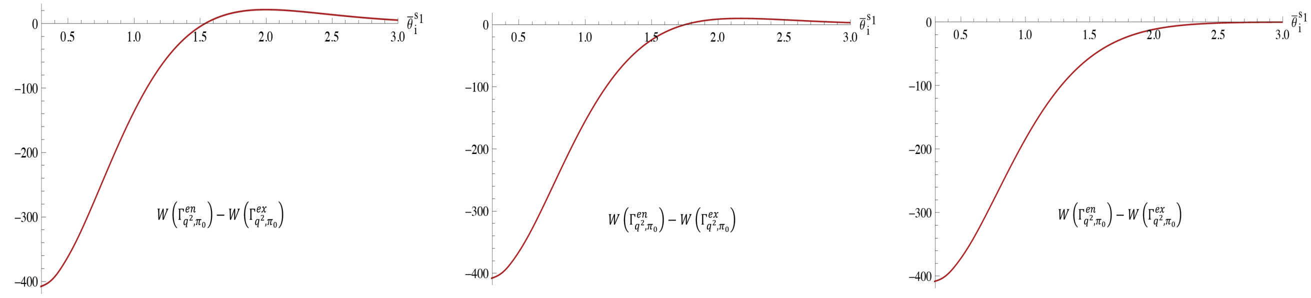
[21] The Taylor rule provides an example of a monetary policy that depends mechanically on observable variables. This rule, first introduced by Taylor (1993), ties the change in the nominal interest rate to the variations of the output, unemployment, and inflation rate.

[22] For instance, welfare policies are typically set in terms of the level of individual benefits and eligibility requirements, so the aggregate level of spending mechanically depends on the unemployment rate, the income distribution, and the age distribution. Most countries also index the benefits' level to the cost of living. Some countries (e.g., the Netherlands) tie pension benefits to the return of the pension fund, and other countries (e.g., Sweden and France) tie the eligibility criteria to life expectancy. See Bikker and Vlaar (2007).

1 below illustrates that when the players are not too polarized, the effect of an evolving status quo on welfare might dominate the detrimental effect of partisanship.

Example 1 *We extend the example solved in Section 4 to two states as follows:* $S = (s_1, s_2)$, $\pi_{s_1,s_1} = \pi_{s_2,s_2} \equiv \pi \geq \frac{1}{2}$, $\theta_i^s = \bar{\theta}_i^s + \varepsilon$, *and* $\theta_j^s = \bar{\theta}_j^s + \varepsilon$, *where* $\varepsilon \sim N(0, 1)$. *That is, the two states are somewhat persistent and players' preferences are perfectly correlated. The players' preference distribution in* s_1 *is such that* $\bar{\theta}_i^{s_1} > \bar{\theta}_j^{s_1}$, $\bar{\theta}_i^{s_1} + \bar{\theta}_j^{s_1} \geq 0$, *and* s_2 *is the symmetric of* s_1*:* $\bar{\theta}_i^{s_2} = -\bar{\theta}_j^{s_1}$ *and* $\bar{\theta}_j^{s_2} = -\bar{\theta}_i^{s_1}$. *Hence, player* i *is more rightist than player* j *in both states, but in state* s_1*, when players' preferences disagree,* R *is socially better, so* R *is the socially optimal status quo in* s_1*. Conversely,* L *is the socially optimal status quo in* s_2*.*

The following figure compares the utilitarian welfare in the least partisan equilibrium of $\Gamma^{en}_{q^S,\pi^0}$ *and* $\Gamma^{ex}_{q^S,\pi^0}$*, where* $\pi^0(s_1) = 1$ *and* $q^s = L$ *in both states. We fix players' initial polarization* $\bar{\theta}_i^{s_1} - \bar{\theta}_j^{s_1}$ *at* 0.5 *and let* $\bar{\theta}_i^{s_1}$ *vary. Note that given our assumptions, when* $\bar{\theta}_i^{s_1} = 0.25$*, both states are identical. As* $\bar{\theta}_i^{s_1}$ *increases, the preferences of both players move to the right in* s_1 *and to the left in* s_2*: the states becomes less similar, but in each state, the probability of preference disagreement decreases. Each panel depicts* $W\left(\Gamma^{en}_{q^S,\pi^0}\right) - W(\Gamma^{ex}_{q^S,\pi^0})$ *as a function of* $\bar{\theta}_i^{s_1}$ *for* $\pi = 0.99$, 0.8, *and* 0.5.



All panels show that the exogenous status quo dominates for small $\bar{\theta}_i^{s_1}$*, but this can reverse itself for large* $\bar{\theta}_i^{s_1}$*. A comparison of panels A, B, and C reveals that the endogenous status quo is more likely to dominate as* π *increases. The intuition for this is as follows. When* $\bar{\theta}_i^{s_1} = 0.25$*, both states are identical, and the exogenous status quo dominates trivially by Proposition 6. As* $\bar{\theta}_i^{s_1}$ *increases, the probability of disagreement in each state decreases; hence, defending the status quo becomes less important, and the degree of partisanship in* $\Gamma^{en}_{q^S,\pi^0}$ *decreases. At the same time, both players become more likely to vote for* L *in* s_1 *and* R *in* s_2*, which guarantee an optimal status quo in the* *next period if the state does not change. The more persistent the states, the more beneficial this effect is.*

7 N-player game

In this section, we extend the model to $N > 2$ players. In an abuse of notation, $N = \{1, ..., n, ..., N\}$ also refers to the set of players. For any generic parameter p, the bold symbol $\boldsymbol{p}$ now refers to the vector $(p_n)_{n \in N}$. As in the two-player case, the payoff of each player is given by Equation (1), where $\left(\boldsymbol{\theta}^t\right)_{t \geq 1}$ follows a stationary and irreducible Markovian process on the finite state space S, with a probability density function f^S. In line with Assumption 1, we assume that in all states, $\theta_1 \geq .. \geq \theta_N$ with probability one, and for any two distinct voters $n, m \in N$, there exists a state in which θ_n and θ_m are of opposite sign with positive probability.

The game proceeds exactly as in $\Gamma^{en}_{q^S, \pi^0}$, but we allow for a broader class of voting rules. A voting rule is characterized by a pair of collections of winning coalitions (Ω_L, Ω_R) , which determine the outcome as follows: if the status quo is L (R) in a given period, then it is replaced by R (L) if and only if the set of players who vote for R (L) in this period is an element of Ω_L (Ω_R). We impose the following conditions on the voting rule.

Definition 3 *A voting rule is a pair of collection of coalitions $\Omega = (\Omega_L, \Omega_R) \in 2^N \times 2^N$ where for all $q \in \{L, R\}$, Ω_q satisfies the following conditions:*
(i) Monotonicity: if $C \in \Omega_q$ and $C \subseteq C'$, then $C' \in \Omega_q$,
(ii) Properness: if $C \in \Omega_q$, then $N \setminus C \notin \Omega_q$,
(iii) Nonemptyness: $\{1..N\} \in \Omega_q$,
(iv) Joint properness: for $q' \neq q$, if $C \in \Omega_q$, then $N \setminus C \notin \Omega_{q'}$.

Conditions (i) to (iii) are standard in the voting literature (see, e.g., Austen-Smith and Banks 2000). Monotonicity ensures that having more votes in favor of R cannot change the outcome from R to L; properness ensures that the outcome of the vote is unique; nonemptyness ensures that the voting rule is Paretian. Condition (iv) means that if a coalition can change the status quo, then the players outside this coalition cannot reverse this change. Conditions $(i) - (iv)$ characterize a large class of voting rules such as majoritarian voting rules, but also other nonanonymous, and nonneutral, voting rules.[23]

The $N-$player game that uses a voting rule Ω and begins with an initial state distribution π^0 and an initial status quo distribution q^S is denoted by $\Gamma^{en}_{q^S,\pi^0}(\Omega)$.

7.1 The equilibrium

Suppose that all players vote myopically for their most preferred policy. Under such behavior, since $\theta_1 \geq .. \geq \theta_N$, if player n votes for R, then all players $i \leq n$ also vote for R. Conditions $(i)-(iii)$ in Definition 3 then imply that there exists a player n_R such that when the status quo is R, L is implemented if and only if that player votes for L. By the same token, there exists a player n_L such that when the status quo is L, R is implemented if and only if that player votes for R. We will call these players *pivotal.* Formally:

Definition 4 *The pivotal players for the voting rule Ω are (n_L, n_R) such that*

$$\begin{aligned} \{1,...,n_L\} &\in \Omega_L \text{ and } \{1,...,n_L-1\} \notin \Omega_L, \\ \{n_R,...,N\} &\in \Omega_R \text{ and } \{n_R+1,...,N\} \notin \Omega_R. \end{aligned}$$

Note that condition (iv) in Definition 3 implies that $n_R \leq n_L$ (equivalently, $\theta_{n_R} \geq \theta_{n_L}$): the player pivotal to implementing a change to L is more rightist than the player pivotal to implementing a change to R. For instance, under the unanimity rule, $n_R = 1$ and $n_L = N$, while under the simple majority rule, $n_R = \lfloor \frac{N+1}{2} \rfloor$ and $n_L = \lceil \frac{N+1}{2} \rceil$.[24]

The following proposition characterizes the equilibria of $\Gamma^{en}_{q^S,\pi^0}(\Omega)$.

Proposition 7 *In all equilibria of $\Gamma^{en}_{q^S,\pi^0}(\Omega)$, the players use state-dependent but status-quo-independent cutoff strategies: there exists $\boldsymbol{c}^S \in \mathbb{R}^{N\times S}$ such that in state $s \in S$, player $n \in N$ votes for R if*

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[23] An example of a nonanonymous voting rule is the combination of a simple majority and a veto player n:

$$\Omega_R = \Omega_L = \left\{ C \subseteq N : |C| > \frac{N}{2} \text{ and } n \in C \right\}.$$

An example of a nonneutral voting rule is a simple majority when $q^t = L$ and unanimity when $q^t = L$:

$$\Omega_L = \left\{ C \subseteq N : |C| > \frac{N}{2} \right\} \text{ and } \Omega_R = \{N\}.$$

[24] For any $x \in \mathbb{R}$, $\lfloor x \rfloor$ is the largest integer smaller than or equal to x, and $\lceil x \rceil$ is the smallest integer greater than or equal to x. When N is odd, $\frac{N+1}{2} = \lfloor \frac{N+1}{2} \rfloor = \lceil \frac{N+1}{2} \rceil$, and the pivotal player is unique.

$\theta_n > c_n^s$ and for L if $\theta_n < c_n^s$. The equilibrium cutoffs are given by the fixed point of the mapping $\boldsymbol{H}^S$, defined as follows: for all $\boldsymbol{c}^S \in \mathbb{R}^{N\times S}$,

$$H_n^s\left(\boldsymbol{c}^S\right) = \delta \sum_{s'} \pi\left(s, s'\right)\left(\int_{\left\{\boldsymbol{\theta}\in\mathbb{R}^N : \theta_{n_L} \leq c_{n_L} \text{ and } \theta_{n_R} \geq c_{n_R}\right\}} \left(c_n^{s'} - \theta_n\right) f^{s'}\left(\boldsymbol{\theta}\right) d\boldsymbol{\theta}\right). \tag{5}$$

Moreover,

- *(i) for all $s \in S$, $c_1^s \leq .. \leq c_N^s$, so in any period, the status quo changes if and only if players n_L and n_R vote against it;*
- *(ii) for all $s \in S$, $c_{n_R}^s \leq 0 \leq c_{n_L}^s$;*
- *(iii) the set of equilibrium cutoffs of the pivotal players $\left(c_{n_L}^S, c_{n_R}^S\right)$ is a complete lattice for the partial order $(\leq, \geq)^S$;*
- *(iv) if $\boldsymbol{c}^S$ and $\boldsymbol{d}^S$ are two equilibria such that the pivotal players are more partisan at $\boldsymbol{d}^S$ than at $\boldsymbol{c}^S$ (i.e., $\left(d_{n_L}^S, d_{n_R}^S\right)(\leq, \geq)^S\left(c_{n_L}^S, c_{n_R}^S\right)$), then all players $n \in \{n_R, ..., n_L\}$ are better off at $\boldsymbol{c}^S$ than at $\boldsymbol{d}^S$. In particular, there exists a Pareto worse and Pareto best equilibrium for those players.*

The proof of Proposition 7 proceeds by showing that stage dominance implies that all voters are partisan, and since θ_n^t is increasing in n, partisanship is also monotonic in n. Therefore, the players who are pivotal according to Definition 4 are also pivotal in every period of $\Gamma_{q^S,\pi^0}^{en}(\Omega)$. Thus, analyzing $\Gamma_{q^S,\pi^0}^{en}(\Omega)$ boils down to analyzing the two-player game Γ_{q^S,π^0}^{en} with the preference distribution $\left(\theta_{n_R}^t, \theta_{n_L}^t\right)$, and all the results from the two-player game follow. In particular, the pivotal voters are partisan in directions that exacerbate their conflicts of interest; therefore, as in the two-player case, the endogeneity of the status quo decreases the responsiveness of the agreements to the shocks.

7.2 Concentration of power and welfare

This more general setup allows us to investigate how the voting rule, and in particular the concentration of power implied by the voting rule, affects equilibrium behavior.

Definition 5 *The concentration of power is greater (i.e., the dispersion of power is lower) under* Ω *than under* Ω' *if* $\Omega_L \subseteq \Omega'_L$ *and* $\Omega_R \subseteq \Omega'_R$*. The concentration of power under* Ω *is maximal if* $n_R = n_L$*.*

In words, the concentration of power increases when the approval of a smaller set of players is required to change the status quo. We show in the appendix (see the proof of Proposition 8) that when the concentration of power under Ω is greater than under Ω', then the pivotal voters are more moderate under $\Omega:\ n'_R \leq n_R \leq n_L \leq n'_L$. When $n_L = n_R$, the concentration of power is maximal because any further increase in the concentration of power must leave the pivotal voters, and thus the equilibrium, unchanged.[25]

The following proposition shows that partisanship decreases with concentration of power.

