Inside-Outside Money Competition[□]

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Abstract

We study how competition from privately supplied currency substitutes affects monetary policy. We focus on regimes where monetary policy must be sequentially optimal. We obtain that the workings of competition between currency and currency substitutes depend critically on government objectives. However, the impact inside money competition has on equilibrium outcomes does not. In fact, we show that, in general, it enhances efficiency and reduces equilibrium inflation rates. Nevertheless, if the supply of “inside monies” is very efficient, then the equilibrium with currency may not be sustainable and only “inside money” will be held in equilibrium.

JEL classification: E40; E50; E58; E60

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1. Introduction

Payment systems have gone through a major transformation in the last decade. In particular, electronic payments have risen in most developed countries and are expected to rise even more in the future.¹ The development of the technology to transfer information electronically has increased substitutability between deposits and currency in transactions and has substantially reduced the cost of transactions using deposits. Deposits used for electronic payments are highly substitutable for currency because the settling value is ultimately a liability of the financial institution that issues the payment (checks, in contrast, are a liability of the purchaser). Clearing electronic transactions is also considerably less expensive than clearing checks (1/3 to 1/2 of the cost of checks, according to Humphrey et al, 1996). In contrast to currency, deposits used for electronic transactions can pay nominal interest on the average balance at a very low cost. The technological developments in electronic transactions are blurring the distinction between different components of M1 (and among components of broader monetary aggregates), fueling competition between currency and deposits used for electronic payments,² that is, between outside and inside money.

In spite of the technological developments leading to a widespread use of different forms of electronic money, there has been little theoretical attention on a range of issues raised by this. How is monetary policy affected by the increased competition from inside money? Does the increased efficiency in the supply of inside money induce lower inflation rates? Can the increasing efficiency in the supply of inside money result in a cash-less economy? In other words, will inside money drive outside money out? In this paper we investigate the theoretical issues associated with competition between currency and currency substitutes.

At first glance, competition between suppliers of currency substitutes and between them and the monopolist supplier of currency should induce lower costs for the use of money and, therefore, lower inflation rates as well as nominal interest rates. However, one reason for high inflation rates, emphasized in the literature, is the time inconsistency of monetary policy; and it is unclear whether exposing monetary authorities to competition will discipline them or whether the time consistency problem will get even worse. In other words, the role of competi-

¹For example, Humphrey et al. (1996) find that "in all (fourteen) developed countries but the United States, electronic payments have been either the sole or the primary reason for the 34 percent rise in total non-cash payments between 1987 and 1993" (p. 935).

²We call these deposits used for electronic payments, electronic money, which are privately issued currency substitutes, a form of inside money.

tion cannot be analyzed independently of the commitment problem. The aim of this paper is to study these issues in the context of a dynamic monetary general equilibrium model where currency is the unique outside money (acting as unit of account, while in circulation) and where competitively supplied inside monies are perfect substitutes for currency. To this end, we consider various alternative monetary regimes. These regimes differ in two dimensions regarding the monetary authority: the objective function (whether the aim is to maximize revenues or welfare) and the ability to commit to a stated policy (whether there is full commitment or policies are sequentially redesigned).

We start the analysis in Section 2 by describing the competitive equilibria for given monetary policies. In Section 3, we show that a government that maximizes revenues (or transfers) and is able to commit to its policies will be induced by competition from currency substitutes to set a low stationary inflation rate. Furthermore, inflation is driven down with the reduction of financial intermediation costs. Since this low inflation equilibrium is time inconsistent we analyze, in Section 4, how competition from currency substitutes affects the set of sustainable equilibria. We show that policies with stationary deflation are not sustainable and that competition from currency substitutes imposes an upper bound on inflation in the set of sustainable equilibria with valued currency. If this upper bound is negative, currency has zero value in the unique sustainable equilibrium.

