Normative Fiscal Policy and Growth: Some Quantitative Implications for the Chilean Economy*

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Abstract

This paper explores the qualitative and quantitative implications of optimal taxation in a developing economy when economic growth is endogenously determined. We differentiate this class of economies from a developed economy in two aspects: 1. the informal sector is quantitatively significant and, 2. tax-collecting technologies are more rudimentary. We characterize competitive equilibrium allocations and Ramsey allocations in the context of a small open economy in which the interest rate is endogenously determined, some workers can be hired in the informal market and imperfect tax-collecting technology can be heterogeneous across types of taxes.

We calibrate the parameters of our model to the Chilean economy. Overall, our results suggest that capital should still be taxed but considerably less than actual taxes (that is, 10.78% versus 18.5%). Labor should be subsidized (to stimulate accumulation of human capital) while consumption taxes should be increased by 50% approximately (from 19% to 28%). As expected, the better collecting technologies, the higher the corresponding taxes. In this context, the resulting growth rate increases only slightly along the balanced growth path.

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1 Introduction

In recent years, the determinants of the divergent paths of development across countries have deserved considerable interest. A body of research has emphasized that differences in development paths is closely related to differences in governments policies (see Jones, Manuelli and Rossi (1993)). Thus, a fundamental question in macroeconomics naturally emerges, how should fiscal policy be optimally set over the long-run?

This paper aims to add to our understanding of the connection between government policies and growth by laying out and extending recent developments within a framework which integrates tools from public finance into macroeconomics and modern growth theory. Specifically, we explore quantitative assessments of the effect of making drastic changes in the structure of fiscal policies relative to the current situation for a representative Latin American economy: Chile. We explore the effect of the switch to an optimal tax scheme on the growth rate and other key variables in a representative-agent calibrated economy.

We study optimal fiscal policy under full commitment to finance an exogenous path of public expenditures in a small open economy in the context of an endogenous growth model in which the degree of efficiency of the tax collecting technology can vary across taxes and activities. We quantify the behavior of the economy along the competitive equilibrium balanced growth path to understand how changes in taxes affect the variables in the long-run. Then, using the characterization of the competitive equilibrium, we study the design of the optimal tax policy. The model economy we propose to study these issues includes some non-standard assumptions to capture particular features of Latin American development countries. In this economy, the labor market includes a formal sector and a less productive informal sector; the technology to collect taxes is neither perfect nor symmetric and the domestic interest rate has an extra-component determined by the level of domestic debt. The solutions that we obtain are time-inconsistent, a common characteristic of this kind of models. This is not unreasonable since this is a normative analysis and these models do not aim at developing testable implications but at providing quantitative guidelines for optimal decision making by governments, which is the main purpose of this study. Once the optimal path for the fiscal variables are designed, one can be interested in defining the institutional environments that can support it.

This is, to the best of our knowledge, the first attempt to asses the quantitative implications of this sort of fiscal reforms that are a key benchmark to discuss any fiscal reform.

The rest of the paper is organized as follows. Next section reviews the literature on optimal fiscal policy. Section 3 describes our model, a small open economy with endogenous
growth, international capital mobility and two labor sectors: informal and formal. The tax collecting technology is neither perfect nor symmetric. That is, one unit taxed to some agent does not necessarily transform in one unit of fiscal income but possible less, and different taxes can have different degrees of imperfection. Section 4 formalizes the competitive equilibrium and quantifies the behavior of the economy along the balanced growth path. The evidence found in this section suggests that the introduction of an informal sector in the economy and increasing labor, capital and consumption taxes have a negative impact over the long-run growth rate. This last effect is expected since distortionary taxes should slow down the economy. Additionally, and again as expected, increasing labor taxes reduce the time devoted to work in the formal sector and increase the time allocated to work in the informal sector of the economy. However, the reduction in working time in the formal sector is larger than the increasing working time in the informal sector resulting in a decline in total time allocated to work. Increasing capital taxes reduce both the time devoted to work in the formal and informal sector. Also, time devoted to leisure increases and time devoted to human capital accumulation diminishes as capital taxes increase. Increasing consumption taxes have similar effects than increasing capital taxes. An increment in the consumption tax rate reduces not only the time devoted to work in both sectors of the economy but also the time allocated to accumulate human capital. This implies that time devoted to non-market activities, like home goods production, increases because agents avoid these increments in taxes by moving to consume much more heavily the untaxed good, leisure. In Section 5, we study the behavior of this economy along the balanced growth path when the government set taxes optimally. The goal is twofold, first to understand how the different tax collecting technologies affect optimal tax rates, growth and time allocation, and second, to measure how much the tax rates observed in the Chilean economy should change to decentralize the Ramsey allocation problem, that is, to switch to the optimal tax policy. Once this last question is answered, we move to the implications of implementing these policies on the growth rate and the allocation of time along the balanced growth path. The empirical evidence of Section 5 indicates that the tax rate changes needed to decentralize the Ramsey allocation are significant compared to the levels observed in the Chilean economy. The policy recommendations stemming from Section 6 suggest that capital, which in standard neoclassical growth models should not be taxed, has to be taxed but considerably less than actual taxes (10.78% compared to the observed 18.5%). Labor should be subsidized (to stimulate accumulation of human capital) while consumption taxes should be increased by 50% approximately (from 19% to 28%). In this context, the resulting growth rate increases only slightly along the balanced growth path despite quite significant changes in both formal and informal labor as well as time devoted
to non-market activities. Section 7 concludes the paper.

2 Literature Review

There is a vast theoretical literature that studies optimal fiscal policy in some version of the neoclassical growth model. Chamley (1986) showed that the long-run tax rate on capital should be zero. This finding was extended to an endogenous growth model by Lucas (1990) and Jones, Manuelli and Rossi (1993). The basic intuition behind this result is that a capital income tax distorts the investment decision, so, in the long run, should be replaced entirely by an income tax. This is an important result since the optimal tax structure that it describes is significantly different from what it is observed in practice. As such, the model on which it is based requires further consideration. In particular, a situation in which the zero tax will not apply is studied by Correia (1996), analyzing a small open economy and assuming that there are one or more factors of production that the government cannot tax (or cannot tax optimally). Then the tax upon capital income will be dependent on the relationship between capital and the non-taxable factors. Our setting shares this extra ingredient.

Given these theoretical results, actual tax systems are apparently far from these prescriptions. This raises the possibility that reforms in these systems can raise the rate of growth and the level of welfare. This suggests a purely quantitative question to whether justify a policy reform that considers a budget-balanced replacement of the capital tax by taxes on consumption or labor.

The first major contribution in this respect was by Lucas (1990), who analyzed an endogenous growth model with investment in human capital driving growth in a representative agent setting that eliminates distributional issues to focus entirely upon efficiency. Using data from the US economy, he measures what would have happened if the tax on capital had been set to zero in 1985 with revenue-neutrality ensured by increasing the tax on labor. With an initial capital tax rate of 36 %, the rate of growth of output per capita before the tax reduction is 1.5 %. In this setting, reducing the capital tax to zero causes a reduction in the growth rate to 1.47 %, an increase of over 30 % in the capital stock, and increases of 6% in consumption and 5.5 % in welfare. Consequently, the policy change results in a significant level effect but an insignificant growth effect. These findings can be explained as follows. Since time is the only input into the production of human capital, the cost (and return) is just the forgone wage. This leaves the human capital choice unaffected by taxation and, since

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1 A comprehensive survey of this area can be found in Chari and Kehoe (1998).
it is this that drives growth, there is no growth effect. The level effect arises simply because of the replacement of a distortionary tax by a non-distortionary one.\footnote{Whereas Lucas considers only the differences between steady states, Laitner (1995) explicitly models the transition process. Along the transition process, there has to be an accumulation of physical capital, and hence a reduction in consumption, until the permanently higher level is achieved. Taking account of this will lower the increase in welfare. The results of Laitner suggest that taking full account of the transition will reduce the welfare gain by about 40 per cent, to give a net increase in welfare of 3.3 per cent.}

The analysis of Lucas is extended by King and Rebelo (1990), who consider both an open economy and a closed economy. The model differs from Lucas’s through having physical capital as an input into the production of human capital. In addition, King and Rebelo also permit depreciation of both capital inputs. In their benchmark case, where the share of physical capital in human capital production is a third, increases in the capital tax and the labor tax from 20\% to 30\% reduce the growth rate by 1.52\% from its initial level of 1.02 to \(-0.5\). The level effect is a 62.7\% decrease in welfare. A 10\% increase in the capital tax alone reduces growth to 0.5\%. When the share of physical capital in human capital production is decreased to 0.20, 0.52\% falls to 0.11\%. In the open-economy version of the model, which is characterized by an interest rate fixed at the world level, the fall in growth is even larger: a 10\% increase in the capital tax reduces growth by 8.6\%.

Jones, Manuelli and Rossi (1993) provide the most general and ambitious quantitative exercises in a setting that combines elements from both Lucas and King and Rebelo, in particular, human capital requires time and goods to be produced. Jones et al. parameterize the utility function significantly different from Lucas.