Proposition 8 *Let* $\boldsymbol{c}^S(\Omega)$ *denote the cutoffs from the least partisan equilibrium of* $\Gamma^{en}_{q^S,\pi^0}(\Omega)$*. If the concentration of power is greater under* Ω *than under* Ω'*, then for all* $s \in S$*,*

$$c^s_{n'_R}\left(\Omega'\right) \leq c^s_{n'_R}\left(\Omega\right) \leq c^s_{n_R}\left(\Omega\right) \leq 0 \leq c^s_{n_L}\left(\Omega\right) \leq c^s_{n'_L}\left(\Omega\right) \leq c^s_{n'_L}\left(\Omega'\right), \tag{6}$$

where (n_L, n_R) *and* (n'_L, n'_R) *denote the pivotal players under* Ω *and* Ω'*, respectively.*
There exists a nonpartisan equilibrium (i.e., $\boldsymbol{c}^S = \boldsymbol{0}^S$*) under the voting rule* Ω *if and only if the concentration of power under* Ω *is maximal.*

The intuition behind Proposition 8 is as follows. As argued before, increased dispersion of power makes more extreme players pivotal. This has two consequences. From Proposition 7, we know that for a given voting rule Ω, more extreme players are more partisan, which explains the four inner inequalities in (6). But since the players determining the agreements are now more extreme, their disagreement is more likely than the disagreement of n_R and n_L. This increases the inertia of the status quo. As a result, players care more about the identity of the status quo and are thus more partisan. This effect explains the two outer inequalities in (6). In other words, increased dispersion of power increases the status quo inertia not only because more players have to agree,

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[25] Conditions $(i) - (iv)$ in Definition 3 imply that $n'_R \leq n'_L$. Therefore, if $n_L = n_R$ and if Ω' increases the concentration of power as compared to Ω, then it necessarily follows that $n'_R = n_R$ and $n'_L = n_L$. The condition $n_L = n_R$ is satisfied, for example, when the voting rule is strong, that is, when for all $q \in \{R, L\}$ and all $C \subseteq N$, if $C \notin \Omega_q$, then $N \setminus C \in \Omega_q$ (see, e.g., Austen-Smith and Banks, 2000).

but also because the pivotal players become more partisan. In this sense, the endogenous status quo exacerbates the inertial effect of dispersion of power.

The second part of Proposition 8 states that partisanship disappears when power is maximally concentrated. This is the case, for instance, under dictatorship, but also under more equitable rules such as a simple majority rule. The reason is that in these two cases, a single player (the dictator and the median player, respectively) is always pivotal, and hence, votes according to her preferences.

The welfare effect of increased dispersion of power may depend on players' ideological positions. The resulting increase in partisanship is clearly detrimental to all players. However, an extreme player may benefit from having pivotal players that are more extreme, because when her most preferred status quo is in place, the policy is chosen by someone with preferences more similar to hers. As shown in the following proposition, the latter effect is absent for players who are more moderate than the pivotal players; hence, increased dispersion of power clearly hurts these players. Moreover, when the average preferences are between those of the pivotal players, utilitarian welfare must decrease with the dispersion of power.

Proposition 9 *If the concentration of power under Ω is greater than under Ω', then all players $n \in \{n_L, ..., n_R\}$ are better off under Ω than under Ω'. Moreover, if $\theta_{n_R} \leq \frac{1}{N}\sum_n \theta_n \leq \theta_{n_L}$ with probability 1, then utilitarian welfare is greater under Ω than under Ω'.*

Note that the condition $\theta_{n_R} \leq \frac{1}{N}\sum_n \theta_n \leq \theta_{n_L}$ is satisfied under standard specifications. For instance, it is satisfied if Ω is a supermajority rule, that is, when the approval of $M > \frac{N+1}{2}$ players is required to change the status quo, and the preference distribution across players is relatively symmetric around $\theta_{\frac{N+1}{2}}$, i.e., $\frac{1}{N}\sum_n \theta_n \approx \theta_{\frac{N+1}{2}} \in \left[\theta_{n_{N-M+1}}, \theta_M\right]$.

Propositions 8 and 9 have important consequences for constitutional design. There exists no modern democracy in which a single decision maker is pivotal in every decision, even when majority rule is used at all stages of the decision process. For instance, even in a purely parliamentarian regime, short of strong party discipline and a sufficient majority in both chambers, bicameralism implies the existence of two distinct pivotal voters. Moreover, in most constitutions, majoritarian decision making is complemented by other rules and institutions, such as presidential veto power, judicial review by a constitutional court, the possibility of public initiatives, or supermajoritarian requirements such as the filibuster tradition in the U.S. Congress.[26]

Admittedly, these checks and balances are not designed to smooth the decision process. Rather, their role is to limit agency costs and abuses of power by any government branch. Our model shows, however, that when checks and balances are introduced in a decision process that uses the endogenous status quo protocol, they tend to make legislators more partisan, which can greatly exacerbate the inherently inertial effect of checks and balances. Hence, in order to avoid welfare-decreasing partisan behavior, a system of checks and balances should be complemented with the use of exogenous status quos or sunset provisions. Interestingly, a very similar argument was made by Thomas Jefferson when he famously argued in favor of laws of limited duration in his correspondence with James Madison.[27]

7.3 Biased voting rules

Under some voting rules, the sets of affirmative votes required to approve a policy change are policy-dependent (i.e., $\Omega_R \neq \Omega_L$). An example of such a rule can be found in the U.S. budget process. This budget process is governed by the Congressional Budget Act of 1974, which prohibits the use of the filibuster against budget resolutions. This act was amended in 1985 (and later in 1990) by the Byrd Rule to allow the use of a filibuster against any provision that increases the deficit beyond the years covered by the reconciliation measure. Effectively, the Byrd Rule requires a higher majority to raise the budget deficit than to lower it, since curbing the deficit was its main rationale.[28] However, the game-theoretic logic highlighted in this model suggests that with an endogenous status quo, this rule might have unintended consequences. Fiscally expansionist legislators may be unwilling to reduce the budget deficit in good times, realizing that the Byrd rule will make it more difficult to increase it in the future. As the example below illustrates, because of

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[26] It should be noted that if we allow for preference reversal, for instance, if $\boldsymbol{\theta}$ has full support on $\mathbb{R}^N$ at least in some state, then even under simple majority rule, all players can be pivotal in every period. Therefore, whenever a player is pivotal, she would consider how her future preferences might conflict with those of the next pivotal players, and she would bias her vote accordingly. This means that partisanship would emerge even under simple majority rule.

[27] "[T]he power of repeal is not an equivalent [to mandatory expiration]. It might indeed be if [...] the will of the majority could always be obtained fairly and without impediment. But this is true of no form. [...] Various checks are opposed to every legislative Proposition [...] and other impediments arise so as to prove to every practical man that a law of limited duration is much more manageable than one which needs a repeal." (See Woods 2009, p. 93).

[28] More precisely, to pass a provision that increases the deficit, the Byrd rule requires a filibuster-proof majority, or a simple majority together with a sunset clause on that provision. We leave that latter possibility aside for the sake of simplicity, since our goal here is to illustrate the incentives generated by biased voting rules rather than to model in detail the U.S. budget process.

this strategic effect, a fiscally conservative voting rule might in fact generate more spending.

Example 2 *We extend the example solved in Section 4 to three players, as follows:* $|S| = 1$, $N = \{1, 2, 3\}$, *and for all* $n \in N$, $\theta_n = \bar{\theta}_n + \varepsilon$, *with* $\bar{\theta}_1 > \bar{\theta}_2 > \bar{\theta}_3$ *and* $\varepsilon \sim N(0, 1)$. *Consider the following two voting rules: the simple-majority rule, under which a policy replaces the status quo if it is approved by two players; and the* $R-$*biased rule, under which a simple majority is needed to replace* L *and unanimity is required to replace* R.

Under the simple majority rule, $n_R = n_L = 2$ *so player* 2 *is always pivotal, while under the* $R-$*biased rule,* $n_R = 1$ *and* $n_L = 2$, *so player* 1 *is pivotal when* $q^t = R$ *and player* 2 *is pivotal when* $q^t = R$. *From Proposition 7, under the simple-majority rule players vote myopically for their most preferred policy, so* R *wins when* $\varepsilon^t > -\bar{\theta}_2$ *and* L *wins when* $\varepsilon^t < -\bar{\theta}_2$. *Under the* $R-$*biased rule, players are partisan, and* R *wins when* $\varepsilon^t > c_1 - \bar{\theta}_1$, L *wins when* $\varepsilon^t < c_2 - \bar{\theta}_2$, *and the status quo stays in place when* $\varepsilon^t \in \left(c_1 - \bar{\theta}_1, c_2 - \bar{\theta}_2\right)$. *Proposition 7 implies that* $c_1 - \bar{\theta}_1 < -\bar{\theta}_2 < c_2 - \bar{\theta}_2$, *so compared to the simple-majority rule,* R *stays in place more often, but so does* L. *The latter effect may dominate if players' partisanship is strong. Assume that* $\bar{\theta}_1 = 0.3$ *and* $\delta = 0.9$. *We show numerically that for* $\bar{\theta}_2 \leq -0.5$, *the probability of* L *being implemented at the invariant distribution is higher under the* $R-$*biased rule.*[29] *For example, when* $\bar{\theta}_2 = -0.5$, *then in the long run, the probability that* L *is implemented under the majority rule is* 0.69. *Under the* $R-$*biased rule, the voting cutoffs are* $c_i = -2.37$ *and* $c_j = 3.43$, *and the probability that* L *is implemented increases to* 0.99.[30]

8 Preference Reversal

We believe that Assumption 1 excluding preference reversals is satisfied in a vast array of environments. However, in this section we discuss when and how the results of Section 5 change when we relax this assumption. That is, we allow f^S to have full support.

Note first that the equilibrium characterization in Proposition 1 was derived without Assumption 1, so for any f^S, players use cutoff strategies in any equilibrium. Moreover, the partisanship

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[29] The invariant distribution is the limit distribution of q^t at the equilibrium as $t \to \infty$.

[30] Observe that with an exogenous status quo, players would vote according to their preferences under both rules, and it should be clear that the $R-$biased rule would favor R since R would be implemented more often than under a simple-majority rule. In fact, for $\bar{\theta}_2 = -0.5$, L would be implemented with probability 0.55.

generated by the endogeneity of the status quo is the rule, not an exception. To see that, observe that from Proposition 1, an equilibrium with zero cutoffs (i.e., $c_i^s = c_j^s = 0$ for all s) exists if and only if for all $s \in S$,

$$\sum_{s' \in S} \pi\left(s, s'\right)\left(\int_{-\infty}^{0} \int_{0}^{\infty} \boldsymbol{\theta} f^{s'}(\boldsymbol{\theta})\, d\theta_i d\theta_j + \int_{0}^{\infty} \int_{-\infty}^{0} \boldsymbol{\theta} f^{s'}(\boldsymbol{\theta})\, d\theta_i d\theta_j\right) = (0, 0)\,. \tag{7}$$

Clearly, condition (7) is satisfied only in special cases. For instance, if $|S| = 1$ and the distribution of $\boldsymbol{\theta}$ is bivariate normal, condition (7) holds if and only if $\bar{\boldsymbol{\theta}} = (0, 0)$.[31]

Second, the polarizing effect of the endogenous status quo is a robust phenomenon. As shown in the appendix, both part (b) of Proposition 4 and Proposition 5 hold for any preference distribution. Hence, even if we allow for preference reversal, it is still true that more polarized players are more partisan; moreover, when they are sufficiently polarized and patient, the endogenous status quo leads to complete gridlock.

The main change in the analysis is the determination of the sign of the equilibrium cutoffs. Without Assumption 1, one cannot unambiguously order players' preferences, so Proposition 2 does not extend automatically. Moreover, as we shall see in Example 3 below, the signs of the cutoffs may even change across equilibria. It turns out, however, that if one player is rightist and the other is leftist in the sense defined below, then there exists an intuitive equilibrium in which the former is partisan for R and the latter is partisan for L. Proposition 10 further shows that such equilibria are more plausible.

Before stating the proposition, we need to introduce some notations. The definition below links a player's ideological position to her preferences over different exogenous status quos. For example, $H_k^s(0, ..., 0) \leq 0$ means that conditional on her preference disagreeing with her opponent's preferences, player k prefers alternative R in state s. Therefore, from Remark 1, she would prefer the exogenous status quo in state s to be R, and we call her a rightist in state s. Formally:

Definition 6 *Player k is rightist (leftist) if for all $s \in S$, $H_k^s(0, ..., 0) \leq 0$ (≥ 0).*

Let $\Gamma^{en}_{q^S,\pi^0}(T)$ denote the finite-horizon game which proceeds as $\Gamma^{en}_{q^S,\pi^0}$ but ends after T periods.

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[31] For the formal proof, see Dziuda and Loeper (2010, Example 5 in the appendix). In that paper, Example 1 shows that (7) can be violated even if the marginal distribution of each player's preferences is symmetric across players and across alternatives.

As shown in the appendix, this game admits a unique stage-undominated equilibrium, which is in cutoff strategies, and we denote by $\boldsymbol{c}^S(t)$ the equilibrium cutoffs t periods before the end of the game.

Proposition 10 *Assume that f^S is such that player i is rightist and player j is leftist. Then, there exists equilibria $\boldsymbol{c}^S$ such that $\boldsymbol{c}^S(\leq,\geq)^S\mathbf{0}^S$. The set of such equilibria forms a complete lattice for the order $(\leq,\geq)^S$. The least partisan of these equilibria in the order $(\leq,\geq)^S$ is equal to* $\lim_{t\to\infty}\boldsymbol{c}^S(t)$. *The comparative statics in Proposition 4 hold for that equilibrium selection.*

If the players do not have opposed ideologies in the sense of Definition 6, the general results on the direction of players' partisanship are more elusive. To understand why, consider the case in which $|S|=1$, $H_i(0,0)>0$, and $H_j(0,0)>0$. In that case, the players' expected preferences conditional on disagreement are congruent: they both prefer L to be the status quo. Hence, one could conjecture that with an endogenous status quo, there is an equilibrium in which both players are partisan for L. However, this might not be true. If i votes often for L, then the disagreement in which i prefers R and j prefers L happens rarely. The reverse disagreement may be more likely, so j may end up partisan for R.[32]

One could conjecture that allowing for preference reversal should decrease partisanship, since no player can be sure which alternative she will prefer when players disagree. As the following example demonstrates, however, this is not necessarily true. Moreover, arbitrarily similar players can behave as if their interests were highly discordant. Hence, the possibility of preference reversal exacerbates the polarizing effect of the endogenous status quo.

Example 3 *We introduce preference reversal in the example solved in Section 4, as follows: $|S|=1$, $\delta=0.9$, $\theta_i=\bar{\theta}_i+\varepsilon_i$, and $\theta_j=\bar{\theta}_j+\varepsilon_j$, where ε_i and ε_j are i.i.d. with ε_k drawn from $N(0,1)$ with probability $\frac{1}{2}$ and from $N(0,10)$ with the remaining probability. The figure below depicts the locus points of the solutions of $c_i=H_i(\boldsymbol{c})$ and $c_j=H_j(\boldsymbol{c})$ for $\bar{\theta}_i=-\bar{\theta}_j=0.1$; the intersections are*

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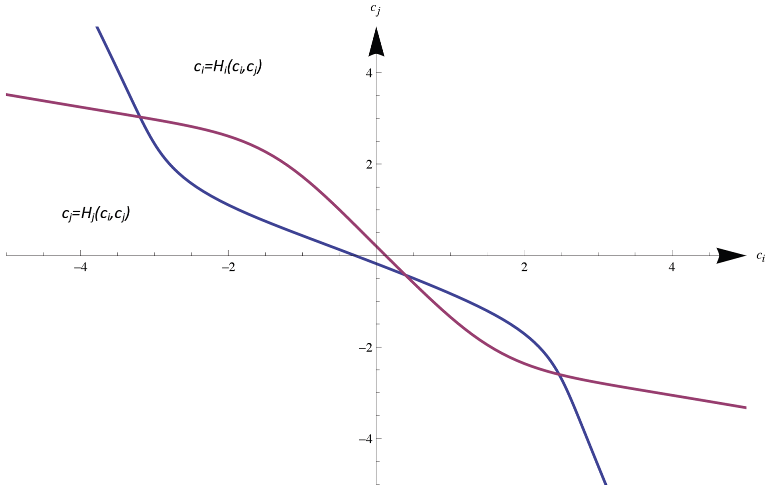
[32] See Section 4.2 in Dziuda and Loeper (2010).

the equilibria. We see that Γ_q^{en} *has two equilibria with* $c_i > 0 > c_j$ *and one with* $c_i < 0 < c_j$.