With a “representative” government, which acts as a Ramsey planner under full commitment, the Friedman rule is the policy chosen by the central bank. Therefore, with commitment, there is no disciplinary role for competition from currency substitutes (Section 5:1). This Ramsey solution is time-inconsistent and only the government’s concern with maintaining its reputation may prevent a deviation from the Friedman rule. However, the presence of competition from currency substitutes weakens the disciplinary role of “reputation” because it raises the value of the worst sustainable equilibrium; the “punishment” is less severe when households can still use inside money. Nevertheless, we show that a benevolent government might be deterred from deviating and might implement the Friedman rule in spite of the lack of commitment and the presence of competition from currency substitutes (see Section 5:2). Also in this context, there can be multiple sustainable equilibria with valued currency, possibly with high inflation rates. But, with competition from currency substitutes and low intermediation costs, high inflation equilibria do not exist. It is in this sense that, even in a “representative” government regime, competition from inside money may enhance efficiency.

In our model we abstract from some aspects that distinguish different “currency substitutes”. Deposits can be used for transactions through electronic payments in many different ways. Our model allows for the use of deposits for transactions through an arrangement that most resembles debit cards, which allow buyers to make purchases directly using funds from some form of deposit account. Reloadable cash cards would also fit our description of currency substitutes, but these cards are not widely used and do not typically pay interest, although they could.

In our set up, currency substitutes are assumed to be fully backed, default free deposits that are perfect substitutes for currency. The suppliers of these deposits are price takers. Relaxing these assumptions can significantly alter the results. If the suppliers of currency substitutes can default on nominal contracts, then they will, unless the resulting loss of future profits prevents them from doing so. If the suppliers of inside monies were not price takers, then they would have an incentive to overissue in order to devalue outstanding balances. In Marimon, Nicolini and Teles (1999b), we analyze a monetary arrangement of this type as an example of how the reputation and the competition mechanisms interact.

This alternative environment is also analyzed in Klein (1974) with “...one dominant money (currency supplied by a government monopoly) and many privately produced nondominant monies (deposits supplied by different commercial banks).” Klein asked most of the questions that we address in this paper. In particular, he made it clear that competition between the private issuers of currency substitutes and between them and the monopolist issuer of currency did not dismiss and could even raise intertemporal consistency problems, a point we make in section 5. However, he did not provide a full characterization of the equilibrium set as we do in this paper. To our knowledge, Taub (1985) has been the only previous attempt to study “currency competition” taking into account reputational aspects using a dynamic general equilibrium framework³.

2. Competitive monetary equilibria

In this section we describe the economic environment and characterize its monetary equilibria. The economy is populated by a large number of identical infinitely lived households, financial intermediaries, and a government. The households have

³In contrast with this paper, Taub (1985) only considers time-consistent stationary policies and therefore can not provide a full characterization of equilibria with reputation and competition.

preferences defined over consumption of a cash good, c_t^1 , consumption of a credit good, c_t^2 , and total labor, n_t , for any period $t \geq 0$.

$$V = \sum_{t=0}^{\infty} \beta^t [u(c_t^1) + u(c_t^2) + \theta n_t] \quad (2.1)$$

Assuming that leisure enters linearly into the utility function is in no way essential, but significantly simplifies the analysis. The function u is increasing, strictly concave, and differentiable.

The households are endowed with a unit of time that can be used for leisure or labor, n_t , and which can be allocated to the production of the consumption good or the production of deposits, n_t^e . The production technology of the consumption good is linear with unitary coefficients. Thus, feasibility requires that

$$c_t^1 + c_t^2 = n_t + n_t^e$$

We use the timing of transactions as found in Svensson (1985). The goods market meets in the beginning of the period. The cash good, c_t^1 , must be purchased using either currency, M_t , or privately issued currency substitutes, E_t , which has been carried over from the previous period. Currency substitutes are deposits that the buyers can easily access, for example using electronic cards. The credit good, wages, and government transfers are paid at the end of the period in the assets market where the households can adjust their portfolios of currency, deposits, and real bonds. In principle, currency and currency substitutes do not have to be traded at par. In other words, there are two nominal units of account corresponding to the two payment systems. Goods are priced in units of these two assets; that is, P_t^m is the price of goods in currency and P_t^e the corresponding price in privately issued currency substitutes. The corresponding exchange rate in period t is $\pi_t = P_t^m / P_t^e$, which denotes the price of deposits in units of currency. We focus our attention on three types of equilibria. In the first type of equilibria, both currency and deposits are valued, meaning that if the supply was positive and finite, the price would be finite. Because of perfect substitutability the exchange rate is indeterminate. We look at equilibria where the two monies trade at par, $\pi_t = 1$, and $E_t = 0$, $t \geq 0$. In the second type of equilibria, currency is never valued, so that if $M_t > 0$, then $\pi_t = 1$. Only deposits are used for transactions. Finally, we also consider a third type of equilibria where deposits are not valued. In this case, if $E_t > 0$, then $\pi_t = 0$. We use P_t to denote the price of goods in the relevant unit of account. Thus, when currency is valued, including the case