Lucas’ intertemporal marginal rate of substitution (IMRS), is 0.5 and the elasticity of labor supply (ELS) is 0.5. In contrast, Jones et al. calibrate the ELS with the data and so, when IMRS is 0.5, the corresponding ELS is 4.99; e.g. labor supply is much more elastic, implying in turn that taxation will have a greater distortionary effect. For $\sigma = 2$, Jones et al. find that the elimination of all taxes (so distortions are completely removed) raises the growth rate from 2\% to 5\% with a welfare gain of 15\% (e.g., 1.15 is the factor by which the consumption path must be raised in order to bring utility under the current system up to the level attained in the Ramsey solution). For higher values of the IMRS, and hence greater values of ELS, the effect is even more dramatic.\footnote{The reason for this increase in growth can be seen in the response of labor supply to the tax changes.}

Summarizing these contributions, Lucas finds no growth effect but a significant level effect. In contrast, King and Rebelo and Jones et al. find very strong growth and level effects. King and Rebelo use a much lower share of human capital in its own production than Lucas and a depreciation rate of 10\%. For human capital especially, this rate would seem excessive. For
Jones et al., it is the higher degree of elasticity of labor supply that leads to the divergence with Lucas.\footnote{The role played by each ingredient to explain the divergence between the results is studied in Stokey and Rebelo (1995), who use a model that encompasses the previous three.}

In this paper we quantify the impact of implementing tax reforms that decentralize the optimal fiscal policy for the Chilean economy. The model economy we propose to study these issues, discussed in detail below, not only encompasses Jones, Manuelli and Rossi (1993) and Correia (1996) but also adds some ingredients that we consider key to study Latin America and the Caribbean (LAC) economies. This is, to the best of our knowledge, the first attempt to assess the quantitative implications of this sort of fiscal reforms that are considered the key benchmark to discuss any reform.

3 A Theoretical Framework

In this section we describe the physical setting, the asset market structure and the government. As noticed above, some non-standard assumptions are made to capture particular features of a prototype LAC economy.

3.1 Technology and Households

There is a neoclassical technology to produce a tradable consumption good in this economy that displays constant returns to scale. Tradable goods are produced using effective units of labor and tradable capital. This technology is represented by

\[
Y_t = F(K_t, L_t^F, L_t^I) = A \left( K_t \right)^{\alpha} \left( \beta \left( L_t^F \right)^{\frac{\eta - 1}{\eta}} + (1 - \beta) \left( L_t^I \right)^{\frac{\eta - 1}{\eta}} \right)^{\frac{\eta(1 - \alpha)}{\eta - 1}}
\]

where \( A \) is a technology parameter, \( \alpha \in (0, 1) \), \( \beta \in (0.5, 1) \) and \( \eta \geq 0 \). The distribution parameter \( \beta \) reflects intensity in units of effective formal labor while \( \alpha \) is the participation of capital. \( \eta \) is the elasticity of substitution between effective formal labor while \( \alpha \) is the participation of capital, \( \eta \) is the elasticity of substitution between effective informal, \( (L_t^F) \), and effective formal, \( (L_t^I) \), labor while the elasticity of substitution between capital and composite labor is 1. Note that when there is strict complementarity \( (\eta < 1) \), other things equal, a rise in \( L_t^F \) leads in equilibrium to an increase in informal labor, and conversely, when there is strict substitutability \( (\eta > 1) \), a rise in \( L_t^F \) induces a decrease in informal labor. If \( \eta = 1 \), this technology reduces the standard Cobb-Douglas production function.

Firms can hire labor either in the formal labor market or in the informal labor market. Informality translates into less productivity, \( (\beta > 0.5) \), and labor hired in the informal market
do not pay labor taxes. At date $t$, firms pay wages $w_t^F$ and $w_t^I$ per unit of effective formal and informal labor, respectively.

Let $C_t$ and $x_t$ denote private consumption and leisure at date $t$, respectively. Leisure in this model should be interpreted in a broad sense, that is, including any non-market activity like home goods production. Representative household preferences are described by time-separable, discounted utility where $\{C_t, x_t\}_{t=0}^{\infty}$ is valuated

$$
\sum_{t=0}^{\infty} \rho^t \frac{\left( C_t (x_t)^{\theta} \right)^{1-\sigma}}{1 - \sigma},
$$

where $\rho \in (0, 1)$, $\sigma > 0$ and $\theta \geq 0$. $\rho$ is the discount rate and $\sigma$ is the intertemporal elasticity of substitution. It is important to mention that, similar to Lucas (1990), if agents do not value leisure ($\theta = 0$) then taxes have no impact on growth rates.

The representative household is endowed with a unit of time every period which must be allocated across three types of activities and leisure. That is, effective units of labor are given by

$$
\begin{align*}
L_t^F &= u_t \cdot H_t \\
L_t^I &= v_t \cdot H_t
\end{align*}
$$

where $u_t$ and $v_t$ is the date–$t$ fraction of time working at the formal and informal sector, respectively and $H_t$ is the date–$t$ stock of human capital that evolves according to

$$
H_{t+1} = A^H \left( 1 - u_t - v_t - x_t \right) H_t + (1 - \delta_H) \cdot H_t
$$

where $A^H > 0$ is a human capital technology parameter and $\delta_H \in (0, 1)$ denotes human capital depreciation rate. Notice that $(1 - u_t - v_t - x_t) \cdot H_t$ is interpreted as the effective units of labor allocated in the human capital sector at date $t$.

### 3.2 Factor Mobility, Asset Market Structure and the Government

Let $G_t$ denote public consumption expenditures at date $t$. We consider a benevolent government that provides public goods, $G_t$, financed levying linear taxes on labor, capital and consumption as well as issuing debt. We assume that $G_t = gY_t$ for all $t$. That is, $g \in (0, 1)$ is the government spending to income ratio.

The government can levy a tax of $\tau_K \in [0, \bar{\tau}_K]$ on the the net return on capital, $(r_t - \delta_K) \cdot K_t$, where $r_t$ denotes the domestic rental price of capital before taxes.\(^5\) Think of $\tau_K$ as a tax

\(^5\)Following the convention in the literature we assume that return on capital after depreciation are taxed.
on corporate profits, levied on firms operating in the country. The government can also tax consumption at the rate $\tau_C$ and taxes on formal labor at the rate $\tau_w$. As stressed above, labor in the informal sector does not pay taxes.

The technology to collect taxes is neither perfect nor symmetric. The first feature means that one unit taxed to some agent does not necessarily transform in one unit of fiscal income but possible less. The second feature means that different taxes can have different degrees of imperfection. Each unit of capital taxes, labor taxes and consumption taxes transforms in $e_k$, $e_w$ and $e_c \in (0,1)$ units of fiscal proceeds, respectively.

There is a one-period bond to trade internationally at the price $q_t = 1/R_t$, where $R_t$ denotes the gross interest rate, which will be determined endogenously. The government and households have access to the credit market. Let $B^p_t$ and $B^g_t$ denote private and government asset holdings, respectively and $B_t = B^p_t + B^g_t$. The government’s budget constraint is

$$B^g_{t+1} + G_t = e_c \tau_C C_t + e_w \tau_W W_t + e_k \tau_K (r_t - \delta_K) K_t + (1 + R_t) B^g_t$$

where $\{B^g_{t+1}\}_{t=0}^{\infty}$ is further restricted by some no-Ponzi condition specified later. We denote $\pi = \{\tau_C, \tau_W, \tau_K, G_t, B^g_{t+1}\}_{t=0}^{\infty}$ as a fiscal policy.

There is no international labor mobility and physical capital is restricted as follows. Investment must be done domestically to produce new capital. Let $K_t$ denote the domestic stock of capital at date $t$, in units of consumption goods. The law of motion for capital is given by

$$K_{t+1} = I_t + (1 - \delta_K) K_t$$

where $I_t$ denotes domestic investment at date $t$ and $\delta_K \in (0,1)$ denotes the depreciation rate. Notice that agents can borrow one unit in the bond market at date $t$ to invest domestically and produce one unit of capital at $t+1$.

The domestic interest rate depends negatively on the domestic asset (debt)-capital ratio

$$R_t = R \left( \frac{B_t}{K_t} \right)$$

where $R' < 0$. Domestic agents take this rate as given; i.e., they do not internalize the impact of alternative debt choices.

The market clearing condition for the labor market reduces to

$$L^F_t = u_t H_t$$
$$L^I_t = v_t H_t$$

for all $t$. 7
4 Competitive Equilibrium Analysis

In this section we formalize the corresponding competitive equilibrium concept (Subsection 4.1) and then we quantify the behavior of the economy along the balanced growth path (Subsection 4.2). The goal of this section is twofold. First, we find useful to understand how changes in taxes affect the variables in the long-run. Secondly, the characterization of the competitive equilibrium is used to study the design of the optimal tax policy in this context by means of the primal approach discussed in Section 5.

4.1 Fiscal Policy and Competitive Equilibrium

Given a fiscal policy \( \pi = \{\tau_C, \tau_W, \tau_K, G_t, B^g_{t+1}\}_{t=0}^{\infty} \) and prices \( \{R_t, w^F_t, w^I_t, r_t\}_{t=0}^{\infty} \), the representative household solves

\[
\max_{\{C_t, u_t, v_t, K_{t+1}, H_{t+1}, B^p_{t+1}\}} \sum_{t=0}^{\infty} \beta^t \left( \frac{C_t(x_t)^{\gamma}}{1-\sigma} \right),
\]

subject to

\[
(1 + \tau_C)C_t + (K_{t+1} - K_t) + B^p_{t+1} = (1 - \tau_W) w^F_t u_t H_t + w^I_t v_t H_t + (1 - \tau_K) (r_t - \delta_K) K_t + (1 + R_t) B^p_t
\]

\[
H_{t+1} = A^H (1 - u_t - v_t - x_t) H_t + (1 - \delta_H) H_t
\]

\[
\lim_{T \to \infty} \prod_{j=0}^{T} \frac{B^p_T}{(1 + R_j)} \geq 0
\]

where \((K_0, H_0, B_0)\) are given (the last condition restricts \(\{B^g_{t+1}\}_{t=0}^{\infty}\) to rule out Ponzi schemes). Notice that consumption as well as labor and capital taxes are paid by the households. In equilibrium, it is indistinct if factor taxes are paid either by firms or workers.