In the intuitive equilibrium, $c_i = -c_j \simeq -3.2$. *As* $\bar{\theta}_i = -\bar{\theta}_j \to 0$, *all three equilibria remain. Only in the middle one does the players' partisanship vanish, but as argued in Proposition 10, the equilibrium in which* $c_i < 0 < c_j$ *is the most plausible. Hence, the endogenous status quo leads arbitrarily similar players to be very partisan for opposite alternatives and to behave as if their interests were highly discordant.*[33]

Moreover, if we restored Assumption 1 by assuming instead that ε_i *and* ε_j *were perfectly correlated with the same marginal distribution, then in equilibrium,* $c_i = -c_j = 0.0042$. *Hence, allowing for preference reversal can magnify the polarizing effect of the endogenous status quo.*

Finally, allowing for preference reversal complicates the equilibrium welfare comparison between the exogenous and endogenous status quos. The reason is that besides the two detrimental effects of partisanship on welfare outlined in Section 6, a third beneficial effect may arise. A partisan player, while voting for her preferred status quo, may defer to her opponent's preferences: if $c_i < \theta_i < 0 < \theta_j$, player i will vote for the alternative preferred by player j. This may be socially beneficial if the opponent's preferences are relatively more intense. In Dziuda and Loeper 2010 (Proposition 7) we show that under some regularity conditions which basically require that the

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[33] This phenomenon cannot occur in the case of no preference reversal: for $|S| = 1$ and for any sequence of preference distribution $\left(\boldsymbol{\theta}^k\right)_{k\geq 1}$ such that $\bar{\boldsymbol{\theta}}_i^k - \bar{\boldsymbol{\theta}}_j^k$ tends to 0, all equilibrium thresholds tend to $(0,0)$. To see this, observe that since $\theta_i^k - \theta_j^k > 0$ with probability 1, it follows that $E\left(\left|\boldsymbol{\theta}_i^k - \boldsymbol{\theta}_j^k\right|\right)$ must tend to 0. So $\left|H_i^k - H_j^k\right|$ tends to 0 uniformly over $\mathbb{R}^2$. Using proposition 2, the fixed points of $\boldsymbol{H}$ must all tend to $(0,0)$.

probability of a preference reversal not be too large, the welfare results in Proposition 6 hold.

9 Conclusion

Negotiations in a changing environment with an endogenous status quo are at the center of many economically relevant situations. They present the negotiating parties with a fundamental trade-off between responding adequately to the current environment and securing a favorable bargaining position for the future. In this paper, we show that this trade-off has a detrimental impact on the efficiency of agreements and their responsiveness to political and economic shocks. Bundling the vote on today's policy and tomorrow's status quo exacerbates the players' conflict of interest and increases the probability of a disagreement, which in turn increases status quo inertia. As a result, Pareto-improving alternatives may not be adopted.

Our analysis sheds light on the effect of important rules governing legislative institutions: we provide a rationale for sunset provisions and we show that checks and balances exacerbate the partisanship and the inertia generated by the endogenous status quo.

This parsimonious model lends itself to several extensions. First, our paper does not model the selection of the bargaining protocol. In particular, an interesting extension would be to allow the negotiating parties to attach optional sunset provisions at any stage.

Second, adding transfers—interpreted as pork-barrel spending—to the $N-$player model would allow us to analyze the trade-off between their positive role as a lubricant for passing efficient policies and the perverse incentives they generate by concentrating benefits and collectivizing costs. Such a model would better approximate the U.S. budget process, which combines two expenditure categories: discretionary spending (targeted programs with an exogenous status quo) and direct spending (less easily targeted programs with an endogenous status quo).

Finally, in many situations, policies implemented affect the future state of the economy, which introduces an additional dynamic linkage. For example, the interest rate set by the monetary policy committee affects the evolution of the economy. Incorporating this observation into our model amounts to letting the state depend on the previous policies. Such an extension would allow us to investigate the impact of the voting rule on the business cycle.

10 Appendix

Throughout the appendix, we use the following notations:

Notation 1 *For any preference distribution* f^S*, any* $\delta \in [0,1]$*, any* $s \in S$*, any* $\boldsymbol{c} \in \mathbb{R}^2$*, and any* $\boldsymbol{c}^S \in \mathbb{R}^{2S}$*, we denote by* $\boldsymbol{G}^s(\delta, f^s, \boldsymbol{c})$ *the map defined by:*

$$\boldsymbol{G}^s(\delta, f^s, \boldsymbol{c}) = \delta \left(\int_{-\infty}^{c_j} \int_{c_i}^{\infty} (\boldsymbol{c} - \boldsymbol{\theta}) f^s(\boldsymbol{\theta}) \, d\theta_i d\theta_j + \int_{c_j}^{\infty} \int_{-\infty}^{c_i} (\boldsymbol{c} - \boldsymbol{\theta}) f^s(\boldsymbol{\theta}) \, d\theta_i d\theta_j \right). \tag{8}$$

We denote by $\boldsymbol{H}^s\left(\delta, f^S, \boldsymbol{c}^S\right)$ *the map defined by:*

$$\boldsymbol{H}^s\left(\delta, f^S, \boldsymbol{c}^S\right) = \sum_{s' \in S} \pi\left(s, s'\right) \boldsymbol{G}^{s'}\left(\delta, f^{s'}, \boldsymbol{c}^{s'}\right). \tag{9}$$

We denote by $\boldsymbol{c}^S\left(\delta, f^S\right)$ *the smallest fixed point of* $\boldsymbol{H}^S\left(\delta, f^S, \boldsymbol{c}^S\right)$ *for the order* $(\leq, \geq)^S$*, when it exists. Finally,* $\mathbf{0}$ *and* $\mathbf{0}^S$ *are the null element of* $\mathbb{R}^2$ *and* $\mathbb{R}^{2S}$*, respectively.*

The map $\boldsymbol{H}^S\left(\delta, f^S, \boldsymbol{c}^S\right)$ is simply the map $\boldsymbol{H}^S\left(\boldsymbol{c}^S\right)$ defined in the main text in (4) with the explicit reference to the preference distribution f^S and the discount factor δ. The next two lemmas a few some important properties of $\boldsymbol{G}^S$ and $\boldsymbol{H}^S$.

Lemma 1 *Using the conventions of Notation 1, for all* $s \in S$*, all* $\boldsymbol{c} \in \mathbb{R}^2$*, and all* $k, k' \in \{i, j\}$ *and* $k \neq k'$*, we have:*

$$0 \leq \frac{\partial G_k^s(\delta, f^s, \boldsymbol{c})}{\partial c_k} \leq \delta, \text{ and } \frac{\partial G_k^s(\delta, f^s, \boldsymbol{c})}{\partial c_{k'}} < 0,$$

and if g^S *is more polarized than* f^S *(see Definition 2), then*

$$G_i^s(\delta, g^s, \boldsymbol{c}) \leq G_i^s(\delta, f^s, \boldsymbol{c}) \text{ and } G_j^s(\delta, g^s, \boldsymbol{c}) \geq G_j^s(\delta, f^s, \boldsymbol{c}).$$

Proof. Using the Leibnitz integral rule on (8), we obtain

$$\begin{aligned}
\frac{\partial G_i^s(\delta, f^s, \boldsymbol{c})}{\partial c_i} &= \delta \left(\int_{-\infty}^{c_j} \int_{c_i}^{\infty} f^s(\boldsymbol{\theta}) \, d\theta_i d\theta_j + \int_{c_j}^{\infty} \int_{-\infty}^{c_i} f^s(\boldsymbol{\theta}) \, d\theta_i d\theta_j \right) \leq \delta, \\
\frac{\partial G_i^s(\delta, f^s, \boldsymbol{c})}{\partial c_j} &= -\delta \int_{-\infty}^{+\infty} |\theta_i - c_i| \, f^s(\theta_i, c_j) \, d\theta_i < 0,
\end{aligned}$$

where the finding that the last inequality is strict comes from the assumption that the marginal distributions of θ_j and θ_i have full support.

Let f^s be the p.d.f. of $\boldsymbol{\theta}^s$. According to Definition 2, if g^s is more polarized than f^s, then g^s is the p.d.f of $\boldsymbol{\theta}^s + \boldsymbol{\varepsilon}^s$, where $\boldsymbol{\varepsilon}^s$ is a random variable with support on $(\mathbb{R}_+ \times \mathbb{R}_-)$. For all $\alpha > 0$, let f_α^s be the p.d.f. of $\boldsymbol{\theta}^s + \alpha\boldsymbol{\varepsilon}^s$. We have $f_0^s = f^s$ and $f_1^s = g^s$. If we denote by h^s the joint p.d.f. of $\boldsymbol{\theta}^s$ and $\boldsymbol{\varepsilon}^s$, then we can rewrite (8) as follows:

$$\begin{aligned} G_i^s\left(\delta, f_\alpha^s, \boldsymbol{c}\right) &= \delta\left(\int_{\boldsymbol{\theta}}\int_{-\infty}^{\frac{c_j-\theta_j}{\alpha}}\int_{\frac{c_i-\theta_i}{\alpha}}^{\infty}\left(c_i-\theta_i-\alpha\varepsilon_i\right)h^s\left(\boldsymbol{\theta},\boldsymbol{\varepsilon}\right)d\varepsilon_i d\varepsilon_j d\theta_i d\theta_j\right. \\ &\quad \left.+\int_{\frac{c_j-\theta_j}{\alpha}}^{\infty}\int_{-\infty}^{\frac{c_i-\theta_i}{\alpha}}\left(c_i-\theta_i-\alpha\varepsilon_i\right)h^s\left(\boldsymbol{\theta},\boldsymbol{\varepsilon}\right)d\varepsilon_i d\varepsilon_j d\theta_i d\theta_j\right). \end{aligned}$$

Using the Leibniz integral rule, we obtain:

$$\begin{aligned} \frac{\partial G_i^s\left(\delta, f_\alpha^s, \boldsymbol{c}\right)}{\partial\alpha} &= -\int_{\boldsymbol{\theta}}\int_{\boldsymbol{\varepsilon}\in\left]-\infty,\frac{c_i-\theta_i}{\alpha}\right[\times\left]\frac{c_j-\theta_j}{\alpha},+\infty\right[\cup\left]-\infty,\frac{c_i-\theta_i}{\alpha}\right[\times\left]\frac{c_j-\theta_j}{\alpha},+\infty\right[}\varepsilon_i^s h^s\left(\boldsymbol{\theta},\boldsymbol{\varepsilon}\right)d\varepsilon_i d\varepsilon_j d\theta_i d\theta_j \\ &\quad -\int_{\boldsymbol{\theta}}\int_{\frac{c_i-\theta_i}{\alpha}}^{\infty}\frac{c_j-\theta_j}{\alpha^2}\left(c_i-\theta_i-\alpha\varepsilon_i\right)h^s\left(\boldsymbol{\theta},\varepsilon_i,\varepsilon_j=\frac{c_j-\theta_j}{\alpha}\right)d\varepsilon_i d\theta_i d\theta_j \\ &\quad +\int_{\boldsymbol{\theta}}\int_{-\infty}^{\frac{c_i-\theta_i}{\alpha}}\frac{c_j-\theta_j}{\alpha^2}\left(c_i-\theta_i-\alpha\varepsilon_i\right)f^s\left(\boldsymbol{\theta},\varepsilon_i,\varepsilon_j=\frac{c_j-\theta_j}{\alpha}\right)d\varepsilon_i d\theta_i d\theta_j \\ &= -\int_{\boldsymbol{\theta}}\int_{\boldsymbol{\varepsilon}\in\left]-\infty,\frac{c_i-\theta_i}{\alpha}\right[\times\left]\frac{c_j-\theta_j}{\alpha},+\infty\right[\cup\left]-\infty,\frac{c_i-\theta_i}{\alpha}\right[\times\left]\frac{c_j-\theta_j}{\alpha},+\infty\right[}\varepsilon_i^s h^s\left(\boldsymbol{\theta},\boldsymbol{\varepsilon}\right)d\boldsymbol{\varepsilon}d\boldsymbol{\theta} \qquad (10) \\ &\quad +\int_{\boldsymbol{\theta}}\left(\int_{-\infty}^{\infty}\frac{c_j-\theta_j}{\alpha^2}\left|c_i-\theta_i-\alpha\varepsilon_i\right|f^s\left(\boldsymbol{\theta},\varepsilon_i,\varepsilon_j=\frac{c_j-\theta_j}{\alpha}\right)d\varepsilon_i\right)d\boldsymbol{\theta}. \qquad (11) \end{aligned}$$

By assumption, $\varepsilon_i \geq 0$ with probability 1, so (10) is negative, and $\varepsilon_j \leq 0$ with probability 1, so (11) is negative as well. Therefore, $G_i^s\left(\delta, f_0^s, \boldsymbol{c}\right) \geq G_i^s\left(\delta, f_1^s, \boldsymbol{c}\right)$. A similar arguments shows that $G_j^s\left(\delta, f_0^s, \boldsymbol{c}\right) \leq G_j^s\left(\delta, f_1^s, \boldsymbol{c}\right)$. ■

Lemma 2 *Using the conventions of Notation 1,* $\boldsymbol{H}^S\left(\delta, f^S, \boldsymbol{c}^S\right)$ *is isotone in* $\boldsymbol{c}^S$ *in the order* $(\leq, \geq)^S$*, and for all* $k \in \{i,j\}$*,* $H_k^S\left(\delta, f^S, \boldsymbol{c}^S\right)$ *is* δ*-Lipschitz continuous in* c_k^S *for the* sup *norm on* $\mathbb{R}^S$*. Let* $\|\theta\| \doteq \max_{s\in S,k\in\{i,j\}}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}|\theta_k| f^s(\boldsymbol{\theta})\,d\boldsymbol{\theta}$*, and let* $A \doteq \left(\left[-\frac{\delta\|\theta\|}{1-\delta}, \frac{\delta\|\theta\|}{1-\delta}\right]^2\right)^S$*. Then all fixed points* $\boldsymbol{c}^S$ *of* $\boldsymbol{H}^S\left(\delta, f^S, \boldsymbol{c}^S\right)$ *are in* A*, and* $\boldsymbol{H}^S(A) \subseteq A$*.*

Proof. That $\boldsymbol{H}^S\left(\delta, f^S, \boldsymbol{c}^S\right)$ is isotone is immediate from Lemma 1 and equation (9). To show Lipschitz continuity, note that differentiating (9) with respect to $c_k^{s'}$ yields $\partial H_k^s / \partial c_k^{s'} = \pi(s, s') \partial G_k^{s'} / \partial c_k^{s'}$. Using Lemma 1, we obtain that $\sum_{s'} \left|\partial H_k^s / \partial c_k^{s'}\right| < \delta \sum_{s'} \pi(s, s') = \delta$.