in which deposits are also valued, $\pi_t = 1$; $t \geq 0$, then $P_t = P_t^m$, and when only inside money circulates, then $P_t = P_t^e$.⁴

The representative household is endowed with initial holdings of money M_0 and of real bonds valued at $R_0 b_0^h$, as well as initial deposits that are assumed to be zero, $I_0^f E_0 = 0$. The household chooses sequences of consumptions and labor $c_t^1; c_t^2; n_t$ and portfolios $M_{t+1}; b_{t+1}^h; E_{t+1}$, $t \geq 0$, treating parametrically prices and interest rates, $P_t; \pi_t; R_{t+1}; I_{t+1}^f$, government transfers $fg_t g_{t=0}^1$, and dividends from financial intermediaries. I_{t+1}^f is the gross interest on deposits held from period t to $t+1$ in units of deposits. If both outside and inside money are valued forever, i.e., $\pi_t = 1$; $t \geq 0$, the household intertemporal budget and cash-in-advance constraints for $t \geq 0$ are

$$M_{t+1} + P_t b_{t+1}^h + E_{t+1} = M_t + P_t R_{t+1} b_t^h + I_{t+1}^f E_t - P_t(c_t^1 + c_t^2) + P_t n_t + P_t g_t + I_t \quad (2.2)$$

$$P_t c_t^1 \leq M_t + E_t \quad (2.3)$$

and a no-Ponzi scheme condition guarantees that the present value budget is satisfied⁵. Notice that there is no nominal interest paid on currency, while deposits E_{t+1} are remunerated by financial intermediaries at the gross nominal interest rate, I_{t+1}^f . The nominal interest rate on bonds is given by $I_{t+1} = \frac{P_{t+1}}{P_t} R_{t+1}$:

Since currency does not pay nominal interest, in the equilibria where both currency and deposits are valued, and $\pi_t = 1$; $t \geq 0$, it must be that $I_{t+1}^f = 1$, $t \geq 0$. Then an equilibrium allocation must satisfy, for $t \geq 0$:

$$\frac{u^0(c_{t+1}^1)}{P_{t+1}} = I_{t+1}^f, \quad (2.4)$$

$$\frac{u^0(c_t^2)}{P_t} = 1; \quad (2.5)$$

$$R_{t+1} = \pi_{t+1} \pi_t^{-1} \quad (2.6)$$

⁴In our environment with a dominant money (currency) and privately issued deposits, it would be natural to assume a one-sided convertibility legal requirement in that deposits are convertible on demand into currency at a one-to-one fixed exchange rate. This convertibility requirement implies that, in equilibrium, $\pi_t \geq 1$. If $\pi_t < 1$; depositors would exercise their option to convert their deposits at par value.

⁵If inside money is not valued, i.e., $\pi_t = 0$; $t \geq 0$; E_n must be replaced by $\pi_n E_n$ in the constraints and, similarly, if currency is not valued, i.e., $\pi_t = 1$; $t \geq 0$; M_n must be replaced by $M_n \pi_n$:

However, in an equilibrium where currency is never valued and where only deposits are used for transactions, the equation (2.4) is replaced with

$$\frac{u^0(c_{t+1}^1)}{R} = 1 + I_{t+1} - I_{t+1}^f \quad (2.7)$$

meaning that the cost of holding inside money is the difference between the return on bonds and the return on deposits. To simplify notation, from now on we use the fact that in equilibrium the real rate of return on bonds always satisfies (2.6).

