Simultaneous vs. Sequential Price Competition with Incomplete Information

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First version: August 2007

This version: January 2008

Abstract

We compare the equilibria that result from sequential and simultaneous moves when two firms compete à la Bertrand in a homogeneous-good market. and firms' unit costs are private information. Alternatively, our setup can be interpreted as a procurement auction with endogenous quantity where the buyer uses a first-price format if moves are simultaneous and she awards one bidder a right of first refusal if moves are sequential. We show that the first mover can be more or less aggressive in the sequential than it would be in a simultaneous game. In addition, in the case of sequential choices there is a second-mover advantage. Finally, we prove that, under some conditions, buyer and total surplus are larger when moves are simultaneous.

Keywords: oligopoly, auctions with endogenous quantity; right of first refusal; second-mover advantage.

JEL classification: C72, D43, D44

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1 Introduction

The consequences of different possible orderings of moves in strategic interaction has been the subject of extensive analysis in the literature, particularly in oligopoly games. In the case of sequential moves, the main issue has been whether first or second movers hold an advantage. In addition, once the equilibria that follow from simultaneous and sequential moves are known, the timing of the game can be made endogenous by adding a prior stage where players choose when to move. Most of these analyses have been carried out in the context of games with complete information.

Here we compare the equilibria that result from sequential and simultaneous choices in the specific case of price competition with incomplete information. Two firms compete à la Bertrand in a homogeneous-good market. Firms' (constant) unit costs are private information. In one possible case, both firms quote their prices simultaneously, so that price competition is a static game. The alternative timing generates a dynamic game: one of the firms sets its price first; its rival observes that choice and then quotes its own price. Our comparison of simultaneous and sequential equilibria can be viewed, then, as a contribution to any attempt to endogenize the timing of moves in Bertrand competition with incomplete information.

There is another possible interpretation for our setup. This form of price competition under incomplete information may be understood as a procurement auction with variable quantities. That is, the buyer announces a demand schedule and then firms compete in an auction where the exact quantity that will be procured depends on the final price according to that schedule. Simultaneous competition corresponds to the case of a first-price auction. Sequential competition will occur whenever one of the bidders has a *right of first refusal*, i.e. the right to observe her rival's bid and match it to win the auction if she desires to do so. Since rights of first refusal are quite common, for instance, in transactions among firms, examining their consequences is an interesting issue. Our analysis attempts to establish the changes in bidding behavior and the buyer's and bidders' profits induced by the introduction of such rights.

In what follows, we characterize the equilibria of the sequential and the simultaneous game and then compare the price-quoting (or bidding) behavior of both firms. In particular, we show that the fact that the rival will move second can make a firm behave more or less aggressively than it would under simultaneous competition. We also provide sufficient conditions on cost distributions and demand for the first mover to be more aggressive in the sequential case. Next, we show that there is a second-mover advantage in the sequential game. In addition, we prove that the first mover is worse off when price-quoting is sequential than when it is simultaneous.

Finally, we establish that, under certain conditions, equilibrium buyer surplus and total surplus are larger in the simultaneous game.

The case of simultaneous competition with incomplete information is not novel. Gal-Or (1986), Spulber (1995), and Lofaro (2002), for example, study oligopolistic competition with incomplete information. Spulber (1995), in particular, examines the case of static Bertrand competition, i.e. our simultaneous game. Hansen (1988) compares first- and second-price variable- quantity auctions with simultaneous bidding.¹ Sequential competition has been examined by a vast literature, most of which concentrates on establishing the existence of first- or second- mover advantages (see, for instance, Gal-Or, 1985, Dowrick, 1986, Anderson and Engers, 1992, Dastidar, 2004 and Amir and Stepanova, 2006). Most of the work in this area, however, focuses on environments with symmetric information.² The comparison between equilibria of simultaneous and sequential competition is examined in the literature on endogenous timing of firm decisions (Hamilton and Slutsky, 1990, Mailath, 1993, Amir and Grilo, 1999, Hurkens and van Damme, 1999, and Amir and Stepanova, 2006, among others). Again, this comparison is carried out in contexts with complete information. Finally, there is a small literature on the consequences of rights of first refusal in auctions (e.g. Bikhchandhani et al., 2005, Burguet and Perry, 2005, Arozamena and Weinschelbaum, 2006a and 2006b, Grosskopf and Roth, forthcoming). All these papers, however, do not allow for endogenous quantities.