To show the last point, for any $\boldsymbol{c}^S \in \mathbb{R}^{2S}$ let $\left\|c^S\right\| = \max_{s \in S, k \in \{i,j\}} c_k^s$. From (8), we see that for all $\boldsymbol{c} \in \mathbb{R}^2$ and all $s \in S$, $\left|G_k^s\left(\delta, f^S \boldsymbol{c}\right)\right|$ is bounded by $\delta\left(\|\theta\| + |c_k|\right)$, so from (9), $\left|H_k^s\left(\delta, f^S, \boldsymbol{c}^S\right)\right|$ is bounded by $\delta\left(\|\theta\| + \left\|c^S\right\|\right)$. Therefore, for all $\boldsymbol{c}^S \in A$,

$$\left|H_k^s\left(\delta, f^S, \boldsymbol{c}^S\right)\right| \leq \delta\left(\|\theta\| + \frac{\delta\|\theta\|}{1-\delta}\right) = \frac{\delta\|\theta\|}{1-\delta}$$

so $\boldsymbol{H}^S\left(\delta, f^S, \boldsymbol{c}^S\right) \in A$. If $\boldsymbol{c}^S$ is a fixed point of $\boldsymbol{H}^S$, then the above implies that $\left\|c^S\right\| \leq \delta\left(\|\theta\| + \left\|c^S\right\|\right)$, which in turn implies that $\boldsymbol{c}^S \in A$. ■

Lemma 3 *Let σ_j^S be a stationary strategy of player j in Γ_{q^S,π^0}^{en}* [34]*.*

(i) There exists a unique cutoff strategy for player i that is a best response to σ_j^S.

(ii) This cutoff strategy is also the unique stage-undominated best response to σ_j^S.

(iii) These cutoffs are stationary and independent of the current status quo.

(iv) Let c_i^S be this cutoff strategy. For the strategy profile $\left(c_i^S, \sigma_j^S\right)$, let $V_i^s(q)$ be the continuation value for player i at the end of a period in which the decision was $q \in \{R, L\}$ and the state was $s \in S$. Then

$$c_i^s = \frac{\delta}{2}\left(V_i^s(L) - V_i^s(R)\right). \tag{12}$$

The same results hold by switching the role of i and j.

Proof. Let σ_i be a possibly nonstationary best response of player i to σ_j^S. Since σ_j^S is stationary, the continuation value of player i from $\left(\sigma_i, \sigma_j^S\right)$ at the end of any period depends only on the state s and the decision q in that period. Let $V_i^s(q)$ denote that continuation value. In any period t, given the continuation play prescribed by $\left(\sigma_i, \sigma_j^S\right)$ from period $t+1$ onwards, player i cannot do better than voting for the alternative that gives him the greatest intertemporal payoff. So if the

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[34] By stationary, we mean that the probability that a player votes for R in any period t is a function of the current status quo q^t, the current state s^t, and the current preference realization $\boldsymbol{\theta}^t$ only.

state and her current preferences in that period are s and θ_i, player i cannot do better than voting for R when

$$\theta_i + \delta V_i^s(R) > -\theta_i + \delta V_i^s(L), \tag{13}$$

and for L when the reverse inequality holds. Therefore, given the continuation play prescribed by $\left(\sigma_i, \sigma_j^S\right)$, using a voting cutoff as given by (12) is an optimal action in period t. Since the marginal distribution of θ_i has full support in every state s, this cutoff is the unique stage-undominated best response. Note that the cutoff defined by (12) is independent of the current status quo.

Construct a new strategy in which player i uses the cutoff from (12) in period t, and follows σ_i in the other periods. By construction, the new strategy is still a best response to σ_j^S. Since $V_i^S(.)$ is the same for all best responses to σ_j^S, by changing the best response strategy σ_i in all periods t for this cutoff strategy, we obtain a status-quo-independent, stationary cutoff strategy which is a best response to σ_j^S. Since any cutoff best response, or any stage-undominated strategy, must satisfy (13) in every period, they must coincide with the strategy we have constructed. ■

Proof of Proposition 1. From Lemma 3, in any stationary, stage-undominated equilibrium, players use status-quo-independent cutoff strategies. Let c_j^S be a stationary and status-quo-independent cutoff strategy of player j, and let c_i^S be the best response of player i characterized in Lemma 3. Since the status quo in a given period affects the payoffs only when players vote for opposite alternatives in that period, we have that for all $s \in S$,

$$\begin{aligned} V_i^s(L) - V_i^s(R) = {} & \delta \sum_{s' \in S} \pi\left(s, s'\right) \cdot \\ & \left(\int_{-\infty}^{c_j^{s'}} \int_{c_i^{s'}}^{\infty} \left(-\theta_i + \delta V_i^{s'}(L) - \left(\theta_i + \delta V_i^{s'}(R)\right)\right) f^{s'}(\boldsymbol{\theta})\, d\theta_i d\theta_j \right. \\ & \left. + \int_{c_j^{s'}}^{\infty} \int_{-\infty}^{c_i^{s'}} \left(-\theta_i + \delta V_i^{s'}(L) - \left(\theta_i + \delta V_i^{s'}(R)\right)\right) f^{s'}(\boldsymbol{\theta})\, d\theta_i d\theta_j \right). \end{aligned} \tag{14}$$

Using (12) in both sides of (14), we obtain $c_i^S = H_i^S\left(c_i^S, c_j^S\right)$, which is simply the Bellman equation of the maximization problem of player i. From Lemma 2 and the Banach fixed point theorem, this equation has a unique solution in c_i^S, which must be player i's best response. Hence, we have shown that the unique solution c_i^S to the equation $c_i^S = H_i^S\left(c_i^S, c_j^S\right)$ is the best response of player i to c_j^S. Clearly, a symmetric result holds by inverting the role of i and j. Therefore, the equilibrium cutoffs are the fixed points of the map $\boldsymbol{H}^S$. From Lemma 2, all fixed points of $\boldsymbol{H}^S$ lie in a certain set A which is a complete lattice for the order $(\leq,\geq)^S$. Hence, Lemma 2 together with Tarski's fixed point theorem imply that the set of fixed points of the restriction of $\boldsymbol{H}^S$ on A (and hence the set of fixed points of $\boldsymbol{H}^S$ on $\mathbb{R}^{2S}$) is a complete lattice in the order $(\leq,\geq)^S$. ■

Proof of Propostion 2.

Let $p^s(\boldsymbol{c}) = \int_{-\infty}^{c_j}\int_{c_i}^{\infty} f^s(\boldsymbol{\theta})\, d\theta_i d\theta_j + \int_{c_j}^{\infty}\int_{-\infty}^{c_i} f^s(\boldsymbol{\theta})\, d\theta_i d\theta_j$. Note that for all $s \in S$, $p^s(\boldsymbol{c}) \in [0,1]$. Using (8), we have that for $k \in \{i,j\}$, the following holds

$$G_k^s\left(\delta, f^S, \boldsymbol{c}\right) - \delta p^s(\boldsymbol{c})\, c_k = -\delta\left(\int_{-\infty}^{c_j}\int_{c_i}^{\infty} \theta_k f^s(\boldsymbol{\theta})\, d\theta_i d\theta_j + \int_{c_j}^{\infty}\int_{-\infty}^{c_i} \theta_k f^s(\boldsymbol{\theta})\, d\theta_i d\theta_j\right).$$

If f^S satisfies Assumption 1, then from the above equation we see that for all $\boldsymbol{c} \in \mathbb{R}^2$ and $s \in S$, the following holds:

$$G_i^s\left(\delta, f^S, \boldsymbol{c}\right) - \delta p^s(\boldsymbol{c})\, c_i \leq G_j^s\left(\delta, f^S, \boldsymbol{c}\right) - \delta p^s(\boldsymbol{c})\, c_j.$$

Multiplying both sides by the appropriate probabilities and summing over states, for all $\boldsymbol{c} \in \mathbb{R}^{2|S|}$ and $s \in S$, we obtain:

$$H_i^s\left(\delta, f^S, \boldsymbol{c}^S\right) - \delta \sum_{s' \in S} \pi\left(s, s'\right) p^{s'}\left(\boldsymbol{c}^{s'}\right) c_i^{s'} \leq H_j^s\left(\delta, f^S, \boldsymbol{c}\right) - \delta \sum_{s' \in S} \pi\left(s, s'\right) p^{s'}\left(\boldsymbol{c}^{s'}\right) c_j^{s'}.$$

If $\boldsymbol{c}^S$ is an equilibrium, then by Proposition 1 it is a fixed point of $\boldsymbol{H}^S$, so the above inequality implies that for all $s \in S$,

$$c_i^s - \delta \sum_{s' \in S} \pi\left(s, s'\right) p^{s'}\left(\boldsymbol{c}^{s'}\right) c_i^{s'} \leq c_j^s - \delta \sum_{s' \in S} \pi\left(s, s'\right) p^{s'}\left(\boldsymbol{c}^{s'}\right) c_j^{s'}. \tag{15}$$

This can be rewritten in matrix form as $\left(I - \delta \Pi \times D\left(\boldsymbol{c}^S\right)\right) \times \left(c_i^S - c_j^S\right) \leq^S 0$ where $\leq^S$ is the product order on $\mathbb{R}^{|S|}$, I is the $|S| \times |S|$ identity matrix, Π is the transition matrix $\left(\pi\left(s, s'\right)\right)_{s,s' \in S}$, and $D\left(\boldsymbol{c}^S\right)$ is a diagonal matrix whose diagonal terms are $\left(p^s\left(\boldsymbol{c}^s\right)\right)_{s \in S}$. Let $\|.\|$ denote the induced norm on matrices. Since Π is a stochastic matrix, $\|\Pi\| = 1$, and since for all $s \in S$, $P^s\left(\boldsymbol{c}^s\right) \in [0,1]$, $\left\|D\left(\boldsymbol{c}^S\right)\right\| \leq 1$. Therefore, $\left\|\Pi \times D\left(\boldsymbol{c}^S\right)\right\| \leq 1$, so the inverse of $I - \delta \Pi \times D\left(\boldsymbol{c}^S\right)$ is $\sum_{n=0}^{\infty} \delta^n \left(\Pi \times D\left(\boldsymbol{c}^S\right)\right)^n$, which has all its entries positive, because Π and $D\left(\boldsymbol{c}^S\right)$ have all their entries positive. Therefore, $\left(I - \delta \Pi \times D\left(\boldsymbol{c}^S\right)\right) \times \left(c_i^S - c_j^S\right) \leq^S 0$ implies that $c_i^S \leq^S c_j^S$.

Together with Assumption 1, $c_i^S \leq^S c_j^S$ implies that for all $s \in S$, the event "$\theta_i \leq c_i^s$ and $\theta_j \geq c_j^s$" has probability 0, so when players vote for opposite alternatives, $c_i - \theta_i \leq 0$ and $c_j - \theta_j \geq 0$. Hence, (8) can be rewritten

$$G_i^s\left(\delta, f^s, \boldsymbol{c}\right) = \delta \int_{-\infty}^{c_j} \int_{c_i}^{\infty} \left(c_i - \theta_i\right) f^s\left(\boldsymbol{\theta}\right) d\theta_i d\theta_j \leq 0.$$

Using this in (9), and using the fact that $\boldsymbol{c}^S$ is a fixed point of (9), we obtain that $\boldsymbol{c}^S\left(\leq, \geq\right)^S \boldsymbol{0}^S$. To prove strict inequality, note that using the weak inequality, we can rewrite:

$$G_i^s\left(\delta, f^s, \boldsymbol{c}\right) = \delta \int_{-\infty}^{c_j} \int_{c_i}^{\infty} \left(c_i - \theta_i\right) f^s\left(\boldsymbol{\theta}\right) d\theta_i d\theta_j \leq \delta \int_{-\infty}^{0} \int_{0}^{\infty} \left(-\theta_i\right) f^s\left(\boldsymbol{\theta}\right) d\theta_i d\theta_j \leq 0.$$

From Assumption 1, we know that for some $s^0 \in S$, θ_i and θ_j are of opposite sign with positive probability, so the last inequality is strict for $s = s^0$. Using this in (9), we obtain that $\boldsymbol{c}^{s^1}\left(<, >\right)^S \boldsymbol{0}$ for all s^1 for which $\pi\left(s^1, s^0\right) > 0$. From Lemma 1, we know that $G_i^s\left(\delta, f^s, \boldsymbol{c}\right)$ is increasing in c_i and strictly decreasing in c_j, hence for s^1, we have $G_i^{s^1}\left(\delta, f^{s^1}, \boldsymbol{c}^{s^1}\right) < G_i^{s^1}\left(\delta, f^{s^1}, \boldsymbol{0}\right) = \delta \int_{-\infty}^{0} \int_{0}^{\infty} \left(-\theta_i\right) f^{s^1}\left(\boldsymbol{\theta}\right) d\theta_i d\theta_j \leq 0$. Hence, $\boldsymbol{c}^{s^2}\left(<, >\right)^S \boldsymbol{0}$ for all s^2 for which $\pi\left(s^2, s^1\right) > 0$. Repeating this reasoning and using the fact that the Markov process is irreducible, we complete the proof.

■

Proof of Proposition 3. Consider the less partisan strategy profile $\boldsymbol{c}^S$. Suppose that both players deviate from $\boldsymbol{c}^S$ to $\boldsymbol{d}^S$ only in the first period of Γ_{q^S, π^0}^{en}, and let s denote the state in period 1. This deviation has an impact on players' welfare only if it changes the vote of at least one player, i.e., if $d_i^s \leq \theta_i^1 \leq c_i^s$, or if $c_j^s \leq \theta_j^1 \leq d_j^s$, respectively.