Given \(\pi\), denote \(\{C_t(\pi), u_t(\pi), v_t(\pi), K_{t+1}(\pi), H_{t+1}(\pi), B^p_{t+1}(\pi)\}_{t=0}^{\infty}\) and \(\{R_t(\pi), w^F_t(\pi), w^I_t(\pi), r_t(\pi)\}_{t=0}^{\infty}\) as the corresponding solution to the representative household’s problem and prices, respectively. We say that a fiscal policy is feasible if

\[
B^g_{t+1} + G_t = e_C \tau_C C_t(\pi) + e_w \tau_W w^F_t(\pi) u_t(\pi) H_t(\pi) + e_k \tau_K (r_t(\pi) - \delta_K) K_t(\pi) + (1 + R_t(\pi)) B^p_t
\]

for all \(t\), that is, a fiscal policy is feasible if satisfies the government budget constraint. We restrict ourselves to feasible fiscal policies without any further reference.
Definition 1 A Competitive Equilibrium is an allocation \( \{C_t, x_t, u_t, v_t, K_{t+1}, H_{t+1}, B_{t+1}^p\}_{t=0}^{\infty} \) and a price system \( \{R_t, w_F^t, w_I^t, r_t\}_{t=0}^{\infty} \) such that, given a fiscal policy \( \pi = \{\tau_C, \tau_W, \tau_K, G_t, B_{t+1}^g\}_{t=0}^{\infty} \), the following conditions are satisfied

CE.1. Given \( \{R_t, w_F^t, w_I^t, r_t\}_{t=0}^{\infty} \), the allocation \( \{C_t, u_t, v_t, K_{t+1}, H_{t+1}, B_{t+1}^p\}_{t=0}^{\infty} \) solves the representative household’s problem.

CE.2. Fiscal policy \( \pi = \{\tau_C, \tau_W, \tau_K, G_t, B_{t+1}^g\}_{t=0}^{\infty} \) is feasible.

CE.3. Firms maximizes static profits; i.e., for all \( t \)

\[
\begin{align*}
    r_t &= F_1(K_t, u_t H_t, v_t H_t) \\
    w_F^t &= F_2(K_t, u_t H_t, v_t H_t) \\
    w_I^t &= F_3(K_t, u_t H_t, v_t H_t)
\end{align*}
\]

CE.3. Consistency of the domestic interest rate; i.e. for all \( t \)

\[ R_t = R(B_t/K_t) . \]

4.2 Balanced Growth Analysis: Competitive Equilibrium

We are particularly interested in studying the balanced growth path. In the Appendix we characterize a competitive equilibrium for this economy and its corresponding balanced growth path. The model displays some features that are not standard for a small open economy. First, the growth rate is endogenously determined by the fact that the interest rate depends on a measure of relative indebtedness. This friction will be critical to close the model for a developing economy like Chile. Secondly, imperfect tax collecting technologies appear only in the aggregate restrictions i.e. budget constraint of the government and the aggregate budget constraint of the open economy. Given this feature, in our exercises we fix \( e_w = e_r = e_c = 1 \) and change the tax rates.

4.2.1 Only Formal Sector

Our first exercise consists in closing the informal sector of the economy \( (\beta = 1) \) and see how \( \tau_K \) and \( \tau_w \) affect the balance growth path. In particular, next figure shows how taxes affect the economy’s growth rate.
As it is clear from Figure 1, increasing both tax rates reduce the growth rate along the BGP. Changes in the labor tax rate have a greater effect over the growth rate than changes in the capital tax rate. Increasing the labor tax rate 30% reduces growth more than 8% while the same increase in the capital tax rate produces a reduction in the growth rate of around 3%. This is expected since distortionary tax rates should slow down the economy.

Figure 2 shows how change in taxes affect the time allocation between leisure and human capital. The panel on the left in the figure shows the effect over the total time devoted to work and leisure and the right panel of the figure shows the effects over time devoted to leisure and time devoted to human capital accumulation. The figure shows that increasing the labor tax rate has a positive effect on the time devoted to leisure, as expected, and a negative effect on the time devoted to human capital accumulation. Increasing the capital tax rate has an imperceptible effect over the time devoted to accumulate human capital and has a positive effect over the time allocated to leisure. Increasing the capital tax rate by 30% induces an increment of around 6% in the time devoted to non-market activities. It is important to understand that, as modeled, leisure should be interpreted in a broad sense which must include non-market activities; that is, non-market production goods consumed by the agents.
In other words, is not that the agents devote less time accumulating human capital to stay at home doing nothing, they could be engaged in non-market activities producing goods. Still, a large increase in those activities deserves further study.

Overall, increasing both tax rates have a positive effect over the time devoted to work in the formal sector of the economy and leisure (non-market activities). Figure 3 disaggregates this last effect showing how taxes affect the time devoted to work. Increasing labor taxes diminish the time devoted to work. On the other hand, increasing 30% the capital tax rate, reduces the time devoted to work by 2.5%. So, from both figures it seems that increasing the capital tax rate has a larger effect over the time devoted to non-market activities than its effects over the time devoted to work. Increasing the labor tax rate by 30% has a positive impact over the time allocated to leisure, it increases by 17%, but a negative impact, 7%, over the time devoted to work and the time allocated to accumulating human capital.
Figure 3
4.2.2 Formal and Informal Sector

The second exercise introduces into the economy the informal sector. Here we extend our analysis to study both the impact of capital and labor taxes as well as the impact of capital and consumption taxes.

The Impact of Capital and Labor Taxes

Figure 4 shows the impact over the growth rate along the BGP. As in an economy without informal sector increasing both tax rates have a negative effect over the growth rate. Overall, the average growth rate in an economy with informal sector is lower than the average growth rate without this sector. Figure 1 and Figure 4 suggests that the introduction of an informal sector in the economy has the effect of diminishing growth. In particular if there is no change in the tax rates the introduction of the informal sector reduces the growth rate of the economy from 4.6 to 4.2%.

As next figure makes clear the time devoted to working in the formal sector of the economy declines as the labor tax rate increases, while the time devoted to work in the informal sector
increases, which are expected features of the model. However, the figure also says that the total time devoted to work decline with this tax rate, implying that the disincentive to work in the formal sector is on average larger than the positive effect over the time allocated to work in the informal sector. Increasing the capital tax rate reduces both the time devoted to work in the formal and informal sector of the economy.

Figure 5

In the same line, the next figure shows how the time devoted to human capital accumulation and leisure change with the labor and capital tax rates. An increment in the labor (capital) tax rate induces an increase in the time allocated to non-market activities and reduces the time devoted to accumulate human capital.
The Impact of Capital and Consumption Taxes

Figure 7 shows the impact of capital and consumption taxes on the growth rate. Both taxes affect negatively the long-run growth rate due to additional distortions. When both taxes are zero, the growth rate is 4.2% while it is 3.9% if both taxes are increased to 10%. Again, this is expected since distortionary taxes should slow down the economy.
Now we study the impact on time allocation. Figure 8 shows that total time devoted to work decreases with taxes. Moreover, time devoted to work in formal as well as in the informal sector decrease with both taxes.
Figure 9 below shows that not only time devoted to work decreases but also time devoted to accumulate human capital. This implies that the time allocated to consume leisure increases significantly. Observe that consumption taxes have an important negative impact on long-run growth because agents avoid this tax moving to consume much more heavily the untaxed good, non-market activities. This will decrease both labor market participation and time devoted to accumulate human capital.
Overall, the evidence found in this section suggests that the introduction of an informal sector in the economy and increasing labor, capital and consumption taxes have a negative impact over the long-run growth rate. This last effect is expected since distortionary taxes should slow down the economy. Additionally, and again as expected, increasing labor taxes reduce the time devoted to work in the formal sector and increase the time allocated to work in the informal sector of the economy. However, the reduction in working time in the formal sector is larger than the increasing working time in the informal sector resulting in a decline in total time allocated to work. Increasing capital taxes reduce both the time devoted to work in the formal and informal sector. Also, time devoted to leisure increases and time devoted to human capital accumulation diminishes as capital taxes increase. Increasing consumption taxes have similar effects than increasing capital taxes. An increment in the consumption tax rate reduces not only the time devoted to work in both sectors of the economy but also the time allocated to accumulate human capital. This implies that time devoted to non-market activities, like home goods production, increases because agents avoid these increments in taxes by moving to consume much more heavily the untaxed good, leisure.

In the next section we study the behavior of this economy along the balanced growth
path when the government set taxes optimally. The goal is twofold, first to understand how the different tax collecting technologies affect optimal tax rates, growth and time allocation, and second, to measure how much the tax rates observed in the Chilean economy should change to decentralize the Ramsey allocation problem, that is, to switch to the optimal tax policy. Once this last question is answered, we move to the implications of implementing these policies on the growth rate and the allocation of time along the balanced growth path.

5 Dynamic Optimal Taxation: The Ramsey Problem

In this section we study a dynamic optimal taxation problem called a Ramsey problem with a solution called a Ramsey plan. The government’s goal is to maximize households’ welfare subject to raising set revenues through distortionary taxation. When designing an optimal policy, the government takes into account the equilibrium reactions by consumers and firms to the tax system.

The nature of efficient taxation arises out of the tension between two principles, both of which are familiar from Ramsey’s original static analysis. One principle is that factors of production in inelastic supply - factors whose income is a pure rent - should be taxed at confiscatory rates. In the present application, for instance, if consumers’ initial capital holdings can be taxed directly via a capital levy, this eases the government constraint and reduces (or possibly eliminates entirely) the need to resort to distorting taxes. In the same way, defaulting on initial government debt and reducing promised transfer payments from government to households will reduce the need to resort to distorting taxes and improve welfare. Insofar as the government’s ability to obtain capital levies in this general sense is left unrestricted, it will increase utility to use these tax sources fully. In our analysis, we will assume away capital levy possibilities.