Section 2 below sets up the basic model and characterizes the equilibria of simultaneous and sequential competition. In Section 3, we compare the aggressiveness of bidding behavior at the equilibria of both games. Section 4 examines the welfare and efficiency implications of moving from simultaneous to sequential competition and Section 5 concludes.

2 The model

Two risk-neutral firms compete à la Bertrand in a homogeneous-product market. Market demand is $Q(p)$, with $Q'(p) < 0$.³ The firm that quotes the lowest price sells the quantity

¹Shi (2006) characterizes the optimal auction in the same setting.

²There are a few papers where the setup includes asymmetric information or some form of uncertainty, but their environments are quite different from ours. Gal-Or (1987) considers sequential quantity competition when the first mover has private information about demand. Bagwell (1985) and Maggi (1999) study the value of commitment (i.e. of moving first) in quantity competition when the leader's choice is imperfectly observable -and, in the latter case, when the leader has private information.

³As usual, we will assume that $Q(p)$ is not "too convex," so that the second-order conditions of the corresponding monopoly profit-maximization problem are satisfied.

that the demand function specifies at that price, while its rival makes no profit. Each firm has a constant-returns production technology. Let c_i be firm i 's constant unit cost ($i = 1, 2$). c_i is firm i 's private information. Unit costs are i.i.d according to the cumulative distribution function F , with support $[\underline{c}, \bar{c}]$. We will assume that F is logconcave⁴ and its density is positive and bounded for all $c \in [\underline{c}, \bar{c}]$. As mentioned above, this setup could be interpreted as well as a variable-quantity procurement auction with independent private values, two risk-neutral bidders and a quantity schedule given by $Q(p)$. Finally, let $p^M(c)$ be the profit-maximizing price in this market for a monopolist with constant unit cost c . To ensure that no firm will ever prefer to choose a price lower than required to beat its rival, we will assume throughout that $p^M(\underline{c}) \geq \bar{c}$.

Suppose first that both firms quote prices simultaneously.⁵ A standard, static Bayesian game obtains. This is the case of simultaneous price competition under incomplete information studied in Spulber (1995) or, alternatively, the first-price variable-quantity auction examined in Hansen (1988). Let $b_i^0(c_i)$ be firm i 's bidding function in this game. Under our assumptions (see Maskin and Riley, 1984, Theorem 2), there is a unique, symmetric equilibrium in strictly increasing strategies. We characterize it in a standard way in what follows. Suppose firm j ($j \neq i$) quotes its price according to the strictly increasing bidding function $b_j^0(c_j)$, and let $\phi_j^0(b)$ be its inverse. Then, firm i 's expected profit maximization problem when its cost is c_i is

$$\max_{b_i} (b_i - c_i) Q(b_i) [1 - F(\phi_j^0(b_i))]$$

The corresponding first-order condition is

$$b_i - c_i = \frac{Q(b_i) [1 - F(\phi_j^0(b_i))]}{Q(b_i) f(\phi_j^0(b_i)) \phi_j^{0'}(b_i) - Q'(b_i) [1 - F(\phi_j^0(b_i))]}$$

Since the equilibrium is symmetric, we have $b_i^0(c) = b_j^0(c) = b^0(c)$, and $\phi_i^0(c) = \phi_j^0(c) = \phi^0(c)$. Then, the equilibrium inverse bidding function, $\phi^0(c)$, solves the differential equation

$$b - \phi^0(b) = \frac{Q(b) [1 - F(\phi^0(b))]}{Q(b) f(\phi^0(b)) \phi^{0'}(b) - Q'(b) [1 - F(\phi^0(b))]} \quad (1)$$

Unfortunately, in general there is no explicit solution to (1), so we will have to work with the differential equation defining $\phi^0(b)$ implicitly.

⁴Logconcavity of the c.d.f. function holds for most well-known distributions. For details see Bagnoli and Bergstrom (2005).