Suppose that $d_i^s \leq \theta_i^1 \leq c_i^s$, that is, the deviation from $\boldsymbol{c}^S$ to $\boldsymbol{d}^S$ in the first period makes player i vote for R instead of L (the proof in the case $c_j^s \leq \theta_j^1 \leq d_j^s$ is symmetric). From Proposition 2, we know that $c_i^s \leq c_j^s$; by assumption, we have $c_j^S \leq d_j^s$; and from Assumption 1, we have $\theta_j^1 \leq \theta_i^1$ with probability one. Therefore, $\theta_j^1 \leq \theta_i^1 \leq c_i^s \leq c_j^s \leq d_j^s$, which means that player j votes for L in the first period. Therefore, player i voting for R instead of L affects the outcome only if $q^s = R :$ L is implemented under $\boldsymbol{c}^s$ and R is implemented under $\boldsymbol{d}^s$. This change therefore increases the payoff of player $k \in \{i, j\}$ in period 1 by $2\theta_k$. Since players play their equilibrium strategy $\boldsymbol{c}^S$ in the subgame starting from period 2 onwards, using the notations of Lemma 3, the net effect of the deviation for player k is $2\theta_k + \delta\left(V_k^s(R) - V_k^s(L)\right)$. Using part (iv) of Lemma 3, we obtain that this effect is equal to $2\left(\theta_k^1 - c_k^s\right)$. And since $\theta_i^1 \leq c_i^s$ and $\theta_j^1 \leq c_j^s$, the first period deviation is detrimental for both players.

To conclude the argument, consider the strategy profile in which players play $\boldsymbol{d}^S$ in period 1 and $\boldsymbol{c}^S$ afterwards, and let players deviate from $\boldsymbol{c}^S$ to $\boldsymbol{d}^S$ also in the second period. The same reasoning as above shows that the net effect of this deviation is negative irrespective of the status quo distribution at the beginning of period 2. By induction on the number of periods in which players deviate from $\boldsymbol{c}^S$ to $\boldsymbol{d}^S$, the proposition follows. ∎

Proof of Proposition 4. Instead of proving this proposition, we state and prove a version (Proposition 11 below) that holds even when Assumption 1 is not satisfied. This will be useful in Section 8, when we relax Assumption 1. When Assumption 1 is relaxed, we no longer can assume that $\boldsymbol{c}^S\left(\delta, f^S\right)(\leq,\geq)^S\,\mathbf{0}^S$. ∎

Proposition 11 *Using the conventions in Notation 1,*

a) If $\boldsymbol{H}^S\left(\delta, f^S, \mathbf{0}^S\right)(\leq,\geq)^S\,\mathbf{0}^S$, and if there exists δ_0 such that for all $\delta \in [\delta_o, 1[$, we have $\boldsymbol{c}^S\left(\delta, f^S\right)(\leq,\geq)^S\,\mathbf{0}^S$, then $\boldsymbol{c}^S\left(\delta, f^S\right)$ is increasing in δ on $[\delta_o, 1[$ in the order $(\leq,\geq)^S$;

b) If g^S is more polarized than f^S (see definition 2), then for the least partisan equilibrium satisfying $\boldsymbol{c}^S\left(\delta, f^S\right)(\leq,\geq)^S\,\mathbf{0}^S$, we have $\boldsymbol{c}^S\left(\delta, g^S\right)(\leq,\geq)^S\,\boldsymbol{c}^S\left(\delta, f^S\right)$.

Proof. From Lemma 2, $\boldsymbol{H}^S$ is isotone in $\boldsymbol{c}^S$ in the order $(\leq,\geq)^S$, so $\boldsymbol{H}^S\left(\delta, f^S, (\mathbb{R}_- \times \mathbb{R}_+)^S\right) \subseteq (\mathbb{R}_- \times \mathbb{R}_+)^S$. If $\boldsymbol{\Gamma}^S\left(\delta, f^S, \boldsymbol{c}^S\right)$ denotes the restriction of the mapping $\boldsymbol{c}^S \to \boldsymbol{H}^S\left(\delta, f^S, \boldsymbol{c}^S\right)$ on $(\mathbb{R}_- \times \mathbb{R}_+)^S$, then for $\delta \in [\delta_0, 1[$, $\boldsymbol{c}\left(\delta, f^S\right)$ is the least fixed point of $\boldsymbol{\Gamma}^S$ for the order $(\leq,\geq)^S$. From (4), for all $\boldsymbol{c}^S(\leq,\geq)^S\,\mathbf{0}^S$, $\frac{\partial \boldsymbol{H}^S}{\partial \delta} = \frac{\boldsymbol{H}^S}{\delta}(\leq,\geq)^S\,\mathbf{0}^S$, so $\boldsymbol{\Gamma}^S$ is increasing in δ for the order $(\leq,\geq)^S$. Using the results on the comparative statics of fixed points in Villas-Boas (1997, Corollary 1), for all δ' and $\delta \in [\delta_o, \delta']$ the mapping $\boldsymbol{c}^S \to \boldsymbol{\Gamma}^S\left(\delta, f^S, \boldsymbol{c}^S\right)$ has a fixed point which is smaller than $\boldsymbol{c}^S\left(\delta', f^S\right)$ for the order $(\leq,\geq)^S$. Therefore, its smallest fixed point $\boldsymbol{c}^S\left(\delta, f^S\right)$ must satisfy $\boldsymbol{c}^S\left(\delta', f^S\right)(\leq,\geq)^S\,\boldsymbol{c}^S\left(\delta, f^S\right)$.

Part b: From Lemma 1 and (4), for all $\boldsymbol{c}^S \in \mathbb{R}^{2|S|}$, $\boldsymbol{H}^S\left(\delta, g^S, \boldsymbol{c}^S\right)(\leq,\geq)^S\,\boldsymbol{H}^S\left(\delta, f^S, \boldsymbol{c}^S\right)$. Using the same argument as in part a), the result follows from Corollary 1 in Villas-Boas (1997) applied to $\boldsymbol{c}^S \to \boldsymbol{H}^S\left(\delta, f^S, \boldsymbol{c}^S\right)$ and $\boldsymbol{c}^S \to \boldsymbol{H}^S\left(\delta, g^S, \boldsymbol{c}^S\right)$ for the order $(\leq, \geq)^S$. ■

Proof of Proposition 5. We will prove this proposition without assuming Assumption 1. This is going to be useful in Section 8, when we relax Assumption 1. Hence, we will not use the fact that $\boldsymbol{c}^S\left(\delta, f^S\right)(\leq, \geq)^S \boldsymbol{0}^S$.

First, note that it suffices to show that there exists f^S such that $\lim_{\delta \to 1} \boldsymbol{c}^S\left(\delta, f^S\right) = (-\infty, +\infty)^S$. Proposition 5 follows then from part (b) of Proposition 11: for any preference distribution g^S which is more polarized than f^S, we have $\lim_{\delta \to 1} \boldsymbol{c}^S\left(\delta, g^S\right)(\leq, \geq)^S \lim_{\delta \to 1} \boldsymbol{c}^S\left(\delta, f^S\right) = (-\infty, +\infty)^S$. Note also that it suffices to prove Proposition 5 for $|S| = 1$. That is, it suffices to show that there exists a p.d.f. f on $\mathbb{R}^2$ such that, if $\boldsymbol{c}(\delta, f)$ denotes the smallest fixed point of $\boldsymbol{c} \to \boldsymbol{G}(\delta, f, \boldsymbol{c})$, where $\boldsymbol{G}$ is defined in (8), then $\lim_{\delta \to 1} \boldsymbol{c}(\delta, f) = (-\infty, +\infty)$.[35] To see why proving the case with $|S| = 1$ is sufficient, let f^S be such that for all $s \in S$, we have $f^s = f$. In this case, all states are identical, and hence the case is equivalent to the case with $|S| = 1$. Hence, if $\lim_{\delta \to 1} \boldsymbol{c}(\delta, f) = (-\infty, +\infty)$, then $\lim_{\delta \to 1} \boldsymbol{c}^S\left(\delta, f^S\right) = (-\infty, +\infty)^S$.

Assume then that $|S| = 1$. Throughout this proof, $(\boldsymbol{m}^n)_{n \geq 0}$ is an arbitrary sequence which tends to $(+\infty, -\infty)$, f is an arbitrary p.d.f., and for all $\boldsymbol{m} \in \mathbb{R}^2$, $f_{\boldsymbol{m}}$ is defined by $f_{\boldsymbol{m}}(\boldsymbol{\theta}) = f(\boldsymbol{\theta} - \boldsymbol{m})$. With a simple change of variable, for all $\boldsymbol{c} \in \mathbb{R}^2$, we can rewrite (8) as follows:

$$\begin{aligned} \boldsymbol{G}(\delta, f_{\boldsymbol{m}}, \boldsymbol{c}) &= \delta \left(\int_{-\infty}^{c_j - m_j} \int_{c_i - m_i}^{\infty} (\boldsymbol{c} - \boldsymbol{\theta} - \boldsymbol{m}) f(\boldsymbol{\theta})\, d\theta_i d\theta_j \right. \\ &\quad \left. + \int_{c_j - m_j}^{\infty} \int_{-\infty}^{c_i - m_i} (\boldsymbol{c} - \boldsymbol{\theta} - \boldsymbol{m}) f(\boldsymbol{\theta})\, d\theta_i d\theta_j . \right) \end{aligned} \tag{16}$$

We will show that for n sufficiently large, $\lim_{\delta \to 1} \boldsymbol{c}(\delta, f_{\boldsymbol{m}^n}) = (-\infty, +\infty)$.

Step 1: For n *sufficiently large,* $\boldsymbol{c} \to \boldsymbol{G}(1, f_{\boldsymbol{m}^n}, \boldsymbol{c})$ *has no fixed point.*

Let $s \in S$. Rewriting (16), for all $\boldsymbol{m}, \boldsymbol{c} \in \mathbb{R}^2$, we obtain:

$$\begin{aligned} c_i - G_i(1, f_{\boldsymbol{m}}, \boldsymbol{c}) &= \left(\int_{-\infty}^{c_j - m_j} \int_{-\infty}^{c_i - m_i} c_i f(\boldsymbol{\theta})\, d\theta_i d\theta_j + \int_{c_j - m_j}^{\infty} \int_{c_i - m_i}^{\infty} c_i f(\boldsymbol{\theta})\, d\theta_i d\theta_j \right. \\ &\quad + \int_{c_j - m_j}^{\infty} \int_{-\infty}^{c_i - m_i} (m_i + \theta_i) f(\boldsymbol{\theta})\, d\theta_i d\theta_j \\ &\quad \left. + \int_{-\infty}^{c_j - m_j} \int_{c_i - m_i}^{\infty} (m_i + \theta_i) f(\boldsymbol{\theta})\, d\theta_i d\theta_j \right). \end{aligned} \tag{17}$$

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[35] The reader can check that Lemma 2 and its proof hold unchanged for $\boldsymbol{G}$ in place of $\boldsymbol{H}^S$ (set π equal to the identity matrix), which implies that $\boldsymbol{c}(\delta, f)$ exists for all $\delta < 1$.

We denote by $A(\boldsymbol{m},\boldsymbol{c})$, $B(\boldsymbol{m},\boldsymbol{c})$, $C(\boldsymbol{m},\boldsymbol{c})$, and $D(\boldsymbol{m},\boldsymbol{c})$ the four integrals in the order they appear on the right-hand side of (17). Suppose, by contradiction, that for all n, $\boldsymbol{G}(1,\boldsymbol{m}^n,\boldsymbol{c})$ has a fixed point, and let $(\boldsymbol{c}^n)_{n\in\mathbb{N}}$ be a selection of these fixed points. From Corollary 1 of Villas-Boas (1997), we know that we can choose $(\boldsymbol{c}^n)_{n\in\mathbb{N}}$ that is increasing in n in the order $(\leq,\geq)$. Then $\boldsymbol{c}^n-\boldsymbol{m}^n$ tends to $(-\infty,+\infty)$.

Let $f_i(.)$ and $f_j(.)$ be the marginal p.d.f.s of f. We know that if g is the p.d.f. of an integrable real random variable, then $\int_{-\infty}^{x}|xg(u)|\,du\to 0$ as $x\to-\infty$, so

$$|A(\boldsymbol{m}^n,\boldsymbol{c}^n)|\leq\int_{-\infty}^{c_i^n-m_i^n}|c_i^n|\,f_i(\theta_i)\,d\theta_i\to 0, \tag{18}$$

$$|C^s(\boldsymbol{m}^n,\boldsymbol{c}^n)|\leq\int_{-\infty}^{c_i^n-m_i^n}(|m_i^n|+|\theta_i|)\,f_i(\theta_i)\,d\theta_i\to 0, \tag{19}$$

and $D^s(\boldsymbol{m}^n,\boldsymbol{c}^n)\to+\infty$. Evaluating (17) at $c_i^n=G_i(1,f_{\boldsymbol{m}^n},\boldsymbol{c}^n)$, and using (18), (19), and $D^s(\boldsymbol{m}^n,\boldsymbol{c}^n)\to+\infty$, we obtain that $B(\boldsymbol{m}^n,\boldsymbol{c}^n)\to\infty$. However,

$$|B(\boldsymbol{m}^n,\boldsymbol{c}^n)|\leq|c_i^n|\int_{c_j^n-m_j^n}^{\infty}f_j(\theta_j)\,d\theta_j=\frac{|c_i^n|}{\left|c_j^n\right|}\times\left|c_j^n\right|\int_{c_j^n-m_j^n}^{\infty}f_j(\theta_j)\,d\theta_j.$$

Since $B(\boldsymbol{m}^n,\boldsymbol{c}^n)\to\infty$, and $\lim_{c_j^n\to\infty}\left|c_j^n\right|\int_{c_j^n-m_j^n}^{\infty}f_j(\theta_j)\,d\theta_j=0$, we need that $|c_i^n|/\left|c_j^n\right|\to+\infty$. The symmetric argument for j implies that $\left|c_j^n\right|/|c_i^n|\to+\infty$, which is a contradiction.

Step 2: Ss $\delta\to 1$, $\boldsymbol{c}(\delta,f_{\boldsymbol{m}^n})$ as a limit *(possibly infinite) which we denote by* $\boldsymbol{c}(1,f_{\boldsymbol{m}^n})$ and*, for n sufficiently large, we have* $\boldsymbol{c}(1,f_{\boldsymbol{m}^n})\in[-\infty,0]\times[0,+\infty]$.

Note that f_{m^n} is more polarized than f_{m^0}. Hence, from part (b) of Proposition 4 we know that for all n, we have $\boldsymbol{c}(\delta,f_{\boldsymbol{m}^n})\,(\leq,\geq)\,\boldsymbol{c}(\delta,f_{\boldsymbol{m}^0})$. One can see from (16) that as $\boldsymbol{m}\to(+\infty,-\infty)$, $\boldsymbol{G}(\delta,f_{\boldsymbol{m}},\boldsymbol{c})\to(-\infty,+\infty)$ uniformly on $\delta\in[0,1]$ and $\boldsymbol{c}\in[-\infty,c_i(\delta,f_{\boldsymbol{m}^0})]\times[c_j(\delta,f_{\boldsymbol{m}^0}),+\infty]$. So for n sufficiently large, $\boldsymbol{c}(\delta,f_{\boldsymbol{m}^n})\,(\leq,\geq)\,\boldsymbol{0}$, and $\boldsymbol{G}(\delta,f_{\boldsymbol{m}^n},\boldsymbol{0})\,(\leq,\geq)\,\boldsymbol{0}$. Therefore, from part (a) of Proposition 4, $\boldsymbol{c}(\delta,f_{\boldsymbol{m}^n})$ is monotonic in δ in the order $(\leq,\geq)$, which shows that $\boldsymbol{c}(1,f_{\boldsymbol{m}^n})\in[-\infty,0]\times[0,+\infty]$.