A second principle in Ramsey’s analysis is that goods that appear symmetrically in consumer preferences should be taxed at the same rate-taxes should be spread evenly over similar goods. In this application, this principle means that taxes should be spread evenly over consumption at different dates. Since capital taxation applied to new investment involves taxing later consumption at heavier rates than early consumption, this second principle implies that capital is a bad thing to tax. In our formulation there is but one tax rate applied to income from old and new capital alike, so these two principles cannot simultaneously be obeyed.

In order to study this taxation problem, we first formalize the so-called *primal approach*. Then, we quantify the behavior of the economy along the balanced growth path in the case
where the implicit taxes are set optimally.

5.1 The Primal Approach

Our approach builds on the primal approach to optimal taxation. [See, for example, Atkinson and Stiglitz (1980), Lucas and Stokey (1983), and Chari et al. (1991)]. This approach characterizes the set of allocations that can be implemented as a competitive equilibrium with distorting taxes by two sets of conditions: resource constraints and implementability constraints. The implementability constraint is the consumer budget constraint in which the consumer and the firm first-order conditions are used to substitute out for prices and taxes. Thus both constraints depend only on allocations. This characterization implies that optimal allocations are solutions to a programming problem. So, the basic idea is to recast the problem of choosing optimal taxes as a problem of choosing allocations subject to constraints which capture the restrictions on the type of allocations that can be supported as a competitive equilibrium for some choice of taxes.

In the Appendix we provide the details of how to solve the Ramsey problem applying the primal approach. Without loss of generality, we normalize $c_c = 1$ (that is, we assume the tax collecting technology for the consumption tax in Chile is more efficient than the other two tax collecting technologies) and denote the wedges that represent labor and capital taxes, respectively, as follows

$$
\text{taow}_t \equiv \left( \frac{F_2(t) - F_3(t)}{F_2(t)} \right) = 1 - \frac{\beta u_t}{\alpha v_t}
$$

$$
\text{taor}_t \equiv \left( \frac{F_1(t) - \delta_K - R_t}{F_1(t) - \delta_K} \right) = \left( 1 - \frac{R_t}{(1 - \alpha - \beta) A(k_t)^{-\alpha - \beta} (u_t)^\alpha (v_t)^\beta - \delta_K} \right)
$$

where $F_j(t)$ stands for the partial derivative of $F$ with respect to the $j$-th argument and $k_t = K_t/H_t$.

Let $\phi$ be the Lagrange multiplier corresponding to the incentive constraint (22) in the Appendix and define

$$
V(C_t; \phi) \equiv U(C_t, x_t) + \phi U_C(C_t, x_t) C_t
$$

where $U_C$ denotes the partial derivative of $U$ with respect to consumption.

The Ramsey problem for this economy reduces to

$$
\max_{(C_1, u_t, v_t, K_{t+1, H_{t+1}, B_{t+1}})} \sum_{t=0}^{\infty} \beta^t V(C_t; \phi)
$$
subject to
\[ U_C(C_t, x_t) = \rho U_C(C_{t+1}, x_{t+1}) \ F_3(t+1) \ [A^H(1 - x_{t+1}) + (1 - \delta_H)] \]

(4)

\[ H_{t+1} = A^H \ (1 - u_t - v_t - x_t) \ H_t + (1 - \delta_H)H_t \]

(5)

\[ C_t + (K_{t+1} - K_t) + B_{t+1} + g \ Y_t \]

(6)

\[ = \ [1 - taow_t \ (1 - e_w)] \ F_2(t) \ u_t \ H_t + F_3(t) \ v_t \ H_t \]

\[ + [1 - taor_t \ (1 - e_k)] \ (F_1(t) - \delta_K) \ K_t + (1 + R_t) \ B_t \]

The objective function (3) stems from coupling the utility function and the implementability constraint. This last constraint should be thought of as an infinite-horizon version of the budget constraint of either the consumer or the government, where the consumer and firm first-order conditions have been used to substitute out the prices and taxes. Next, restriction (4) captures the idea that the planner must allocate resources taking into account that agents choose optimally intertemporally. Restriction (6) represents the period-by-period resource constraint in a small open economy. Observe that this constraint is adapted to accommodate alternative tax collecting technologies which imply that there are additional wedges that need to be optimally manipulated by the planner. In particular, taow and taor disappear if \( e_w = e_k = 1 \); i.e. if tax collecting technologies are perfect.

In the Appendix we show how to characterize a Ramsey allocation in this setting with the non-standard features.

5.2 Balanced Growth Analysis: Ramsey Allocation

The conditions that characterize a Ramsey allocation along the balanced growth path are detailed in the Appendix. Imperfect tax collecting technologies are important features that distinguishes our setting from the literature. In particular, they make evident that the limiting capital tax (as well as labor taxes) will not be necessarily zero. To see this, notice that the steady state optimal taxes on capital and labor are given by

\[ taor^* \equiv 1 - \frac{R(b^*/k^*)}{(1 - \alpha - \beta) A(k^*)^{-(\alpha+\beta)} (u^*)^\alpha (v^*)^\beta - \delta_K} \]

\[ taow^* \equiv 1 - \frac{\beta u^*}{\alpha v^*} \]

where "*" variables denote their levels along the balanced growth path.

Notice that taor is equal to zero along the balanced growth path only when the net return on capital equals the gross interest rate. In this setting, the government might choose
to optimally distort that margin given the effects of different tax collecting technologies. On the other hand, $taow^*$ equals to zero implies that in equilibrium both sectors are treated equally; that is, there is no informal sector since informality here stems from the fact that both firms and workers avoid paying labor taxes. Again, the government might optimally choose to distort this margin as well.

Also, notice that the balanced growth rate is determined by

$$\gamma^* = A^H (1 - u^* - v^* - x^*) + (1 - \delta^H)$$

So, the higher the amount of time devoted to accumulate human capital, $(1 - u^* - v^* - x^*)$, the higher the growth rate. Importantly, the growth rate of economy does not depend on taxes if agents do not value leisure (see Lucas, 1990 for a similar result).

The tax collecting technologies parameters for Chile are calibrated to $e_K = 0.69$ and $e_W = 0.82$ where $e_C$ is normalized to 1 (see Jorrat, 2012) in order to set the tax collecting technology for consumption taxes as the more efficient of the three technologies. This calibration implies that the efficiency of the tax collecting technology for capital taxes is around the 70% of the efficiency of the tax collecting technology for consumption taxes while the efficiency of the labor tax collecting technology is around 80% of the one for consumption taxes. In what follows, we describe the results of our computed examples to analyze the behavior and predictions of the Ramsey allocation along the balanced growth path. The exercises will illustrate the impact of changing the tax collecting technologies around these levels.

Figure 11 illustrates the behavior of the optimal tax on capital as a function of the tax collecting efficiency parameters, $e_w \in (0.80, 0.84)$ and $e_k \in (0.66, 0.70)$. These parameters are critical as they introduce novel effects on the optimal choice of variables (and so on optimal taxes) that are absent in setting with perfect tax collecting technologies.

The most important observation is that the relationship is non-monotonic. One would conjecture that the higher the efficiency level of the tax collecting technology for capital, the higher the optimal capital tax. This is not always the case due to several interacting effects. Importantly, depending on $(e_w, e_k)$, optimal limiting taxes on capital can be either positive or negative. Zero capital taxes are rarely optimal.
Similarly, optimal labor and consumption taxes vary significantly and they can also be optimally set positive or negative depending on tax collecting technologies. It is important to notice that the impact on taxes is larger for $e_k$ than for $e_w$. Our intuition is that $e_k$ impacts capital accumulation directly while $e_w$ impact human capital accumulation only indirectly.
Now we discuss participation in both labor markets, that is, formal and informal, and the corresponding optimal growth rate along the balanced growth path. Again, the relationship is non-monotonic and the impact on labor market participation is larger for $e_k$ than for $e_w$. 
Growth rate behaves non-monotonically but in general better collecting technologies predict higher growth. This peculiar behavior needs further analysis but the difficulty can be grasped when one inspects the conditions characterizing the Ramsey solution (see Appendix 2). When $e_W = 1, e_K = 1$, there are several effects that are shut down. When $e_W \neq 1$ and/or $e_K \neq 1$, on the other hand, all these interacting effects become active and some further analysis is needed to identify exactly their effects.
6 Policy Implications

In this section, we provide some policy implications of our quantitative exercises. We target to provide answers to the following two questions:

1. what are the changes in the tax system needed to implement the Ramsey allocation?;
2. what are the implications of implementing these policies on the growth rate and the allocation of time along the balanced growth path?

Throughout this exercise, we keep the tax collecting technologies where the coefficients are calibrated to (see Jorrat, 2012)

\[ e_K = 0.69, \quad e_W = 0.82 \]

Table 1 summarizes our quantitative findings and make evident three facts along the balance growth path. First, we argue that the tax rate changes needed to decentralize the Ramsey allocation are significant compared to the levels observed in the Chilean economy. The optimal tax on capital along the balanced growth path is lower than the observed rate
(from 18.5% to 10.78%) while formal labor must be heavily subsidized (the labor tax rate moves from 2% to -9.2%). On the other hand, consumption must be more heavily taxed. Some features are important to mention. First, unlike some previous results, capital should be taxed in the long-run and, as a matter of fact, relatively high. Labor taxation seems a bad idea in this setting since, among other things, moves away workers from the formal to the informal sector. Finally, consumption taxes should be used more intensively since it is less harmful in terms of distortions. Also, as expected, efficiency dictates to increase tax rates from those goods with better collecting technologies and decrease the others.