⁵Ties (which will happen with zero probability at the equilibrium) are solved randomly in this case.

Consider now the case of sequential competition. One firm (say, firm 1) quotes its price b_1 . Its rival, firm 2, observes b_1 and then chooses its own price b_2 . As mentioned in the previous section, this could happen because of price leadership in a duopolistic market or, in the case of an auction, because firm 2 was awarded a right of first refusal. To avoid technical complications, we will assume that firm 2 wins the competition if there is a tie.

The equilibrium behavior of firm 2 in the sequential game is easy to establish. Given b_1 , firm 2 has to match that bid to win. It will want to do so whenever $b_1 \geq c_2$, and will thus set $b_2 = b_1$. If $b_1 < c_2$, firm 2 will not match but rather set some price $b_2 > b_1$ so as to lose. Any strategy that generates this behavior will strictly dominate any strategy that does not.⁶

Given firm 2's behavior, firm 1 effectively competes against firm 2's cost: it has to quote a price lower than c_2 to win. Hence, given c_1 , firm 1's expected profit maximization problem is

$$\max_b (b - c_1)Q(b)[1 - F(b)]$$

The resulting first-order condition is

$$b - c_1 = \frac{Q(b)[1 - F(b)]}{Q(b)f(b) - Q'(b)[1 - F(b)]}$$

We define

$$H(b) \equiv \frac{Q(b)[1 - F(b)]}{Q(b)f(b) - Q'(b)[1 - F(b)]} = \frac{1}{\frac{f(b)}{1 - F(b)} - \frac{Q'(b)}{Q(b)}}$$

In what follows, we will assume that H is strictly decreasing.⁷ Then, the equilibrium bidding function that solves the first-order condition, which we denote by $b^1(c_1)$, is strictly increasing. Let $\phi^1(b)$ be its inverse, which is defined by

$$b - \phi^1(b) = \frac{Q(b)[1 - F(b)]}{Q(b)f(b) - Q'(b)[1 - F(b)]} \quad (2)$$

Having presented both possible timings in price competition, in the next section we compare the equilibria of both games.

3 Bidding aggressiveness

Suppose we move from simultaneous to sequential bidding. How does equilibrium bidding behavior change? In the case of firm 2, the answer is straightforward. From firm 1's perspective,

⁶Since when $b_1 < c_2$ any $b_1 > b_2$ is optimal, there isn't a strictly dominant strategy.

⁷Where convenient, we will assume as well that it is differentiable. It is easily verified that H is strictly decreasing if the hazard rate of F is not too decreasing and the demand function is not too convex.

in the sequential case firm 2 behaves as if it was bidding its own cost. Since in the simultaneous case firm 2 bids above its cost, firm 1 faces a more aggressive rival in sequential competition.

The comparison of equilibrium bidding behavior, however, is more interesting in the case of firm 1. When moving from simultaneous to sequential competition, does the fact that the rival will quote its price last make firm 1 become more or less aggressive? Is it possible that firm 1 become uniformly more aggressive (i.e. $b^0(c) > b^1(c)$ for all $c < \bar{c}$) or uniformly less aggressive (i.e. $b^0(c) < b^1(c)$ for all $c < \bar{c}$)? The following proposition provides sufficient conditions for firm 1 to be uniformly more aggressive in the sequential than in the simultaneous case.

Let $\gamma(b) = -\frac{(1-F(b))/f(b)}{Q(b)/Q'(b)}$. Then,

Proposition 1 *If $H(b)$ is convex and $\gamma(b)$ is decreasing (one of them strictly), then $b^0(c) > b^1(c)$ for all $c < \bar{c}$.*

Proof. It will be easier to present the result in terms of inverse bidding functions. That is, we have to show that $\phi^0(b) < \phi^1(b)$ for all $b < \bar{c}$. The proof proceeds in two steps. We first show that if the inverse bidding functions intersect at some bid lower than $\bar{c}$, then it has to be true that $\phi^0(b)$ crosses $\phi^1(b)$ from below. Next, we prove that, for b close enough to $\bar{c}$, it has to be the case that $\phi^0(b) < \phi^1(b)$. Then, result follows.