Private issuers of inside money The financial intermediation sector is competitive. A representative issuer of inside money offers deposits E_{t+1} at a gross interest rate I_{t+1}^f . We assume that their contracts are enforceable, possibly through banking regulations. Financial intermediation technology is such that they must pay a real cost—in units of labor—for the supply of deposits—at redemption time—as a fraction of the real value of the outstanding deposits: $n_t^e = \mu \frac{E_t}{P_t}$. The financial intermediary must hold the total amount deposited, E_{t+1} , as bonds, $P_t b_{t+1}^e$, which pay gross interest I_{t+1} . The cash flow of the financial intermediary in period $t \geq 0$ is

$$\pi_t = E_{t+1} - E_t I_t^f - P_t b_{t+1}^e + P_t b_t^{e-1} - P_t n_t^e$$

Free entry in the financial intermediation sector results in $\pi_t = 0$; which, given that $E_{t+1} = P_t b_{t+1}^e$ and $P_t n_t^e = E_t \mu$; implies

$$I_{t+1} - I_{t+1}^f = \mu, \quad t \geq 0 \quad (2.8)$$

Recall that we assume that financial intermediaries are price takers and honor their liabilities. If they were not price takers, then it would be optimal to surprise the households and overissue. This overissuing would have an impact on the price level and would reduce the value of the nominal liabilities of the financial intermediary⁶. Similarly, if the deposit contracts could not be enforced then they would have an incentive, in any given period, to default on deposits.⁷

Government Given $(M_0; R_0 d_0)$; a government policy consists of a sequence of transfers $f g_t g_{t=0}^1$ and a monetary and debt policy $f M_{t+1}; d_{t+1} g_{t=0}^1$. The government is not subject to a cash-in-advance constraint. For now, we abstract from sources

⁶This is the standard time inconsistency problem that monopolist issuers of currency face.

⁷Marimon, Nicolini and Teles (1999) studies the case of private issuers of currency who are neither price takers nor necessarily credible.

of revenues other than seigniorage; therefore, in the equilibria where both currency and deposits are valued, the intertemporal budget constraint of the government is

$$M_{t+1} + P_t d_{t+1} = M_t + P_t R_t d_t + P_t g_t \quad (2.9)$$

together with a no-Ponzi games condition.

In setting its policy, the government takes into account the competitive behavior of the private sector. Using (2.6), the present value budget constraint can be written as

$$\sum_{t=0}^{\infty} \beta^{-t} g_t = \sum_{t=0}^{\infty} \beta^{-t+1} (I_{t+1} - 1) m(I_{t+1}) + \frac{M_0}{P_0^m} + R_0 d_0 \quad (2.10)$$

where $I_{t+1} = \beta^{-1} \frac{P_{t+1}}{P_t}$ and $m(I)$ is defined implicitly by (2.4) together with the cash-in-advance constraint, i.e., $m(I) = u^{0,1}(\cdot)$. In the equilibria where currency is not valued the budget constraint would be

$$\sum_{t=0}^{\infty} \beta^{-t} g_t = R_0 d_0 \quad (2.11)$$

Therefore, as long as currency is valued (i.e., $I_{t+1} = 1 + \mu$; $t \geq 0$), the monetary and debt policy can also be thought of as setting $(P_0^m; \{I_{t+1}\}_t)$ that satisfies (2.10).

Competitive equilibria Given $M_0; R_0 d_0; R_0 b_0^h; R_0 b_0^e; I_0^f E_0$ and a prespecified government policy $\{M_{t+1}; d_{t+1}; g_t; g_{t=0}^1\}$; a competitive equilibrium where inside and outside money are valued (i.e., $\pi_t = 1$; $t \geq 0$)⁸ consists of sequences of prices and interest rates $\{P_t; R_{t+1}; I_{t+1}^f\}_{t=0}^{\infty}$; households' allocations $\{c_t^1; c_t^2; n_t; M_{t+1}; E_{t+1}; b_{t+1}^h; g_{t=0}^1\}_{t=0}^{\infty}$; and financial intermediaries' allocations $\{n_t^e; E_{t+1}; b_{t+1}^e\}_{t=0}^{\infty}$, such that households maximize their utility subject to their budget constraints, financial intermediaries maximize their revenues, and markets clear; that is, for $t \geq 0$, $c_t^1 + c_t^2 = n_t + n_t^e$; $n_t^e = \mu \frac{E_t}{P_t}$; $d_t = b_t^h + b_t^e$.