<table>
<thead>
<tr>
<th>Competitive Equilibrium*</th>
<th>Competitive Equilibrium*</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \tau_K ) (%)</td>
<td>18.5</td>
</tr>
<tr>
<td>( \tau_W ) (%)</td>
<td>2</td>
</tr>
<tr>
<td>( \tau_C ) (%)</td>
<td>19</td>
</tr>
<tr>
<td>( \gamma ) (%)</td>
<td>3.73</td>
</tr>
<tr>
<td>( u ) (%)</td>
<td>33.28</td>
</tr>
<tr>
<td>( v ) (%)</td>
<td>14.55</td>
</tr>
<tr>
<td>( x ) (%)</td>
<td>7.5</td>
</tr>
<tr>
<td>( 1 - (u + v + x) ) (%)</td>
<td>44.67</td>
</tr>
<tr>
<td>Ramsey Allocation</td>
<td>Ramsey Allocation</td>
</tr>
<tr>
<td>( \tau_K ) (%)</td>
<td>10.78</td>
</tr>
<tr>
<td>( \tau_W ) (%)</td>
<td>-9.2</td>
</tr>
<tr>
<td>( \tau_C ) (%)</td>
<td>28.06</td>
</tr>
<tr>
<td>( \gamma ) (%)</td>
<td>3.82</td>
</tr>
<tr>
<td>( u ) (%)</td>
<td>2.13</td>
</tr>
<tr>
<td>( v ) (%)</td>
<td>4.55</td>
</tr>
<tr>
<td>( x ) (%)</td>
<td>47.53</td>
</tr>
<tr>
<td>( 1 - (u + v + x) ) (%)</td>
<td>45.79</td>
</tr>
</tbody>
</table>

Table 1

\(^*\) See Appendix A.3

Secondly, optimal allocation translates into a significant reallocation of time not only between formal and informal work but also leisure. However the huge reallocation is from total hours worked towards non-market activities. What is remarkable is the amount of time devoted to accumulate human capital along both balanced growth path is almost unchanged. This leads to the key aspect: we observe that, in spite of considerable tax rate changes needed, the growth rate only increases slightly, 0.09%, from 3.73 to 3.82. A similar effect was early alerted by Lucas (1990).

This may appear puzzling a priory because one could conjecture that the needed tax changes would foster both physical and human capital accumulation and then economic growth. However, the significant raise needed in the consumption tax leads to consume more untaxed leisure. This makes the time devoted to accumulate human capital (and so the growth rate along the balanced growth path) remain basically unchanged. It is important to remember that, as modeled, leisure should be interpreted in a broad sense which must include non-market activities; i.e. non-market production goods consumed by the agents. Still, the large increase in those activities deserves further study.

We consider these results key to understand from a different point of view some of the ideas behind some radical fiscal reforms and so it encourages further discussions. For instance, Anton, Hernández and Levy (2012) propose a provocative fiscal reform for Mexico to mitigate the nocive effects of informality in the labor market. Surprisingly, since our setting is not a priori targeted to match any important feature of the Mexican economy, our predictions are in line with their proposal not only qualitatively but also quantitatively in some sense.
They propose a reform that shifts taxation for social insurance from labor to consumption eliminating labor taxes and setting a uniform value added tax rate of 16 %. Our results indicates that this proposal, under some circumstances, might indeed fall short. Of course, this is simply indicative and it needs to be studied with careful details.

Marginal Tax-Collecting Technological Changes

To conclude this section, we present two exercises in which we vary only marginally the parameters of the tax collecting technologies at the time. Table 2 displays zoomed sections of Figures 7 and 8 in which $\epsilon_W$ is kept at its calibrated value, 0.82, and $b = -0.13$ while $\epsilon_K$ moves around its calibrated value 0.69. As the technology to collect capital taxes enhances (i.e. $\epsilon_K$ increases), the optimal capital tax increases, labor gets even more subsidized and consumption is substantially more taxed.

<table>
<thead>
<tr>
<th>$\epsilon_W = 0.82$</th>
<th>$\epsilon_K = ...$</th>
<th>$\tau_K$ (%)</th>
<th>$\tau_W$ (%)</th>
<th>$\tau_C$ (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.683</td>
<td>10.26</td>
<td>-8.19</td>
<td>20.63</td>
<td></td>
</tr>
<tr>
<td>0.686</td>
<td>10.53</td>
<td>-8.73</td>
<td>24.07</td>
<td></td>
</tr>
<tr>
<td><strong>0.690</strong></td>
<td><strong>10.78</strong></td>
<td><strong>-9.20</strong></td>
<td><strong>28.06</strong></td>
<td></td>
</tr>
<tr>
<td>0.692</td>
<td>10.93</td>
<td>-9.49</td>
<td>30.17</td>
<td></td>
</tr>
<tr>
<td>0.695</td>
<td>11.14</td>
<td>-9.92</td>
<td>33.32</td>
<td></td>
</tr>
</tbody>
</table>

Table 2
Similarly, Table 3 displays zoomed sections of Figures 7 and 8 in which $e_K = 0.69$ and $b = -0.13$ while $e_K$ moves marginally around its calibrated value 0.82. As the technology to collect optimal taxes gets better, the optimal capital tax decreases, labor gets less subsidized and consumption is taxed less heavily.

$$e_K = 0.69 \mid e_W = \ldots$$

<table>
<thead>
<tr>
<th>$\tau^K$ (%)</th>
<th>$\tau^W$ (%)</th>
<th>$\tau^C$ (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.790</td>
<td>11.18</td>
<td>-10.1</td>
</tr>
<tr>
<td>0.796</td>
<td>11.07</td>
<td>-9.82</td>
</tr>
<tr>
<td>0.806</td>
<td>10.98</td>
<td>-9.53</td>
</tr>
<tr>
<td>0.815</td>
<td>10.86</td>
<td>-9.34</td>
</tr>
<tr>
<td><strong>0.820</strong></td>
<td><strong>10.78</strong></td>
<td><strong>-9.20</strong></td>
</tr>
<tr>
<td>0.831</td>
<td>10.65</td>
<td>-8.93</td>
</tr>
<tr>
<td>0.839</td>
<td>10.54</td>
<td>-8.71</td>
</tr>
</tbody>
</table>

Table 3

Around the calibrated levels for $e_K$ and $e_W$, the changes in optimal taxes behave as expected. The relatively better tax collecting technology, the relatively higher the corresponding tax. Moreover, these exercises show that fiscal reforms that affect the efficiency to collect taxes would have a significant impact on all optimal taxes but it would have a major impact on consumption taxes. So the proposals for fiscal reforms to make radical changes would indeed be exacerbated.

In particular, both exercises shed light on two remarkably facts for this set of parameters. First, capital and consumption taxes are complementary; that is, changes in tax collecting technologies will make both capital and consumption taxes move in the same direction. Secondly, efficiency dictates that formal jobs should be significantly subsidized. This reflects the fact that formal workers are more productive and so the government choose optimally allocate more resources in those jobs.

7 Final Remarks

This paper has made progress characterizing competitive equilibrium allocations and Ramsey allocations in the context of a small open economy in which the interest rate is endogenously determined, some workers can be hired in the informal market and the tax collecting technologies are imperfect and heterogeneous for different taxes. That is, we ask two questions in
this setting. The first is Ramsey’s (1927) normative question: What choice of tax rates will maximize consumer utility, consistent with given government consumption and with market determination of quantities and prices? The second is positive and quantitative: How much difference does it make?

Our quantitative exercises show that from a baseline economy, the inclusion of an informal sector reduces the growth rate over the balance growth path. Increasing labor taxes produces a reduction in the time devoted to work in the formal sector of the economy and in the time devoted to accumulate human capital. Increasing capital taxes have a less pronounced effects in the same direction.

Optimal taxes stemming from the Ramsey allocation hints that capital and labor taxes are increasing in the degree of efficiency of its corresponding tax collecting technology. On the other hand, optimal consumption taxes move in the same direction as capital taxes as a response to changes in efficiency.

Summarizing, the policy recommendations stemming from our quantitative results for the parameters calibrated to Chilean economy suggest that capital, which in standard neoclassical growth models should not be taxed, has to be taxed but considerably less than actual taxes (that is, 10.78% compared to 18.5%). Labor should be subsidized (to stimulate accumulation of human capital) while consumption taxes should be increased by 50% approximately (from 19% to 28%). The resulting growth rate increases only slightly along the balanced growth path despite quite significant changes in both formal and informal labor as well as time devoted to leisure.
Appendix

In this Appendix, we first characterize a competitive equilibrium (Subsection A.1) and then the Ramsey problem (Subsection A.2).