Step 1: If $\phi^0(\hat{b}) = \phi^1(\hat{b})$ for some $\hat{b} < \bar{c}$, then $\phi^{0'}(\hat{b}) > \phi^{1'}(\hat{b})$.

From the definitions of $H(b)$ and $\gamma(b)$, we have

$$H(b) = \frac{(1 - F(b))/f(b)}{1 + \gamma(b)}$$

Hence, for any b ,

$$\frac{H(\phi^1(b))}{H(b)} = \frac{\frac{1-F(\phi^1(b))}{f(\phi^1(b))}}{\frac{1-F(b)}{f(b)}} \left[\frac{1 + \gamma(b)}{1 + \gamma(\phi^1(b))} \right] \quad (3)$$

If, for some $\hat{b} < \bar{c}$, we have $\phi^1(\hat{b}) = \phi^0(\hat{b}) = \hat{c}$, then, from (1) and (2)

$$\phi^{0'}(\hat{b}) = \frac{\frac{1-F(\hat{c})}{f(\hat{c})}}{\frac{1-F(\hat{b})}{f(\hat{b})}} \quad (4)$$

Differentiating both sides of (2) and subtracting from (4) we have

$$\begin{aligned} \phi^{0'}(\hat{b}) - \phi^{1'}(\hat{b}) &= \frac{\frac{1-F(\hat{c})}{f(\hat{c})}}{\frac{1-F(\hat{b})}{f(\hat{b})}} - \left(1 - H'(\hat{b}) \right) = \\ &= \frac{H(\hat{c})}{H(\hat{b})} \left[\frac{1 + \gamma(\hat{c})}{1 + \gamma(\hat{b})} \right] - 1 + H'(\hat{b}) \end{aligned}$$

where the last equality follows from (3). Since $\gamma(b)$ is decreasing and $\widehat{c} < \widehat{b}$,

$$\begin{aligned}\phi^{0'}(\widehat{b}) - \phi^{1'}(\widehat{b}) &\geq \frac{H(\widehat{c})}{H(\widehat{b})} - 1 + H'(\widehat{b}) = \frac{H(\widehat{c}) - H(\widehat{b})}{H(\widehat{b})} + H'(\widehat{b}) \\ &= \frac{H(\widehat{c}) - H(\widehat{b})}{\widehat{b} - \widehat{c}} + H'(\widehat{b})\end{aligned}$$

where the inequality is strict if $\gamma(b)$ is strictly decreasing and the last equality follows from (2). Note that the first term in the last expression is positive, while the second is negative. If $H(b)$ is convex (strictly convex), then

$$\frac{H(\widehat{c}) - H(\widehat{b})}{\widehat{c} - \widehat{b}} < (\leq) H'(\widehat{b})$$

Hence, $\frac{H(\widehat{c}) - H(\widehat{b})}{\widehat{b} - \widehat{c}} + H'(\widehat{b}) > 0$.

Step 2: If $b \in (\bar{c} - \delta, \bar{c})$ for δ small enough, $\phi^0(b) < \phi^1(b)$.

If $\phi^0(\widetilde{b}) = \phi^1(\widetilde{b})$ for some $\widetilde{b}$ in that interval, we know from the first step that for b slightly above $\widetilde{b}$ it has to be the case that $\phi^0(b) > \phi^1(b)$. Then, we focus on showing that this last inequality leads to a contradiction

Suppose then that $\phi^0(b) > \phi^1(b)$ for some $b \in (\bar{c} - \delta, \bar{c})$. Since $\phi^0(\bar{c}) = \phi^1(\bar{c}) = \bar{c}$, it has to be true that, for some b^* close to $\bar{c}$, $\phi^{1'}(b^*) > \phi^{0'}(b^*)$ and $\phi^0(b^*) > \phi^1(b^*)$. Then, from (2), and the fact that $H(b)$ is convex,

$$\phi^{1'}(b^*) = 1 - H'(b^*) \leq 1 - \frac{H(b^*) - H(\phi^0(b^*))}{b^* - \phi^0(b^*)} = 1 - \frac{b^* - \phi^1(b^*)}{b^* - \phi^0(b^*)} + \frac{H(\phi^0(b^*))}{b^* - \phi^0(b^*)} < \frac{H(\phi^0(b^*))}{b^* - \phi^0(b^*)}$$