Step 3: For some $k\in\{i,j\}$, $c_k(1,f_{\boldsymbol{m}^n})$ *is infinite.*

If $\boldsymbol{c}(1,f_{\boldsymbol{m}^n})$ was finite, then by continuity of $\boldsymbol{G}(\delta,f_{\boldsymbol{m}},\boldsymbol{c})$ in δ and in $\boldsymbol{c}$, $\boldsymbol{c}(1,f_{\boldsymbol{m}^n})$ would be a fixed point of $\boldsymbol{G}(1,f_{\boldsymbol{m}^n},\boldsymbol{c})$, which is impossible from step 1.

Step 4: for n sufficiently large, $\boldsymbol{c}(1, f_{\boldsymbol{m}^n}) = (-\infty, +\infty)$.

Suppose that $c_k(1, f_{\boldsymbol{m}^n})$ is finite for all n and for some k. To fix ideas, let $k = j$. From step 3, for n sufficiently large, $c_i(1, f_{\boldsymbol{m}^n}) = -\infty$. By the continuity of G_j^n, for all n, $c_j(1, f_{\boldsymbol{m}^n})$ must be a fixed point of the map $G_j^n(-\infty, c_j)$, where $G_j^n(-\infty, c_j)$ is defined by:

$$G_j^n(-\infty, c_j) = \lim_{c_i \to -\infty} G_j(1, f_{\boldsymbol{m}^n}, c_i, c_j) = \int_{-\infty}^{c_j - m_j^n} \left(c_j - \theta_j - m_j^n\right) f_j(\theta_j)\, d\theta_j.$$

This equation shows that for n sufficiently large n, $G_j^n(-\infty, c_j) - c_j > 0$ when $c_j = 0$, and $G_j^n(-\infty, c_j) - c_j > 0$ as $c_j \to \infty$. So, if a fixed point exists, $G_j^n(-\infty, c_j)$ must cross c_j from below at least once. Simple calculus shows that $\frac{dG_j^n(-\infty, c_j)}{dc_j} = \int_{-\infty}^{c_j - m_j^n} f_j(\theta_j)\, d\theta_j \leq 1$. Therefore, $G_j^n(-\infty, c_j)$ can cross c_j only from above, which is a contradiction. ■

Proof of Corollary 2. Since ε has a symmetric distribution, $\boldsymbol{c}(\delta, f) = (-c(\delta, f), c(\delta, f))$. From Proposition 2, we know that $c(\delta, f) \geq 0$, and from Proposition 1, $c(\delta, f)$ is the smallest fixed point of $H(\delta, f, c) = \delta \int_{-c-\bar{\theta}}^{c+\bar{\theta}} \left(c + \bar{\theta} - \varepsilon\right) f(\varepsilon)\, d\varepsilon$. From the proof of Proposition 5, to show that $\lim_{\delta \to 1} \boldsymbol{c}(\delta, f) = (-\infty, +\infty)$, we need to show that $H(1, f, c)$ has no fixed point for $\bar{\theta} \geq \sigma$. Using successively the symmetry of ε, for all $c > 0$, we obtain

$$\begin{aligned}
H(1, f, c) - c &= \int_{-c-\bar{\theta}}^{c+\bar{\theta}} \left(c + \bar{\theta}\right) f(\varepsilon)\, d\varepsilon - \int_{-\infty}^{\infty} c f(\varepsilon)\, d\varepsilon \\
&= -\int_{-\infty}^{-c-\bar{\theta}} \left(c + \bar{\theta}\right) f(\varepsilon)\, d\varepsilon - \int_{c+\bar{\theta}}^{+\infty} \left(c + \bar{\theta}\right) f(\varepsilon)\, d\varepsilon + \bar{\theta} \\
&> -\int_{-\infty}^{-c-\bar{\theta}} |\varepsilon| f(\varepsilon)\, d\varepsilon - \int_{c+\bar{\theta}}^{+\infty} |\varepsilon| f(\varepsilon)\, d\varepsilon + \bar{\theta} \\
&> -\int_{-\infty}^{+\infty} |\varepsilon| f(\varepsilon)\, d\varepsilon + \bar{\theta}.
\end{aligned}$$

■

Proof of Propostion 6. Let $\boldsymbol{c}^S$ be an equilibrium of Γ_{q^S,π^0}^{en}. We shall compare the equilibrium payoffs in every period t in Γ_{q^S,π^0}^{en} and in $\Gamma_{q'^S,\pi^0}^{ex}$. Let s be the state in period t. There are 5 possible cases to consider:

Case 1: $\theta_i^t < c_i^s$. In this case, we have $\theta_j^t < \theta_i^t < c_i^s < 0 < c_j^s$, so both players vote for L in Γ_{q^S,π^0}^{en} and in $\Gamma_{q'^S,\pi^0}^{ex}$. Therefore, the period t payoffs in the two games are the same.

Case 2: $\theta_j^t > c_j^s$. In this case, we have $c_i^s < 0 < c_j^s < \theta_j^t < \theta_i^t$, so both players vote for R in $\Gamma^{en}_{q^S,\pi^0}$ and in $\Gamma^{ex}_{q'^S,\pi^0}$. Therefore, the period t payoffs in the two games are the same.

Case 3: $c_i^s < \theta_i^t < 0$. In this case, we have $\theta_j^t < \theta_i^t < 0 < c_j^s$, so both players vote for L in $\Gamma^{ex}_{q'^S,\pi^0}$, but they disagree in $\Gamma^{en}_{q^S,\pi^0}$. Since $\theta_i^t + \theta_j^t < 0$, the sum of players' period t payoffs is weakly higher in $\Gamma^{ex}_{q'^S,\pi^0}$ than in $\Gamma^{en}_{q^S,\pi^0}$, and it is strictly higher when $q^s = R$.

Case 4: $0 < \theta_j^t < c_j^s$. In this case, we have $c_i^s < 0 < \theta_j^t < \theta_i^t$, so both players vote for R in $\Gamma^{ex}_{q'^S,\pi^0}$, but they disagree in $\Gamma^{en}_{q^S,\pi^0}$. Since $\theta_i^t + \theta_j^t > 0$, the sum of players' period t payoffs is weakly higher in $\Gamma^{ex}_{q'^S,\pi^0}$ than in $\Gamma^{en}_{q^S,\pi^0}$, and it is strictly higher when $q^s = L$.

Case 5: $\theta_j^t < 0 < \theta_i^t$. In this case, we have $c_i^s < \theta_i^t$ and $\theta_j^t < c_i^s$. That means that in period t, players disagree in both games, and the status quo prevails. So which game yields the highest social welfare depends on the distribution of the status quo in period t of $\Gamma^{ex}_{q'^S,\pi^0}$ and $\Gamma^{en}_{q^S,\pi^0}$ and on $\theta_j^t + \theta_i^t$. Suppose, without loss of generality, that R is the status quo that results in higher expected welfare in period t in state s in case 5. If we set $q'^s = R$ in $\Gamma^{ex}_{q'^S,\pi^0}$, the welfare in state s will be weakly higher on average in $\Gamma^{ex}_{q'^S,\pi^0}$ than in $\Gamma^{en}_{q^S,\pi^0}$.

Since the marginal distributions of θ_i and θ_j are assumed to have full support, Proposition 2 implies that cases 3 and 4 occur with strictly positive probability. Depending on the identity of q^s, in at least one of these two cases welfare in $\Gamma^{en}_{q^S,\pi^0}$ is strictly lower than in $\Gamma^{ex}_{q'^S,\pi^0}$. ■

Proof of Proposition 7. One can easily check that Lemma 3 and its proof hold unchanged for the game $\Gamma^{en}_{q^S,\pi^0}(\Omega)$ if we replace player i and j by player $n \in N$ and all the other players, respectively. Hence, a stationary, stage undominated equilibrium must be in cutoff strategies with some cutoffs $\boldsymbol{c}^S$. If $\boldsymbol{V}^S(L)$ and $\boldsymbol{V}^S(R)$ denote the continuation values for the strategy profile $\boldsymbol{c}^S$, then by Lemma 3, c_n^s must satisfy $c_n^s = \frac{\delta}{2}\left(V_n^s(L) - V_n^s(R)\right)$ for all $n \in N$ and all $s \in S$.

For all $\boldsymbol{c}^s \in \mathbb{R}^N$, let $D(\boldsymbol{c}^s) \subseteq \mathbb{R}^N$ be the set of preference realizations $\boldsymbol{\theta}$ such that if players vote according to the strategy profile $\boldsymbol{c}^s$ in state s, the outcome of the vote is different when $q^s = L$ than when $q^s = R$. Condition (iv) in Definition 3 implies that if $\{i \in N : \theta_i \geq c_i\} \in \Omega_L$, then it cannot be that $\{i \in N : \theta_i \leq c_i\} \in \Omega_R$. Therefore, the outcome of the vote is different depending on q^s only when the status quo prevails. That is,

$$D(\boldsymbol{c}) = \left\{\boldsymbol{\theta} \in \mathbb{R}^N : \{i \in N : \theta_i \geq c_i\} \notin \Omega_L \text{ and } \{i \in N : \theta_i \leq c_i\} \notin \Omega_R\right\}.$$

The status quo matters in some period t with state s only if $\boldsymbol{\theta}^t \in D(\boldsymbol{c}^s)$, so

$$V_n^s(L) - V_n^s(R) = \delta \sum_{s' \in S} \pi(s, s') \left(\int_{D\left(\boldsymbol{c}^{s'}\right)} \left(-\theta_n + \delta V_n^{s'}(L) - \left(\theta_n + \delta V_n^{s'}(R) \right) \right) f^{s'}(\boldsymbol{\theta})\, d\boldsymbol{\theta} \right).$$

If we substitute (12) into both sides of the above equation, we obtain

$$c_n^s = \delta \sum_{s' \in S} \pi(s, s') \int_{D\left(\boldsymbol{c}^{s'}\right)} \left(-\theta_n + c_n^{s'} \right) f^{s'}(\boldsymbol{\theta})\, d\boldsymbol{\theta}. \tag{20}$$

Since $\theta_1 \geq .. \geq \theta_N$ with probability one, for all $s \in S$, the expression $\int_{D(\boldsymbol{c}^s)} \theta_n f^s(\boldsymbol{\theta})\, d\boldsymbol{\theta}$ is weakly decreasing in n. Together with (20), this implies that $c_n^s - \delta \sum_{s' \in S} \pi(s, s') \Pr\left(D\left(\boldsymbol{c}^{s'}\right)\right) c_n^{s'}$ is weakly increasing in n. As shown in the proof of Proposition 2, this implies in turn that c_n^s is weakly increasing in n, which proves part (i).

From what precedes, for all state s, $\theta_1 - c_1^s \geq .. \geq \theta_N - c_N^s$ with probability one. So for any $\boldsymbol{\theta}$, there exists $n \in \{0, ..N\}$ such that

$$\begin{aligned} \{i \in N : \theta_i \geq c_i^s\} &= \{1, ..., n\}, \\ \{i \in N : \theta_i \leq c_i^s\} &= \{n+1, ..., N\}. \end{aligned}$$

Therefore, up to a zero measure set, we can rewrite $D(\boldsymbol{c}^s)$ as a function of the preference realization of the pivotal players only (see Definition 4):

$$D(\boldsymbol{c}^s) = \left\{ \boldsymbol{\theta} \in \mathbb{R}^N : \theta_{n_L} \leq c_{n_L}^s \text{ and } \theta_{n_R} \geq c_{n_R}^s \right\}.$$

Substituting $D(\boldsymbol{c}^s)$ into (20), we obtain (5). Since players n_L and n_R are always pivotal, $\Gamma_{q^S, \pi^0}^{en}(\Omega)$ is equivalent to the two-player game Γ_{q^S, π^0}^{en} played by the two pivotal players. Since $n_R \leq n_L$, we have $\theta_{n_R} \geq \theta_{n_L}$ with probability one, so Proposition 2 implies that for all $s \in S$, $c_{n_R}^s \leq 0 \leq c_{n_L}^s$ (part ii). The lattice structure (part iii) follows from Proposition 1.

To prove part (iv), we will use the proof of Proposition 3. Consider two equilibria of $\Gamma_{q^S, \pi^0}^{en}(\Omega)$: $\boldsymbol{c}^S$ and $\boldsymbol{d}^S$ such that $\boldsymbol{d}^S (\leq, \geq)^S \boldsymbol{c}^S$. We already know that $c_d^s = \frac{\delta}{2}\left(V_d^s(L) - V_d^s(R)\right)$ and $c_i^s \leq c_d^s \leq c_j^s$. Now consider the two-player equivalent of $\Gamma_{q^S, \pi^0}^{en}(\Omega)$, in which only the votes of the pivotal players affect the outcome, but all other players receive payoffs. Assume that the pivotal players play according to $\boldsymbol{c}^S$, and suppose that they deviate to $\boldsymbol{d}^S$ at time t only. As argued in the proof of Proposition 3, without loss of generality we can look only at the case in which $d_i^s \leq \theta_i^t \leq c_i^s$. In this case, period t deviation may change the outcome from L to R only. Player d's payoff changes then from $-\theta_d^t + \delta V_d^s (L)$ to $\theta_d^t + \delta V_d^s (R)$. Hence, her gain is $2\theta_d^t + \delta \left(V_d^s (R) - V_d^s (L)\right) = 2\left(\theta_d^t - c_d^s\right)$, where the last equality comes from the fact that from the next period on, the pivotal players adhere to $\boldsymbol{c}^S$. Since $\theta_d^t \leq \theta_i^t$ with probability 1, $\theta_i^t \leq c_i^s$ implies that $\theta_d^t - c_d^s \leq 0$ with probability 1, so this deviation is detrimental to player d. The rest of the proof follows directly the same steps as in the proof of Proposition 3. ■

Proof of Proposition 8. The inequalities $c_{n_R}^s (\Omega) \leq 0 \leq c_{n_L}^s (\Omega)$ are established in Proposition 7. From Definition 4,

$$\left\{1, ..., n_L'\right\} \in \Omega_L' \text{ and } \left\{n_R', ..., N\right\} \in \Omega_R'.$$

Since the concentration of power is greater under Ω than under Ω', we have

$$\left\{1, ..., n_L'\right\} \in \Omega_L \text{ and } \left\{n_R', ..., N\right\} \in \Omega_R.$$

It follows from Definition 4 and 3 that $n_R' \leq n_R$ and $n_L \leq n_L'$. Proposition 7 implies then that for all s, $c_{n'_R}^s (\Omega) \leq c_{n_R}^s (\Omega)$ and $c_{n_L}^s (\Omega) \leq c_{n_L'}^s (\Omega)$.