A.1 Competitive Equilibrium

The representative household solves

\[
\max_{\{C_t,u_t,v_t,K_{t+1},H_{t+1},B_{t+1}^p\}} \sum_{t=0}^{\infty} \rho^t \frac{(C_t (x_t)^{\theta})^{1-\sigma}}{1-\sigma}, \tag{7}
\]

subject to

\[
[\lambda_t] : (1 + \tau_C) C_t + (K_{t+1} - K_t) + B_{t+1}^p
\]

\[
= (1 - \tau_W) w_t^F u_t H_t + w_t^I v_t H_t
\]

\[
+ (1 - \tau_K) (r_t - \delta_K) K_t + (1 + R_t) B_t^p
\]

\[
[\mu_t] : H_{t+1} = A^H (1 - u_t - v_t - x_t) H_t + (1 - \delta_H) H_t \tag{9}
\]

\[
\lim_{T \to \infty} \prod_{j=0}^{T} \frac{B_T^p}{(1 + R_j)} \geq 0
\]

where \((K_0, H_0, B_0)\) are given and the last condition restricts \(\{B_{t+1}^p\}_{t=0}^{\infty}\) to rule out Ponzi schemes. In brackets we wrote the associated Lagrange multipliers. The conditions characterizing a solution are

\[
(C_t) : \rho^t \left[ C_t x_t^\theta \right]^{-\sigma} x_t^\theta = (1 + \tau_C^t) \lambda_t \tag{10}
\]

\[
(x_t) : \rho^t \left[ C_t x_t^\theta \right]^{-\sigma} C_t x_t^{\theta-1} = \mu_t A^H H_t
\]

\[
(K_{t+1}) : \lambda_t = \lambda_{t+1} \left[ 1 + (r_{t+1} - \delta_K) (1 - \tau^K) \right] \tag{11}
\]

\[
(u_t) : \mu_t A^H H_t = \lambda_t (1 - \tau^w) H_t w_t^F \tag{12}
\]

\[
(v_t) : \mu_t A^H H_t = \lambda_t H_t w_t^I \tag{13}
\]

\[
(H_{t+1}) : \mu_t = \mu_{t+1} \left[ A^H (1 - u_{t+1} - v_{t+1} - x_{t+1}) + (1 - \delta_H) \right] \tag{14}
\]

\[
+ \lambda_{t+1} \left[ (1 - \tau_{t+1}^w) w_{t+1}^F u_{t+1} + w_{t+1}^I v_{t+1} \right]
\]

\[
(B_{t+1}^p) : \lambda_t = \lambda_{t+1} (1 + R_{t+1}) \tag{15}
\]
\[(TCB_{t+1}^p) : \lim_{T \to \infty} \lambda_T B_T^p = \lim_{T \to \infty} \prod_{j=0}^{T} \frac{B_T^p}{(1 + R_j)} = 0 \quad (16)\]

\[(TCK_{t+1}) : \lim_{T \to \infty} \lambda_T K_T = \lim_{T \to \infty} \prod_{j=0}^{T} \frac{K_T}{(1 + R_j)} = 0 \quad (17)\]

\[\left(\lambda_t \right) : (1 + \tau^c_t)C_t + (K_{t+1} - K_t) + B_{t+1}^p = (1 - \tau^w_t)w_t^F H_t + w_t H_t + (1 - \tau^K_t) (r_t - \delta_K) K_t + (1 + R_t) B_t^p \quad (18)\]

\[\left(\mu_t \right) : H_{t+1} = A^H (1 - u_t - v_t - x_t) H_t + (1 - \delta_H) H_t \quad (19)\]

where

\[w_t^F = F_2(K_t, u_t H_t, v_t H_t) = A (K_t)^\alpha (1 - \alpha) \left( \beta (u_t H_t)^{\frac{n-1}{n}} + (1 - \beta) (v_t H_t)^{\frac{n-1}{n}} \right)^{\frac{1}{n}} - \left( u_t H_t \right)^{-\frac{1}{n}} \]

\[w_t^I = F_2(K_t, u_t H_t, v_t H_t) = A (K_t)^\alpha (1 - \alpha) \left( \beta (u_t H_t)^{\frac{n-1}{n}} + (1 - \beta) (v_t H_t)^{\frac{n-1}{n}} \right)^{\frac{1}{n}} - \left( v_t H_t \right)^{-\frac{1}{n}} \]

Notice that from (12) and (13) we have that

\[(1 - \beta) (v_t)^{-\frac{1}{n}} = (1 - \tau^w) \beta (u_t)^{-\frac{1}{n}} \]

The agent’s and government’s budget constraints are couple together to obtain

\[(1 + \tau^c (1 - e_c)) \frac{C_t}{H_t} + \left( \frac{K_{t+1}}{K_t} - 1 \right) \frac{K_t}{H_t} + \frac{B_{t+1}}{B_t} \frac{B_t}{H_t} + g \frac{Y_t}{H_t} = (1 - \tau^w (1 - e_c)) w_t^F u_t + w_t^I v_t + (1 - \tau^K (1 - e_c)) (r_t - \delta_K) \frac{K_t}{H_t} + (1 + R_t) \frac{B_t}{H_t} \quad (20)\]

Conditions (10) - (20) characterize a competitive equilibrium.

**Balanced Growth Analysis: Competitive Equilibrium**

The balanced growth path for the economy characterized above is characterized by the following system of equations
\[ \rho (\gamma)^{-\sigma} = \left(1 + R \left( \frac{b^p + b^g}{k} \right) \right)^{-1} \]

\[ \left( \frac{\mu}{\lambda} \right) x = c \frac{(1 + \tau^c)}{A^H} \theta \]

\[ 1 = \left(1 + R \left( \frac{b^p + b^g}{k} \right) \right)^{-1} [1 + (r - \delta_K) (1 - \tau^K)] \]

\[ \left( \frac{\mu}{\lambda} \right) = \frac{A}{A^H} (1 - \tau^w) w^F \]

\[ (1 - \beta)^n u = (1 - \tau^w)^n \beta^n v \]

\[ 1 = \gamma^* \left(1 + R \left( \frac{b^p + b^g}{k} \right) \right)^{-1} \]

\[ + \left(1 + R \left( \frac{b^p + b^g}{k} \right) \right)^{-1} \frac{\lambda}{\mu} ((1 - \tau^w) w^F u + w^I v) \]

\[ \gamma^* = A^H (1 - u - v - x) + (1 - \delta_H) \]

\[ r = \alpha A k^\alpha \left( \beta \left( u \right)^{\frac{n-1}{n}} + (1 - \beta) \left( v \right)^{\frac{n-1}{n}} \right)^{\frac{n(1-\alpha)}{n-1}} \]

\[ (1 + \tau^c (1 - e_w)) c + (\gamma - 1) k + g A k^\alpha \left( \beta \left( u \right)^{\frac{n-1}{n}} + (1 - \beta) \left( v \right)^{\frac{n-1}{n}} \right)^{\frac{n(1-\alpha)}{n-1}} \]

\[ = (1 - \tau^w (1 - e_w)) w^F u + w^I v + (1 - \tau^k (1 - e_k)) (r - \delta_K) k \]

\[ + \left(1 + R \left( \frac{b^p + b^g}{k} \right) \right) - \gamma \right) (b^p + b^g) \]

\[ g A k^\alpha \left( \beta \left( u \right)^{\frac{n-1}{n}} + (1 - \beta) \left( v \right)^{\frac{n-1}{n}} \right)^{\frac{n(1-\alpha)}{n-1}} \]

\[ = e_w \tau^w c + e_w \tau^w w^F u + e_k \tau^K (r - \delta_K) k \]

\[ + \left(1 + R \left( \frac{b^p + b^g}{k} \right) \right) - \gamma \right) b^g \]

where

\[ w^F = A k^\alpha (1 - \alpha) \left( \beta \left( u \right)^{\frac{n-1}{n}} + (1 - \beta) \left( v \right)^{\frac{n-1}{n}} \right)^{\frac{n(1-\alpha)}{n-1}} \beta (u)^{-\frac{1}{n}} \]

\[ w^I = A k^\alpha (1 - \alpha) \left( \beta \left( u \right)^{\frac{n-1}{n}} + (1 - \beta) \left( v \right)^{\frac{n-1}{n}} \right)^{\frac{n(1-\alpha)}{n-1}} (1 - \beta) (v)^{-\frac{1}{n}} \]

The unknowns of this system are \( (\gamma, \frac{\mu}{\lambda}, r, k, x, u, v, c, b^p, b^g) \). Lower case letters denote the corresponding variables in terms of \( H \).
A.2 The Ramsey Problem

In order to apply the so-called \textit{primal approach}, multiply the budget constraint (18) by $\lambda_t$ and add them up to date $T$ to get

$$
\sum_{t=0}^{T} \lambda_t (1 + \tau_t^w) C_t + \sum_{t=1}^{T-1} (\lambda_{t-1} - \lambda_t [1 + (1 - \tau_t^K) (r_t - \delta_K)]) K_t + \lambda_T K_{T+1} \\
+ \sum_{t=1}^{T-1} (\lambda_{t-1} - \lambda_t (1 + R_t)) B_t^p + \lambda_T B_{T+1}^p
$$

$$
= \sum_{t=0}^{T} \lambda_t \left((1 - \tau_t^w) w_t^F u_t + w_t^I v_t\right) H_t + \lambda_0 \left[1 + (1 - \tau_0^K) (r_0 - \delta_K)\right] K_0 + \lambda_0 (1 + R_0) B_0^p
$$

Notice that

$$
\lambda_{t+1} \left((1 - \tau_{t+1}^w) w_{t+1}^F u_{t+1} + w_{t+1}^I v_{t+1}\right) H_{t+1} = \mu_{t+1} \left[A^H (1 - u_{t+1} - v_{t+1} - x_{t+1}) + (1 - \delta_H)\right] H_{t+1} - \mu_t H_{t+1}
$$

$$
= \mu_{t+1} H_{t+2} - \mu_t H_{t+1}
$$

Hence, using the conditions characterizing a CE and taking the limit as $T \to \infty$ the last expression reduces to

$$
\sum_{t=0}^{\infty} \rho^t \ U_C(C_t, x_t) C_t
$$

$$
= \frac{U_C(C_0, x_0)}{(1 + \tau_0^K)} \left[F_3 (K_0, u_0 H_0, v_0 H_0) \ (u_0 + v_0) H_0\right. \\
+ \left[1 + (1 - \tau_0^K) (r_0 - \delta_K)\right] K_0 + \left(1 + R_0\right) B_0^p\right]
$$

$$
= W_0
$$

where $U_C(C_t, x_t) = (C_t \ x_t^\sigma)^{-\sigma} x_t^\sigma$ for all $t$.