But substituting from (1) in the last expression, we obtain

$$\phi^{1'}(b^*) < \frac{\phi^{0'}(b^*) - \frac{(1-F(\phi^0(b^*))) / f(\phi^0(b^*))}{Q(b)/Q'(b)}}{1 + \gamma(\phi^0(b^*))} \quad (5)$$

In addition, since $\phi^0(b^*) > \phi^1(b^*)$, following the same reasoning as in the first step, it has to be true that

$$\phi^{0'}(b^*) > \frac{\frac{1-F(\phi^0(b^*))}{f(\phi^0(b^*))}}{\frac{1-F(b^*)}{f(b^*)}}$$

Therefore,

$$\frac{\phi^{0'}(b^*) - \frac{(1-F(\phi^0(b^*))) / f(\phi^0(b^*))}{Q(b^*)/Q'(b^*)}}{1 + \gamma(\phi^0(b^*))} < \frac{\phi^{0'}(b^*) - \frac{(1-F(b^*)) / f(b^*)}{Q(b^*)/Q'(b^*)} \phi^{0'}(b^*)}{1 + \gamma(\phi^0(b^*))} = \phi^{0'}(b^*) \frac{1 + \gamma(b^*)}{1 + \gamma(\phi^0(b^*))}$$

Given that $\gamma(b)$ is decreasing, it follows that

$$\phi^{0'}(b^*) \frac{1 + \gamma(b^*)}{1 + \gamma(\phi^0(b^*))} \leq \phi^{0'}(b^*)$$

Then, $\phi^{1'}(b^*) < \phi^{0'}(b^*)$. We conclude that $\phi^0(b^*) > \phi^1(b^*)$ and $\phi^{1'}(b^*) > \phi^{0'}(b^*)$ cannot both hold and a contradiction obtains. ■

Remark 1 *We could use an analogous proof to show that if $H(b)$ is concave and $\gamma(b)$ is increasing (one of them strictly), then firm 1 is uniformly less aggressive in sequential competition than it is in simultaneous competition. Similarly, we could show that if $H(b)$ is linear and $\gamma(b)$ is constant, firm 1's bidding behavior is unaltered by the timing of price quotes. Note, however, that $\gamma(b)$ is positive for all $b < \bar{c}$ and $\gamma(\bar{c}) = 0$.⁸ Then, $\gamma(b)$ has to be decreasing in some subinterval of $[\underline{c}, \bar{c}]$.*

The conditions mentioned in Proposition 1 can be satisfied, as Example 1 shows. Then, it is possible that the first mover in sequential competition is uniformly more aggressive than it would be in simultaneous competition.

Example 1 *If $F(c) = 2c^2$ with support $[0, 1/\sqrt{2}]$ and $Q(p) = 3 - p^2$, it can be easily checked that $H(b)$ is strictly convex and $\gamma(b)$ is strictly decreasing.*

It could also be the case, however, that firm 1 is uniformly *less* aggressive in sequential than in simultaneous bidding.

Example 2 *Hansen (1988) proved that in the case of simultaneous competition, the symmetric equilibrium bidding function is more aggressive in a variable-quantity first-price auction than the analogous bidding function in a fixed-quantity first-price auction. Let $\phi^F(b)$ be the equilibrium inverse bidding function in the first-price auction with a fixed quantity. Then, $\phi^F(b) < \phi^0(b)$ for all $b < \bar{c}$.*

Suppose $F(c) = 1 - \frac{e^{-1}}{1-e^{-1}}(e^{1-c} - 1)$ with support $[0, 1]$ and $Q(p) = 2 - p$. Figure 1 depicts $\phi^F(b)$ (the continuous line) and $\phi^1(b)$ (the broken line). Since $\phi^1(b) < \phi^F(b)$ for all $b < \bar{c}$, it follows that $\phi^1(b) < \phi^0(b)$ for all $b < \bar{c}$.