To complete the proof, it remains to show that $c_{n'_R}^s (\Omega') \leq c_{n_R'}^s (\Omega)$ and $c_{n_L'}^s (\Omega) \leq c_{n_L'}^s (\Omega')$. Since $\theta_1 \geq ... \geq \theta_N$ with probability 1, the distribution of $\left(\theta_{n_L'}, \theta_{n_R'}\right)$ is more polarized than the distribution of $(\theta_{n_L}, \theta_{n_R})$ in the sense of Definition 2. Proposition 4 implies then that for all $s \in S$, in the least partisan equilibria of $\Gamma_{q^S,\pi^0}^{en} (\Omega)$ and $\Gamma_{q^S,\pi^0}^{en} (\Omega')$, the following holds:

$$c_{n_R'}^s \left(\Omega'\right) \leq c_{n_R}^s (\Omega) \leq \text{ and } c_{n_L}^s (\Omega) \leq c_{n_L'}^s \left(\Omega'\right). \tag{21}$$

From Proposition 7,

$$c_{n_L'}^s \left(\Omega'\right) = \delta \sum_{r \in S} \pi (s, r) \int_{\left\{\boldsymbol{\theta} \in \mathbb{R}^N \colon \theta_{n_R'} \geq c_{n_R'}^r (\Omega') \text{ and } \theta_{n_L'} \leq c_{n_L'}^r (\Omega')\right\}} \left(c_{n_L'}^r \left(\Omega'\right) - \theta_{n_L'}\right) f^r (\boldsymbol{\theta})\, d\boldsymbol{\theta}.$$

Hence, $c^s_{n'_L}(\Omega')$ is a sum of integrals whose integrands are nonnegative on their respective domains. Moreover, (21) together with $n'_R \leq n_R$ and $n_L \leq n'_L$ imply that for all $r \in S$, with probability one,

$$\left\{\theta_{n'_R} \geq c^r_{n'_R}(\Omega') \text{ and } \theta_{n'_L} \leq c^r_{n'_L}(\Omega')\right\} \Rightarrow \left\{\theta_{n_R} \geq c^r_{n_R}(\Omega) \text{ and } \theta_{n_L} \leq c^r_{n_L}(\Omega)\right\}.$$

Therefore,

$$c^s_{n'_L}(\Omega') \leq \delta \sum_{r \in S} \pi(s, s') \int_{\left\{\boldsymbol{\theta} \in \mathbb{R}^N:\, \theta_{n_R} \geq c^r_{n_R}(\Omega) \text{ and } \theta_{n_L} \leq c^r_{n_L}(\Omega)\right\}} \left(c^{s'}_{n'_L}(\Omega') - \theta_{n'_L}\right) f^{s'}(\boldsymbol{\theta})\, d\boldsymbol{\theta}.$$

From Proposition 7, the right hand-side of the above equation is simply $c^s_{n'_L}(\Omega)$. A symmetric argument shows that $c^s_{n'_R}(\Omega) \leq c^s_{n'_R}(\Omega')$. ■

In what follows, with a slight abuse of notation, the term "preference distribution" refers to a state dependent p.d.f. f^S, but also to the corresponding sequence of random variable $\left(\theta^t_i, \theta^t_j\right)_{t \geq 1}$.

Definition 7 *For any $k \in \{i, j\}$, a strategy σ_k of the game $\Gamma^{en}_{q^S, \pi^0}$ is more leftist (rightist) than another strategy σ'_k if after any history, σ_k votes for left (right) whenever σ'_k does.*
A preference distribution $\left(\theta^t_k\right)_{t \geq 1}$ for a player $k \in \{i, j\}$ is more leftist (rightist) than another preference distribution $\left(\theta'^t_k\right)_{t \geq 1}$ if for all t, $\theta^t_k \leq \theta'^t_k$ ($\theta^t_k \geq \theta'^t_k$) with probability 1.

The following Lemma establishes some monotone comparative statics results for the game $\Gamma^{en}_{q^S, \pi^0}$: the best response to a more rightist strategy is a more leftist strategy (part (*ii*)), and more rightist players prefer more rightist strategies (part (*i*) and (*iii*)). These results will be instrumental in proving Proposition 9 below. In that lemma and in the proof of Proposition 9, we let the players' preference distribution vary. In particular, notice that if a player's strategy σ_k is a cutoff strategy c^S_k for a given preference distribution $\left(\theta^t_k\right)_{t \geq 1}$, the same strategy σ_k might correspond to different cutoffs c'^S_k for a different preference distribution $\left(\theta'^t_k\right)_{t \geq 1}$, or it might not be a cutoff strategy anymore for $\left(\theta'^t_k\right)_{t \geq 1}$. Therefore, when the players' preference distributions are not clear from the context, a cutoff strategy is a referred to by a general random variable σ_k instead of an element c^S_k of $\mathbb{R}^S$.

Lemma 4 *In the following claims, the underlying game is $\Gamma^{en}_{q^S, \pi^0}$, and more rightist and more leftist are used in the sense of Definition 7. The same results hold if we switch the role of i and j.*

(*i*) *If* σ_i *and* σ'_i *are the cutoff best responses to a status quo independent, stationary cutoff strategy* σ_j *for two preference distributions* $\left(\theta_i^t, \theta_j^t\right)_{t\geq 1}$ *and* $\left(\theta_i'^t, \theta_j^t\right)_{t\geq 1}$*, respectively, and if* $\left(\theta_i'^t\right)_{t\geq 1}$ *is more rightist (leftist) than* $\left(\theta_i^t\right)_{t\geq 1}$*, then* σ'_i *is more rightist (leftist) than* σ_i*.*

(*ii*) *Let* σ_i *and* σ'_i *be two status quo independent, stationary cutoff strategies, possibly for different preference distribution of player* i*. If* σ'_i *is more rightist (leftist) than* σ_i*, and if* σ_j *and* σ'_j *are the cutoff best response to* σ_i *and* σ'_i*, respectively, for a given preference distribution of player* j*, then* σ'_i *is more leftist (rightist) than* σ_i*.*

(*iii*) *If the strategy profile* $\boldsymbol{\sigma}'$ *is more rightist (leftist) than* $\boldsymbol{\sigma}$*, and if* $\boldsymbol{\sigma}'$ *achieves a higher expected payoff than* $\boldsymbol{\sigma}$ *for a preference distribution* $\left(\theta_k^t\right)_{t\geq 1}$*, then the same is true for a preference distribution* $\left(\theta_k'^t\right)_{t\geq 1}$ *which is more rightist (leftist) than* $\left(\theta_k^t\right)_{t\geq 1}$*.*

Proof. Throughout this proof, we use the fact that, as shown in the proof of proposition 1, for a given preference distribution f^S, the cutoff best response of player i to c_j^S is given by the fixed point of $c_i^S \to H_i^S\left(\delta, f^S, \boldsymbol{c}^S\right)$, and that this mapping is a contraction. The order $\geq^S$ refers to the component-wise order on $\mathbb{R}^S$.

(*i*) Let g^S and h^S be the p.d.f. of the preference distributions $\left(\theta_i^t, \theta_j^t\right)_{t\geq 1}$ and $\left(\theta_i'^t, \theta_j^t\right)_{t\geq 1}$, respectively. Under our assumption, h^S is more polarized than g^S in the sense of Definition 2. From Lemma 1 and (4), for all $\boldsymbol{c}^S \in \left(\mathbb{R}^2\right)^S$, $H_i^S\left(\delta, g^S, \boldsymbol{c}^S\right) \geq^S H_i^S\left(\delta, h^S, \boldsymbol{c}^S\right)$. Theorem 2 in Villas-Boas 1997 implies then that the fixed point of $c_i^S \to H_i^S\left(\delta, g^S, \boldsymbol{c}^S\right)$ must be greater than the fixed point of $c_i^S \to H_i^S\left(\delta, h^S, \boldsymbol{c}^S\right)$ for the order $\geq^S$, which means that the latter strategy will be more rightist than the former.

(*ii*) Suppose first that σ_i and σ'_i are cutoff strategies with respect to the same preference distribution $\left(\theta_i^t\right)_{t\geq 1}$. Let c_i^S and $c_i'^S$ denote these cutoffs. Since σ'_i is more rightist than σ_i, $c_i^S \geq^S c_i'^S$. From Lemma 1 and (4), this implies that for all $c_j^S \in \mathbb{R}^S$, $H_j^S\left(\delta, f^S, \left(c_i'^S, c_j^S\right)\right) \geq^S H_j^S\left(\delta, f^S, \left(c_i^S, c_j^S\right)\right)$. Theorem 2 in Villas-Boas 1997 implies then that the fixed point of $c_j^S \to H_j^S\left(\delta, f^S, \left(c_i'^S, c_j^S\right)\right)$ must be greater than the fixed point of $c_j^S \to H_j^S\left(\delta, f^S, \left(c_i^S, c_j^S\right)\right)$ for the order $\geq^S$. This means that the best response to σ'_i is more leftist than the best response to σ_i.

Suppose now that σ_i and σ'_i are cutoff strategies with respect to two different preference distribution $\left(\theta_i^t\right)_{t\geq 1}$ and $\left(\theta_i'^t\right)_{t\geq 1}$, and let c_i^S and $c_i'^S$ be the corresponding cutoff. To complete the proof, it suffices to construct a preference distribution $\left(\theta_i''^t\right)_{t\geq 1}$ such that σ_i and σ_i' are both cutoff strategies for $\left(\theta_i''^t\right)_{t\geq 1}$. Since σ_i' is more rightist than σ_i, in any period t with state s, $\theta_i'^t \leq c_i'^s \Rightarrow \theta_i^t \leq c_i^s$. For all $s \in S$, consider the map $\Phi^s\left(\theta_i, \theta_i'\right)$ defined by

$$\Phi^s\left(\theta_i,\theta_i'\right) = \begin{cases} \theta_i' - c_i'^s \text{ when } \theta_i' \leq c_i'^s, \\ \exp\left(\min\left(\frac{\theta_i - c_i^s}{\theta_i' - c_i'^s}, 0\right)\right) \text{ when } \theta_i' > c_i'^s \text{ and } \theta_i < c_i^s, \\ \theta_i - c_i^s + 1 \text{ when } \theta_i \geq c_i^s. \end{cases}$$

The map Φ^s is well defined and continuous on $\left\{\left(\theta_i,\theta_i'\right) \in \mathbb{R}^2 : \theta_i' \leq c_i'^s \Rightarrow \theta_i \leq c_i^s\right\}$ except at $\left(\theta_i,\theta_i'\right) = \left(c_i^s, c_i'^s\right)$ and on that domain,

$$\Phi^s\left(\theta_i,\theta_i'\right) = \begin{cases} \lesseqgtr 0 \text{ when } \theta_i' \lesseqgtr c_i'^s \\ \lesseqgtr 1 \text{ when } \theta_i \lesseqgtr c_i^s \end{cases}.$$

This means that for the preference distribution $\left(\Phi^{s^t}\left(\theta_i^t, \theta_i'^t\right)\right)_{t\geq 1}$, up to a zero measure set, σ_i and σ_i' are the cutoff strategies $(1,...,1)$ and $(0,...,0)$ respectively.

(iii) Let $\left(y^t\right)_{t\geq 1}$ and $\left(y'^t\right)_{t\geq 1}$ be the outcomes of $\Gamma^{en}_{q^S,\pi^0}$ under the strategy profiles $\boldsymbol{\sigma}$ and $\boldsymbol{\sigma}'$, respectively. For convenience, R will be denoted by 1 and L by -1, so that for all t, y^t and y'^t are in $\{-1,1\}$. Since $\boldsymbol{\sigma}'$ is more rightist than $\boldsymbol{\sigma}$, a straightforward induction argument shows that with probability 1, $y^t \leq y'^t$, and therefore,

$$\sum_{k\in\{i,j\}} \theta_k^t\left(y'^t - y^t\right) \leq \sum_{k\in\{i,j\}} \theta_k'^t\left(y'^t - y^t\right).$$

Taking expectations, the above inequality implies that

$$E\left[\sum_{k\in\{i,j\}} \theta_k^t y'^t\right] \geq E\left[\sum_{k\in\{i,j\}} \theta_k^t y^t\right] \Longrightarrow E\left[\sum_{k\in\{i,j\}} \theta_k'^t y'^t\right] \geq E\left[\sum_{k\in\{i,j\}} \theta_k'^t y^t\right].$$

■

Proof of Proposition 9. For all $n_i, n_j \in N$, let $\Gamma^{en}_{q^S,\pi^0}\left(n_i,n_j\right)$ denote the 2-player game $\Gamma^{en}_{q^S,\pi^0}$ in which players i and j have the same preference distribution as players n_i and n_j in $\Gamma^{en}_{q^S,\pi^0}(\Omega)$.

In what follows, for all $n_i, n_j \in N$, we allow the strategies σ_i and σ_j of $\Gamma^{en}_{q^S,\pi^0}(n_i, n_j)$ to depend on the whole profile of preference $\left(\left(\theta^t_n\right)_{t\geq 1}\right)_{n\in N}$. So a strategy of $\Gamma^{en}_{q^S,\pi^0}(n_i, n_j)$ is a function of $\left(\left(\theta^t_n\right)_{t\geq 1}\right)_{n\in N}$ and of the history of the game. This function will be assumed to stay constant as we change the identity of the players n_i and n_j. Under this convention, the preferences of a player $n \in N$ over strategy profiles are independent of n_i and n_j, and a strategy is said to be a status quo independent, stationary cutoff strategy if it is a status quo independent, stationary cutoff with respect to the preference distribution of some player $n \in N$.[36]

Consider the sequence of status quo independent, cutoff stationary strategy profiles $(\boldsymbol{\sigma}^m)_{m\geq 0}$ defined as follows:

- $\boldsymbol{\sigma}^0$ is a (stationary, stage undominated) equilibrium of $\Gamma^{en}_{q^S,\pi^0}(n'_R, n'_L)$,
- for all $m \geq 1$, σ^{m+1}_i is player i's best response to σ^m_j in $\Gamma^{en}_{q^S,\pi^0}(n_R, n_L)$ and σ^{m+1}_j is player j's best response to σ^{m+1}_i in $\Gamma^{en}_{q^S,\pi^0}(n_R, n_L)$, as characterized in Lemma 3.