Also notice now that equation (21) reduces to

$$
U_C(C_t, x_t) = \rho U_C(C_{t+1}, x_{t+1}) w_{t+1}^F \left[A^H (1 - x_{t+1}) + (1 - \delta_H)\right]
$$

and

$$
\tau_t^w = \left(\frac{w_t^F - w_t^I}{w_t^F}\right) \\
\tau_t^K = \left(\frac{r_t - \delta_K - R_t}{r_t - \delta_K}\right)
$$
Without loss of generality, we normalize $e_c = 1$ and denote

\[
\begin{align*}
\text{taow}_t & \equiv \left( \frac{F_2(t) - F_3(t)}{F_2(t)} \right) \\
\text{taor}_t & \equiv \left( \frac{F_1(t) - \delta_K - R_t}{F_1(t) - \delta_K} \right)
\end{align*}
\]

Let $\phi$ be the Lagrange multiplier corresponding to the incentive constraint (22) and define

\[
V(C_t, x_t; \phi) \equiv U(C_t, x_t) + \phi U_C(C_t, x_t) C_t
\]

\[
= (1 + (1 - \sigma) \phi) \frac{C_t (x_t)^{\theta}}{1 - \sigma}
\]

The Ramsey problem for this economy is

\[
\max_{(C_t, u_t, v_t, K_{t+1}, H_{t+1}, B_{t+1})} \sum_{t=0}^{\infty} \rho^t V(C_t, x_t; \phi) - \phi W_0
\]

subject to

\[
U_C(C_t, x_t) = \rho U_C(C_{t+1}, x_{t+1}) w_{t+1}^f \left[ A^H (1 - x_{t+1}) + (1 - \delta_H) \right]
\]

\[
H_{t+1} = A^H (1 - u_t - v_t - x_t) H_t + (1 - \delta_H) H_t
\]

\[
C_t + (K_{t+1} - K_t) + B_{t+1} + g Y_t
\]

\[
= \left[ 1 - \text{taow}_t \left( 1 - e_w \right) \right] F_2(t) \ u_t \ H_t + \ F_3(t) \ v_t \ H_t + \left[ 1 - \text{taor}_t \left( 1 - e_k \right) \right] \ (F_1(t) - \delta_K) \ K_t
\]

\[
+ (1 + R_t) \ B_t
\]

Let us denote the corresponding (date $t$) Lagrange multipliers by $\rho^t \chi_t^1$, $\rho^t \chi_t^2$ and $\rho^t \chi_t^3$, respectively. First order conditions are given by

\[
C_t : \chi_t^3 = V_C(C_t, x_t; \phi) + U_{CC}(C_t, x_t) \left[ \chi_t^1 - \chi_{t-1} F_3(t) \right] \left[ A^H (1 - x_t) + (1 - \delta_H) \right]
\]

\[
x_t : \chi_t^2 A^H H_t
\]

\[
= V_x(C_t, x_t; \phi) + \chi_t^1 U_{C,x}(C_t, x_t)
\]

\[
- \chi_{t-1} \left[ U_{C,x}(C_t, x_t) \left( A^H (1 - x_t) + (1 - \delta_H) \right) - A^H U_C(C_t, x_t) \right]
\]

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\[ u_t : \chi_t^2 A^H H_t + \chi_{t-1}^1 \frac{\partial F_3(t)}{\partial u_t} U_C(C_t, x_t) \left[ A^H (1 - x_t) + (1 - \delta_H) \right] = \\
+ \chi_t^3 \left[ (1 - \text{taow}_t (1 - e_w)) \left( \frac{\partial F_2(t)}{\partial u_t} u_t H_t + F_2(t) H_t \right) \right. \\
\left. - \frac{\partial \text{taow}_t}{\partial u_t} (1 - e_w) F_2(t) u_t H_t \right. \\
\left. \frac{\partial F_3(t)}{\partial u_t} v_t H_t - \frac{\partial \text{taor}_t}{\partial v_t} (1 - e_c) (F_1(t) - \delta_K) K_t \right. \\
\left. + (1 - \text{taor}_t (1 - e_c)) \frac{\partial F_1(t)}{\partial u_t} K_t - g \frac{\partial Y(t)}{\partial u_t} \right] \]
\[ H_{t+1} = \chi_t^2 + \chi_t^1 \frac{\partial F_3(t+1)}{\partial H_{t+1}} \rho U_C(C_{t+1}, x_{t+1}) \left( A^H (1 - x_{t+1}) + (1 - \delta_H) \right) = \\
\rho \chi_t^2 \left( A^H (1 - u_{t+1} - v_{t+1} - x_{t+1}) + (1 - \delta_H) \right) \\
+ \rho \chi_t^3 \left[ - \frac{\partial taor_{t+1}}{\partial H_{t+1}} (1 - e_w) F_2(t + 1) u_{t+1} H_{t+1} \\
+ (1 - taor_{t+1} (1 - e_w)) \left( \frac{\partial F_2(t + 1)}{\partial H_{t+1}} u_{t+1} H_{t+1} + F_2(t + 1) u_{t+1} \right) + v_{t+1} H_{t+1} \frac{\partial F_3(t + 1)}{\partial H_{t+1}} + v_{t+1} F_3(t + 1) - \frac{\partial taor_{t+1}}{\partial H_{t+1}} (1 - e_c) \ (F_1(t + 1) - \delta_K) \ K_{t+1} \\
+ (1 - taor_{t+1} (1 - e_c)) \frac{\partial F_1(t + 1)}{\partial H_{t+1}} K_{t+1} = g \ \frac{\partial Y(t + 1)}{\partial H_{t+1}} \right] \]

\[ B_{t+1} = \chi_t^3 = \rho \chi_t^3 \left( 1 + R \left( \frac{B_{t+1}}{K_{t+1}} \right) \right) \]

\[ U_C(C_t, x_t) = \rho U_C(C_{t+1}, x_{t+1}) w_{t+1}^j \left[ A^H (1 - x_{t+1}) + (1 - \delta_H) \right] \\
H_{t+1} = A^H (1 - u_t - v_t - x_t) H_t + (1 - \delta_H) H_t \\
C_t + (K_{t+1} - K_t) + B_{t+1} + g Y_t \\
= [1 - taor_t (1 - e_w)] F_2(t) u_t H_t + F_3(t) v_t H_t + [1 - taor_t (1 - e_c)] (F_1(t) - \delta_K) \ K_t + (1 + R_t) B_t \]

**Balanced Growth Analysis: Ramsey Allocation**

Notice that

\[ U(C_t, x_t) = \frac{\left( C_t \ (x_t)^\theta \right)^{1-\sigma}}{1-\sigma} \]

\[ U_C(C_t, x_t) = (C_t)^{-\sigma} \ (x_t)^{\theta(1-\sigma)} \]

\[ U_x(C_t, x_t) = \theta \ (C_t)^{1-\sigma} \ (x_t)^{\theta(1-\sigma)-1} = \theta \frac{C_t}{x_t} U_C(C_t, x_t) \]

\[ U_{CC}(C_t, x_t) = -\sigma \ (C_t)^{-\sigma-1} \ (x_t)^{\theta(1-\sigma)} = -\sigma \ (C_t)^{-1} U_C(C_t, x_t) \]

\[ U_{C,x}(C_t, x_t) = \theta (1-\sigma) (C_t)^{-\sigma} \ (x_t)^{\theta(1-\sigma)-1} = \theta (1-\sigma) (x_t)^{-1} U_C(C_t, x_t) \]
and

\[ V(C_t, x_t; \phi) = (1 + (1 - \sigma) \phi) \left( \frac{C_t (x_t)^\theta}{1 - \sigma} \right) \]

\[ V_C(C_t, x_t; \phi) = (1 + (1 - \sigma) \phi) \ U_C(C_t, x_t) \]

\[ V_x(C_t, x_t; \phi) = (1 + (1 - \sigma) \phi) \ U_x(C_t, x_t) = \theta \frac{C_t}{x_t} U_C(C_t, x_t) \]

\[ V_{C,x}(C_t, x_t; \phi) = (1 + (1 - \sigma) \phi) \ \theta (1 - \sigma) (x_t)^{-1} U_C(C_t, x_t) \]

An close inspection of the conditions needed to characterize a Ramsey allocation delivers the following conditions for a balanced growth path of the Ramsey allocation

\[ z^* = (1 + (1 - \sigma) \phi) - \frac{1}{c^*} p^* \left( 1 - \frac{1}{\gamma^*} F_3^* \left[ A^H(1 - x^*) + (1 - \delta_H) \right] \right) \]

\[ m^* A^H = \theta c^* (x^*)^{-1} + p^* \theta (1 - \sigma) (x^*)^{-1} \]
\[ -\frac{1}{\gamma^*} p^* [\theta (1 - \sigma) (x^*)^{-1} (A^H (1 - x^*) + (1 - \delta_H)) - A^H] \]

\[ m^* A^H + \frac{1}{\gamma^*} p^* (F_3 H)^* \left[ A^H(1 - x^*) + (1 - \delta_H) \right] \]

\[ = z^* \left[ (1 - taow^* (1 - e_w)) \ ((F_{22} H)^* u^* + F_2^*) - \left( \frac{\partial taow}{\partial u} \right)^* (1 - e_w) F_2^* u^* \right. \]
\[ \left. (F_{32} H)^* v^* - \left( \frac{\partial taor}{\partial v} \right)^* (1 - e_c) (F_1^* - \delta_K) k^* \right] + (1 - taor^* (1 - e_c)) (F_{12} H)^* k^* - g F_2^* \]

\[ m^* A^H + \frac{1}{\gamma^*} p^* (F_{33} H_t)^* \left[ A^H(1 - x^*) + (1 - \delta_H) \right] \]