⁸Notice as well that $-Q(b)/Q'(b)$ has to be bounded below for $p^M(c)$ to be well defined. Then, it has to be true that $\gamma(\bar{c}) = 0$.

Figure 1

4 Welfare and efficiency

We shift our attention now to the firms' expected profits, buyer surplus and efficiency at the equilibria of the simultaneous and the sequential case. There are at least two interesting questions to pose. The first question focuses exclusively on the case of sequential competition, and follows the lines proposed by the literature on first- vs. second-mover advantages mentioned in the introduction. It sounds intuitively plausible that the second mover fares better than the first mover. We would like to ascertain whether that is true in our model or not. A second issue links sequential with simultaneous competition. It would be interesting to know how expected seller profits, buyer surplus and efficiency compare between the equilibria corresponding to both timings. We deal with these two questions in what follows.

Is there a second-mover advantage in the sequential game? The next proposition provides a positive answer.

Proposition 2 *Let $U_i^1(c)$ be firm i 's interim expected profit in the sequential game, $i = 1, 2$. Then, $U_2^1(c) > U_1^1(c)$ for all $c < \bar{c}$.*

Proof. For any c ,

$$U_2^1(c) = \int_{\phi^1(c)}^{\bar{c}} [b^1(s) - c]Q(b^1(s))f(s)ds$$

Then, since $\phi^1(c) < c$ for any $c < \bar{c}$,

$$U_2^1(c) > \int_c^{\bar{c}} [b^1(s) - c]Q(b^1(s))f(s)ds > [b^1(c) - c]Q(b^1(c))[1 - F(c)]$$

where the last inequality follows from the fact that $(b - c)Q(b)$ is increasing in b and $b^1(c)$ is increasing. Since

$$U_1^1(c) = [b^1(c) - c]Q(b^1(c))[1 - F(b^1(c))]$$

the result follows. ■

The intuition for this result is the following. There are two effects. First, firm 2 has the opportunity to see firm 1's bid. Then, it will win whenever it is profitable and it will never be more aggressive than necessary to do so. The second effect is that firm 1 effectively faces a rival that bids its own cost, while firm 2 has to beat firm 1's bid, which is higher than her cost. Both effects improve the second mover's position relative to the first mover's.

Let us now compare the equilibria of the simultaneous and the sequential game in terms of buyer and bidder welfare and efficiency. This comparison could be relevant for any model that tried to endogenize the game's timing, as stated above. Let $U_i^0(c)$ be firm i 's interim expected utility when its cost is c in the case of simultaneous competition.

As for the first mover, it is straightforward that it is $U_1^1(c) < U_1^0(c)$ for all $c < \bar{c}$. Indeed, the situation firm 1 faces in the sequential case is the same it would face in the simultaneous case if firm 2 bid its own cost, as mentioned above. Since at the equilibrium of the simultaneous game firm 2 bids above its cost, it has to be true that firm 1 is better off.

The comparison, however, is not as clear in the case of the second mover. If it is the case that firm 1 is not more aggressive in the sequential game than it is in the simultaneous game for any cost level (i.e. if $b^1(c) \geq b^0(c)$ for all c), then $U_2^1(c) > U_2^0(c)$ follows: not only does firm 2 hold the advantage of moving second, but it also faces a less aggressive rival. In other words, for every cost realization (c_1, c_2) , (i) if firm 2 wins in the simultaneous game, it wins as well in the sequential game, and it does so at a higher price; and (ii) it is possible that firm 2 loses in the simultaneous game and wins in the sequential game. Still, as we have shown, there is a whole class of cases where $b^1(c) < b^0(c)$ for all $c < \bar{c}$. Hence, we cannot rule out the possibility that firm 1 becomes so much more aggressive in the sequential game that firm 2 ends up being worse off.