Hence, the strategy profiles in the sequence $(\boldsymbol{\sigma}^m)_{m\geq 0}$ have players n_R and n_L best respond myopically and successively to each other, and the starting point of that best respond dynamics is some equilibrium of $\Gamma^{en}_{q^S,\pi^0}$ played by players n'_R and n'_L. The first step of the proof is to show by induction on m that for all $m \geq 0$,

$P1$ σ^{m+1}_i is more leftist than σ^m_i

$P2$ σ^{m+1}_j is more rightist than σ^m_j

$P3$ For all $n \in \{n_R, ..., n_L\}$, player n prefers the outcome of $\boldsymbol{\sigma}^{m+1}$ to the outcome of $\boldsymbol{\sigma}^m$.

Let (n_R, n_L) and (n'_R, n'_L) be the pivotal players of $\Gamma^{en}_{q^S,\pi^0}(\Omega)$ and $\Gamma^{en}_{q^S,\pi^0}(\Omega')$, respectively. As shown in the proof of Proposition 8, with probability one,

$$\theta_{n'_R} \geq \theta_{n_R} \geq \theta_{n_L} \geq \theta_{n'_L}. \tag{22}$$

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[36] Observe that since conditional on the current state, a players' strategy is independent of the history of the game, the fact that a player's strategy in $\Gamma^{en}_{q^S,\pi^0}(n, m)$ depend on the preferences of another player of $\Gamma^{en}_{q^S,\pi^0}(\Omega)$ does not affect whether it is stationary or not.

By construction, σ_i^1 is player i's best response to σ_j^0 in $\Gamma_{q^S,\pi^0}^{en}(n_R, n_L)$ (and hence in $\Gamma_{q^S,\pi^0}^{en}(n_R, n'_L)$), while σ_i^0 is player i's best response to σ_j^0 in $\Gamma_{q^S,\pi^0}^{en}(n'_R, n'_L)$. From (22), $\theta_{n'_R} \geq \theta_{n_R}$ with probability one, so from Lemma 4 (*i*), σ_i^1 is more leftist than σ_i^0, which shows property $P1$ at $m = 1$.

Likewise, σ_j^0 is player j's best response to σ_i^0 in $\Gamma_{q^S,\pi^0}^{en}(n'_R, n'_L)$. Since $\theta_{n_L} \geq \theta_{n'_L}$, Lemma 4 (*i*) implies that player j's best response to σ_i^0 in $\Gamma_{q^S,\pi^0}^{en}(n'_R, n_L)$ must be more rightist than σ_j^0. Observe that σ_j^1 is also player j's best response to σ_i^1 in $\Gamma_{q^S,\pi^0}^{en}(n'_R, n_L)$. Since σ_i^1 more leftist than σ_i^0, Lemma 4 (*ii*) implies that σ_j^1 must be more rightist than player j's best response to σ_i^0 in the game $\Gamma_{q^S,\pi^0}^{en}(n'_R, n_L)$, and thus more rightist than σ_j^0. This shows property $P2$ for $m = 1$.

To prove property $P3$ for $m = 1$, observe that since she is best responding, player i in $\Gamma_{q^S,\pi^0}^{en}(n_R, n'_L)$ prefers the strategy profile $\left(\sigma_i^1, \sigma_j^0\right)$ to $\boldsymbol{\sigma}^0$. Since σ_i^1 is more leftist than σ_i^0, Lemma 4 (*iii*) implies that any player who is more leftist than n_R, and thus any player $n \geq n_R$, is also better-off at $\left(\sigma_i^1, \sigma_j^0\right)$ than at $\boldsymbol{\sigma}^0$. Likewise, since she is best responding, player j prefers the strategy profile $\boldsymbol{\sigma}^1$ to $\left(\sigma_i^1, \sigma_j^0\right)$ in $\Gamma_{q^S,\pi^0}^{en}(n_R, n_L)$, and since σ_j^1 is more rightist than σ_j^0, so do any player who is more rightist than n_L, and thus any player $n \leq n_L$.

Now suppose that properties $P1$, $P2$, and $P3$ hold at some $m \geq 1$. From $P2$, σ_j^{m+1} is more leftist than σ_j^m, so from Lemma 4 (*ii*), player i's best response to σ_j^{m+1} is more rightist than her best response to σ_j^m in $\Gamma_{q^S,\pi^0}^{en}(n_R, n_L)$, which implies property $P1$ at $m+1$. A symmetric argument shows that properties $P1$ at m implies property $P2$ at $m + 1$.

To prove property $P3$ at $m+1$, observe that since she is best responding, player i in $\Gamma_{q^S,\pi^0}^{en}(n_R, n_L)$ prefers the strategy profile $\left(\sigma_i^{m+2}, \sigma_j^{m+1}\right)$ to $\boldsymbol{\sigma}^{m+1}$. From property $P1$ at $m + 1$ and from Lemma 4 (*iii*), any player $n \geq n_R$, is also better-off at $\left(\sigma_i^{m+2}, \sigma_j^{m+1}\right)$ than at $\boldsymbol{\sigma}^{m+1}$. Likewise, since she is best responding, player j in $\Gamma_{q^S,\pi^0}^{en}(n_R, n_L)$ prefers the strategy profile $\boldsymbol{\sigma}^{m+2}$ to $\left(\sigma_i^{m+2}, \sigma_j^{m+1}\right)$. From property $P2$ at $m + 1$ and from Lemma 4 (*iii*), any player $n \leq n_L$, is also better-off at $\boldsymbol{\sigma}^{m+2}$ than at $\left(\sigma_i^{m+2}, \sigma_j^{m+1}\right)$.

By construction, for all $m \geq 1$, $\boldsymbol{\sigma}^m$ is a cutoff strategy profile for players n_R and n_L, so for all $m \geq 1$, $\boldsymbol{\sigma}^m$ can be represented as an element of $\left(\mathbb{R}^2\right)^S$. Property $P1$ and $P2$ imply that for all $m \geq 1$, $\boldsymbol{\sigma}^{m+1} (\leq, \geq)^S \boldsymbol{\sigma}^m$, and from Proposition 2, we know that $\mathbf{0}^S (\leq, \geq)^S \boldsymbol{\sigma}^m$. Therefore, $(\boldsymbol{\sigma}^m)_{m \geq 0}$ has a limit $\boldsymbol{\sigma}^\infty$. Since payoffs are continuous in stationary cutoffs, the maximum theorem implies that $\boldsymbol{\sigma}^\infty$ is a stationary cutoff equilibrium of $\Gamma_{q^S,\pi^0}^{en}(n_R, n_L)$. Together with Proposition 7, this shows that there exists an equilibrium of $\Gamma_{q^S,\pi^0}^{en}(\Omega)$ in which the pivotal players n_R and n_L

play $\boldsymbol{\sigma}^{\infty}$.

Now suppose that $\boldsymbol{\sigma}^0$ is the strategy profile played by the pivotal players n'_R and n'_L at the least partisan equilibrium of $\Gamma^{en}_{q^S,\pi^0}(\Omega')$. Property $P3$ implies that all players $n \in \{n_R, n_L\}$ are better-off at $\boldsymbol{\sigma}^{\infty}$ than at $\boldsymbol{\sigma}^0$ in the two player game $\Gamma^{en}_{q^S,\pi^0}$, and thus are also better off at the equilibrium of $\Gamma^{en}_{q^S,\pi^0}(\Omega)$ corresponding to $\boldsymbol{\sigma}^{\infty}$ than at the least partisan equilibrium of $\Gamma^{en}_{q^S,\pi^0}(\Omega')$. From Proposition 7, these players are better off at the least partisan equilibrium of $\Gamma^{en}_{q^S,\pi^0}(\Omega)$ than at the equilibrium of $\Gamma^{en}_{q^S,\pi^0}(\Omega)$ corresponding to $\boldsymbol{\sigma}^{\infty}$, which concludes the proof of the first point.

If further $\theta_{n_R} \leq \frac{1}{N}\sum_n \theta_n \leq \theta_{n_L}$ with probability 1, then the preference distribution corresponding to the utilitarian social welfare is also more leftist than n_R and more rightist than n_L, so property $P3$ also holds for that preference distribution, so the least partisan equilibrium of $\Gamma^{en}_{q^S,\pi^0}(\Omega)$ achieves a greater expected utilitarian payoff than the least partisan equilibrium of $\Gamma^{en}_{q^S,\pi^0}(\Omega')$. ■

Proof of Proposition 10. It will be convenient to denote the first period of $\Gamma^{en}_{q^S,\pi^0}(T)$ by $t = T$ and the last by $t = 1$. First, we show that for all $T \geq 1$, $\Gamma^{en}_{q^S,\pi^0}(T)$ has a unique stage undominated equilibrium, that this equilibrium is in cutoff strategies, and that the cutoffs are given by $\boldsymbol{c}^S(1) = \mathbf{0}^S$ and $\boldsymbol{c}^S(t+1) = \boldsymbol{H}\left(\boldsymbol{c}^S(t)\right)$. Let $V_k^{s,q}(t)$ denote the continuation value of $\Gamma^{en}_{q^S,\pi^0}(t)$ for player k for $q^S = (q, ..., q)$ and $\pi^0 = \pi(s, .)$ (with the convention that $V_k^{s,L}(0) = V_k^{s,R}(0) = 0$). We also show that

$$c_k^t = \frac{\delta}{2}\left(V_k^{s,L}(t-1) - V_k^{s,R}(t-1)\right). \tag{23}$$

We prove the above claims by induction. Consider first $\Gamma^{en}_{q^S,\pi^0}(1)$. Since the policy implemented in the last period $t = 1$ determines only the current payoff in that period, each player votes according to her current preferences, so $\boldsymbol{c}^S(1) = \mathbf{0}^S$, and the cutoffs are unique. Suppose now that for some $T > 0$, the game $\Gamma^{en}_{q^S,\pi^0}(T)$ has a unique cutoff-strategy equilibrium with the cutoffs determined by $\boldsymbol{c}^S(t+1) = \boldsymbol{H}\left(\boldsymbol{c}^S(t)\right)$. Consider the game $\Gamma^{en}_{q^S,\pi^0}(T+1)$. If the initial state realization in this game is s, and the first period's outcome is y^{T+1}, then the continuation game is $\Gamma^{en}_{(y^{T+1},...,y^{T+1}),\pi(s,.)}(T)$. Therefore, the equilibrium cutoffs in the last T periods of $\Gamma^{en}_{q^S,\pi^0}(T+1)$ are the same as in the game $\Gamma^{en}_{q^S,\pi^0}(T)$, and hence, by the induction hypothesis, they are unique. Stage undomination implies then that in period $T+1$ of $\Gamma^{en}_{q^S,\pi^0}(T+1)$, player $k \in \{i, j\}$ votes for R if

$$\theta_k^{T+1} + \delta V_k^{s,L}(T) > -\theta_k^{T+1} + \delta V_k^{s,R}(T),$$

and for L if the reverse inequality holds. The status quo in period T matters only when players vote for opposite alternatives. This happens when players' preferences θ_i^T and θ_j^T are on opposite sides of their respective thresholds $c_i^s(T)$ and $c_j^s(T)$. Therefore, we can expand the right-hand side of (23) for $T+1$ as follows:

$$\begin{aligned} c_k^s(T+1) &= \frac{\delta}{2}\sum_{s'\in s}\pi\left(s,s'\right)\cdot \\ &\quad \left(\int_{-\infty}^{c_i^{s'}(T)}\int_{c_j^{s'}(T)}^{\infty}\left(-\theta_k+\delta V_k^{s,L}(T-1)-\left(\theta_k+\delta V_k^{s,R}(T-1)\right)\right)f^{s'}(\boldsymbol{\theta})\,d\theta_i d\theta_j\right. \\ &\quad \left.+\int_{c_i^{s'}(T)}^{\infty}\int_{-\infty}^{c_j^{s'}(T)}\left(-\theta_k+\delta V_k^{s,L}(T-1)-\left(\theta_k+\delta V_k^{s,R}(T-1)\right)\right)f^{s'}(\boldsymbol{\theta})\,d\theta_i d\theta_j\right). \end{aligned}$$

Substituting (23) inside the integrals, we get the recursive relation $\boldsymbol{c}^s(T+1)=\boldsymbol{H}(\boldsymbol{c}^s(T))$, which completes the induction argument.

Suppose now that $\boldsymbol{H}^S\left(\mathbf{0}^S\right)(\leq,\geq)^S\mathbf{0}^S$. Since $\boldsymbol{H}^S$ is isotone for $(\leq,\geq)^S$, this implies that for all $\boldsymbol{c}^S(\leq,\geq)^S\mathbf{0}^S$, $\boldsymbol{H}\left(\boldsymbol{c}^S\right)(\leq,\geq)^S\mathbf{0}^S$. Let $\boldsymbol{\Gamma}^S$ denote the restriction of $\boldsymbol{H}^S$ on $(\mathbb{R}_-\times\mathbb{R}_+)^S$. Lemma 2 with Tarski's theorem imply that the set of fixed points of $\boldsymbol{\Gamma}^S$ is a complete lattice for the order $(\leq,\geq)^S$. Let $\hat{\boldsymbol{c}}^S$ be the least of them. One can readily check that the proof of Proposition 11 holds unchanged for $\boldsymbol{\Gamma}^S$, so the same comparative statics hold for $\hat{\boldsymbol{c}}^S$.

To complete the proof, it remains to show that $\hat{\boldsymbol{c}}^S$ is the limit of $\boldsymbol{c}^S(t)$ as $t\to\infty$. Since $\boldsymbol{c}^S(1)=\mathbf{0}^S$, it follows that $\hat{\boldsymbol{c}}^S(\leq,\geq)\,\boldsymbol{c}^S(1)$. Applying the map $\boldsymbol{H}^S$ to both sides of the inequality and using its monotonicity, we obtain that $\hat{\boldsymbol{c}}^S(\leq,\geq)\,\boldsymbol{c}^S(2)$. By assumption, $\boldsymbol{H}^S\left(\mathbf{0}^S\right)(\leq,\geq)^S\mathbf{0}^S$, so $\boldsymbol{c}^S(2)(\leq,\geq)^S\boldsymbol{c}^S(1)$. Hence, the following inequalities hold for $t=1$:

$$\hat{\boldsymbol{c}}^S(\leq,\geq)^S\boldsymbol{c}^S(t+1)(\leq,\geq)^S\boldsymbol{c}^S(t)(\leq,\geq)^S\mathbf{0}^S.$$

Suppose that it holds for some $t>1$; by applying $\boldsymbol{H}^S$, the same inequalities hold for $t+1$. Hence, by induction, we have shown that $\boldsymbol{c}^S(t)$ is increasing in t for the order $(\leq,\geq)^S$ and it is bounded above by $\hat{\boldsymbol{c}}^S$. Therefore, it has a limit, and the limit must be a fixed point of $\boldsymbol{H}^S$ no greater than $\hat{\boldsymbol{c}}^S$. Since $\hat{\boldsymbol{c}}^S$ is the least fixed point of $\boldsymbol{H}^S$, the limit must be $\hat{\boldsymbol{c}}^S$. ■

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