\[ = z^* \left[ (1 - taow^* (1 - e_w)) \ (F_{22} H)^* u^* - \left( \frac{\partial taow}{\partial v} \right)^* (1 - e_w) F_2^* u^* \right. \]
\[ \left. + (F_{33} H_t)^* v^* + F_3^* - \left( \frac{\partial taor}{\partial v} \right)^* (1 - e_c) (F_1^* - \delta_K) k^* \right] + (1 - taor^* (1 - e_c)) (F_{13} H)^* k^* - g F_3^* \]
\[ z^* + \frac{1}{\gamma^*} p^* (F_{31} H)^* \rho (A^H (1 - x^*) + (1 - \delta_H)) = \rho z^* [(1 - taow^* (1 - e_w)) (F_{21} H_{t+1})^* u^* + (F_{31} H)^* v^* - \left( \frac{\partial taor}{\partial K} K \right)^* (1 - e_c) (F_1^* - \delta_K) + (1 - taor^* (1 - e_c)) ((F_{11} K)^* + (F_1^* - \delta_K)) + 1 - g F_1^*] \]

\[(\gamma^*)^\sigma m^* + \rho \frac{1}{\gamma^*} p^* ((F_{32} H)^* u^* + (F_{33} H)^* v^*) \left( A^H (1 - x^*) + (1 - \delta_H) \right) = \rho m^* \gamma^* + \rho z^* [(1 - taow^* (1 - e_w)) \left( (F_{22} H)^* u^* + (F_{23} H)^* v^* + F_2^* u^* \right) + ((F_{32} H)^* u^* + (F_{33} H)^* v^* + F_3^* v^*) v^* - \left( \frac{\partial taor}{\partial H} H \right)^* (1 - e_c) (F_1^* - \delta_K) k^* + (1 - taor^* (1 - e_c)) ((F_{12} H)^* u^* + (F_{13} H)^* v^*) k^* - g (F_2^* u^* + F_3^* v^*)] \]

\[(\gamma^*)^\sigma = \rho \left( 1 + R \left( \frac{b^*}{k^*} \right) \right) \]

\[(\gamma^*)^\sigma = \rho F_3^* [A^H(1 - x^*) + (1 - \delta_H)] \]

\[\gamma^* = A^H (1 - u^* - v^* - x^*) + (1 - \delta_H)\]

\[c^* + (\gamma^* - 1) k^* + g y^* = [1 - taow^* (1 - e_w)] F_2^* u^* + F_3^* v^* + [1 - taor^* (1 - e_c)] (F_1^* - \delta_K) k^* + \left( 1 + R \left( \frac{b^*}{k^*} \right) - \gamma^* \right) b^* \]
where

\[ F_1^* = (1 - \alpha - \beta) A (k^*)^{-\alpha - \beta} (u^*)^\alpha (v^*)^\beta \]

\[ F_2^* = \alpha A (k^*)^{1 - \alpha - \beta} (u^*)^{\alpha - 1} (v^*)^\beta \]

\[ F_3^* = \beta A (k^*)^{1 - \alpha - \beta} (u^*)^\alpha (v^*)^{\beta - 1} \]

\[ (F_{11} K)^* = - (1 - \alpha - \beta) (\alpha + \beta) A (k^*)^{- (\alpha + \beta)} (u^*)^\alpha (v^*)^\beta \]

\[ (F_{12} H)^* = \alpha (1 - \alpha - \beta) A (k^*)^{- \alpha - \beta} (u^*)^{\alpha - 1} (v^*)^\beta \]

\[ (F_{13} H)^* = \beta (1 - \alpha - \beta) A (k^*)^{- \alpha - \beta} (u^*)^\alpha (v^*)^\beta \]

\[ (F_{22} H)^* = - \alpha (1 - \alpha) A (k^*)^{1 - \alpha - \beta} (u^*)^{\alpha - 2} (v^*)^\beta \]

\[ (F_{33} H)^* = - \beta (1 - \alpha) A (k^*)^{1 - \alpha - \beta} (u^*)^\alpha (v^*)^{\beta - 2} \]

\[ (F_{32} H)^* = \alpha \beta A (k^*)^{1 - \alpha - \beta} (u^*)^{\alpha - 1} (v^*)^{\beta - 1} \]

\[ \frac{\partial \tau_{ow}^*}{\partial u} = \frac{\beta}{\alpha v^*} \]

\[ \frac{\partial \tau_{ow}^*}{\partial v} = \frac{\beta u^*}{\alpha (v^*)^2} \]

\[ \left( \frac{\partial \tau_{ow}^*}{\partial K_t} \right) = \frac{\partial \tau_{ow}^*}{\partial H_t} = 0 \]

\[ \tau_{or}^* = \frac{(1 - \alpha - \beta) A (k^*)^{-\alpha - \beta} (u^*)^\alpha (v^*)^\beta - \delta_K - R^*}{(1 - \alpha - \beta) A (k^*)^{-\alpha - \beta} (u^*)^\alpha (v^*)^\beta - \delta_K} \]

\[ \frac{\partial \tau_{or}^*}{\partial u} = \frac{R^* \alpha (1 - \alpha - \beta) A (k^*)^{-\alpha - \beta} (u^*)^{\alpha - 1} (v^*)^\beta}{(1 - \alpha - \beta) A (k^*)^{-\alpha - \beta} (u^*)^\alpha (v^*)^\beta - \delta_K} \]

\[ \left( \frac{\partial \tau_{or}^*}{\partial v} \right) = \frac{R^* \beta (1 - \alpha - \beta) A (k^*)^{-\alpha - \beta} (u^*)^\alpha (v^*)^{\beta - 1}}{(1 - \alpha - \beta) A (k^*)^{-\alpha - \beta} (u^*)^\alpha (v^*)^\beta - \delta_K} \]

\[ \left( \frac{\partial \tau_{or}^*}{\partial K} \right) = \frac{- R^* (1 - \alpha - \beta) (\alpha + \beta) A (k^*)^{-\alpha - \beta} (u^*)^\alpha (v^*)^\beta}{(1 - \alpha - \beta) A (k^*)^{-\alpha - \beta} (u^*)^\alpha (v^*)^\beta - \delta_K} \]

\[ \left( \frac{\partial \tau_{or}^*}{\partial H} \right) = \frac{R^* (\alpha (1 - \alpha - \beta) A (k^*)^{-\alpha - \beta} (u)^\alpha (v)^\beta + \beta (1 - \alpha - \beta) A (k^*)^{-\alpha - \beta} (u^*)^\alpha (v^*)^\beta)}{(1 - \alpha - \beta) A (k^*)^{-\alpha - \beta} (u^*)^\alpha (v^*)^\beta - \delta_K} \]
and

\[ R^* = R\left(\frac{b^*}{k^*}\right) \]

\[ p^* = \frac{\chi^1}{H} \]

\[ m^* = \frac{\chi^2}{U_C} \]

\[ z^* = \frac{\chi^3}{U_C} \]

We have a system of 10 equations and 10 unknowns; namely \((p^*, m^*, z^*, c^*, \gamma^*, k^*, u^*, v^*, x^*, b^*)\). Alternatively, we calibrate \(b^*\) and include \(\phi^*\) as an unknown (i.e., the Lagrange multiplier corresponding to the implementation constraint).

### A.3 Calibration

Before presenting them we describe the parameter values that were used. The following table summarizes the calibration done for Chile:

<table>
<thead>
<tr>
<th>(A)</th>
<th>(A^H)</th>
<th>(\alpha)</th>
<th>(\beta)</th>
<th>(\delta)</th>
<th>(\delta^H)</th>
<th>(g)</th>
<th>(R^*)</th>
<th>(\phi)</th>
<th>(b)</th>
<th>(\sigma)</th>
<th>(\rho)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.00</td>
<td>0.02</td>
<td>0.02</td>
<td>0.00</td>
<td>0.02</td>
<td>0.04</td>
<td>1.12</td>
<td>0.12</td>
<td>0.04</td>
<td>1.12</td>
<td>0.04</td>
<td>1.12</td>
</tr>
</tbody>
</table>

The value of \(A\) was just a normalization. We chose \(A^H\) to match an annual growth rate of 3%. The value of \(1 - \alpha\) was chosen so as to match the share of labor income in GNP according to national accounts data. The value of \(\beta\) is standard in the literature. For \(g\) we used the average of the government spending to GDP ratio from 1960 to 2000. As for \(\delta\), it was calculated using the gross and net capital stock series presented in Perez Toledo (2003).

The variables \(\phi\) and \(\tilde{b}\) that appear in the table belong to the particular specification that was used of the function \(R(\cdot)\) that takes the form:

\[ R\left(\frac{B^p + B^g}{K}\right) = R^* + \phi \left[e^{\tilde{b}-\frac{B^p + B^g}{K}} - 1\right] \]

where \(R^*\) is the international interest rate. This is the same function that appears in Schmitt-Grohe and Uribe (2003).

The value of \(\phi\) is chosen to be the smallest value that is consistent with have closed the economy. For \(\tilde{b}\) we used the average of the Net International Investment Position to GDP ratio from 1997 to 2008. Finally, for \(R^*\) we used the average 1-year Treasury Bill rate as we needed some measure of the international interest rate the economy faced in an annual basis (we are calibrating the parameters to match the annual interpretation we give to the model’s
periods). We chose $\rho$ in order to make valid the statement $\rho (1 + R^*) = 1$ which is standard in the small open economy literature. Finally, the value of $\sigma$ was taken from Arrau (1990) and the value of $\eta$ was chosen to be one.
References


• Perez Toledo (2003), "Stock de capital de la economía chilena y su distribución sectorial", Central Bank of Chile Working Papers 233