Comparing expected buyer surplus and efficiency between the two games presents similar complications. Note that both equilibria lead to market inefficiency: for all cost pairs (c_1, c_2) , the final price is higher than $\min\{c_1, c_2\}$. But finding out which game generates a higher buyer

surplus or more efficiency hinges upon how prices compare in both cases. Just as above, if $b^1(c) \geq b^0(c)$ for all c it has to be the case that, for all cost pairs, the corresponding price is higher in the sequential game. Then, expected buyer surplus is lower when competition is sequential, and simultaneous competition is more efficient. But it may be the case that $b^1(c) < b^0(c)$ for all $c < \bar{c}$, so that, for some cost pairs, the corresponding price is lower in the sequential game than in the simultaneous one. Then, we cannot make a general assertion. Proposition 3 shows that if $H(b)$ is convex, however, prices are stochastically lower with simultaneous price competition. Note that the class of cases the proposition refers to includes all those that satisfy the sufficient conditions specified in Proposition 1.

Proposition 3 *If $H(b)$ is convex, then expected buyer surplus and expected total surplus are higher in the simultaneous than in the sequential game.*

Proof. Hansen (1988) showed that, under our assumptions, both expected buyer surplus and expected total surplus are lower in a second-price than in a first-price auction. Then, it suffices to show that both surpluses are lower in our sequential game than in a second-price auction. To do so, let $G^{SPA}(b)$ be the cumulative distribution function of the equilibrium price in the case of a variable-quantity, second-price auction, and let $G^1(b)$ be its analog in our sequential game. We will show that $G^{SPA}(b) > G^1(b)$ for all $c < \bar{c}$. Given this first-order stochastic dominance relation, and the fact that buyer and total surplus are decreasing in price, the result follows.

Since in a second-price auction each firm bids its own cost, $G^{SPA}(b) = (F(b))^2$. In addition, $G^1(b) = F(\phi^1(b))$. For all $b \in (\underline{c}, b^1(0))$, we have $G^{SPA}(b) > G^1(b) = 0$. Clearly, then, if both distributions cross at some $\hat{b} < \bar{c}$, it has to be the case that $G^{SPA}(\hat{b}) < G^1(\hat{b})$. But, since $G^{SPA}(\bar{c}) = G^1(\bar{c}) = 1$, this implies that, for some $\tilde{b} \in (\hat{b}, \bar{c})$, we must have $G^{SPA}(\tilde{b}) = G^1(\tilde{b})$ and $G^{SPA}(\tilde{b}) < G^1(\tilde{b})$. Hence,

$$\frac{G^1(\tilde{b})}{G^1(\tilde{b})} < \frac{G^{SPA}(\tilde{b})}{G^{SPA}(\tilde{b})}$$

or

$$\frac{f'(\phi^1(\tilde{b}))\phi^1(\tilde{b})}{F(\phi^1(\tilde{b}))} < \frac{2f(\tilde{b})}{F(\tilde{b})}$$

As $F(c)$ is logconcave, $f(b)/F(b)$ is decreasing. Then, the last inequality can only hold if $\phi^1(\tilde{b}) < 2$. However, from (2), $\phi^1(b) = 1 - H'(b)$. It can be easily checked that $H'(\bar{c}) = -1$. Given that $H(b)$ is convex, $\phi^1(\tilde{b}) \geq 2$, a contradiction. ■

In the case of the second-price auction, both in the simultaneous and the sequential game, each firm has a weakly dominant strategy: bidding its own cost. Then, the proof of Proposition

3 also shows that, in the sequential case, when $H(b)$ is convex the first-price auction generates a lower expected buyer and total surplus. This ranking is the opposite of the one obtained by Hansen (1988) for the simultaneous case.

5 Conclusion

We have examined a simple model with two possible interpretations: (i) Bertrand competition when firms' costs are private information, and (ii) first-price procurement auctions with endogenous quantity. By comparing the equilibria that follow from simultaneous and sequential price-quoting, we conclude that moving first may lead a firm to bid more or less aggressively than it would in a simultaneous game. The second mover, however, holds an advantage. Moreover, shifting from simultaneous to sequential competition has, under some conditions, negative consequences for buyer surplus and efficiency.

If we take the oligopoly interpretation of our games, and along the lines of the literature that examines the endogenous timing of moves in symmetric-information cases, it would then be interesting to add a first stage where firms strategically determine the order of moves when there is incomplete information. This, however, remains to be done.

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