⁸A competitive equilibrium where only outside money is valued (i.e., $\pi_t = 0$; $t \geq 0$) is defined analogously, except that the condition corresponding to $E_{t+1} = E_{t+1}^s$ is obviously satisfied since, in fact, it is replaced by $\pi_{t+1} E_{t+1} = \pi_{t+1} E_{t+1}^s = 0$. Similarly, when currency is not valued $M_{t+1} = \pi_{t+1} M_{t+1} = \pi_{t+1} M_{t+1}^s = 0$.

Equilibria with valued currency As previously mentioned, we are interested in the characterization of stationary equilibria (from period one on) where one or both forms of monies are valued. In equilibria where currency is held, the cost of holding currency must be less than or equal to the cost of holding deposits;

$$I_{t+1}^f \cdot 1$$

must hold. Since the cost of holding deposits is I_{t+1} ; $I_{t+1}^f = \mu$, in a stationary equilibrium with valued currency it must be that either $I_{t+1} = I < 1 + \mu$, $t \geq 0$ or $I_{t+1} = 1 + \mu$, $t \geq 0$. In equilibria with $I < 1 + \mu$, $t \geq 0$, deposits do not have value. This means that if the supply of deposits is positive, $E_t > 0$, the exchange rate must be $\pi_t = 0$. When $I_{t+1} = 1 + \mu$, $t \geq 0$, there is no difference for households between holding currency and deposits. We will assume that only currency is being held, so that $E_t = 0$. Such an assumption is consistent with having a revenue maximizing government interested in capturing all cash-in-advance demand and, hence, willing to lower interest rates if needed.

In what follows, we will focus our attention to equilibria with constant rates of money growth. It is well known that in this model there exists a unique stationary equilibrium and potentially a continuum of bubble equilibria. Following McCallum (2001) and many others, we will only consider the stationary equilibrium.

Incidentally, note that assuming that $\pi_t = 1$ in an equilibrium where both means of payment have value is without loss of generality. Indeed, allowing for exchange rates to be different from one, in an equilibrium where both currency and deposits have value, must satisfy

$$\begin{aligned} \frac{\pi_{t+1}}{\pi_t} I_{t+1}^f &= 1 \\ I_{t+1} &= 1 + \mu \end{aligned}$$

Thus, both I_{t+1} and $\frac{\pi_{t+1}}{\pi_t} I_{t+1}^f$ are pinned down in equilibrium, but neither $\frac{\pi_{t+1}}{\pi_t}$ or I_{t+1}^f are. In fact, any arbitrary sequence I_{t+1}^f is an equilibrium interest rate, as long as $\frac{\pi_t}{\pi_{t+1}} = I_{t+1}^f$: Note, however, that in all these equilibria $E_t = 0$, so this nominal indeterminacy does not imply a real indeterminacy.

Cash-less equilibria In an equilibrium where currency is not valued and only deposits are used for transactions, the price level, $P_t = P_t^e$, and both the nominal interest rate on deposits, I_t^f , and on bonds, I_t , are in units of deposits. In an appendix, available from the authors upon request, we formally show that a competitive equilibrium exists and it is unique, up to the determination of all nominal

variables, an issue that we informally discuss below⁹. In this equilibrium, if the supply of currency is positive, $M_t > 0$, then $\pi_t = 1$. The incremental cost of the cash good is equal to the difference between the interest rate on bonds and the one on deposits, that is, the intermediation cost, μ (i.e., $I_t - I_t^f = \mu$).

In this economy with inside money, the supply of deposits is indeterminate. It follows that price levels are also indeterminate. The interest rates are also indeterminate, but the difference between the two interest rates is not, and that is what is relevant to determining the allocations. All of the real quantities except the initial consumption of the cash good are determined by $u^0(c_t^2) = u^0(c_{t+1}^1) = u^0(c_t^1(1 + \mu))$ and $n_t = c_t^2 + c_t^1(1 + \mu)$, $t \geq 0$. Since $c_0^1 = \frac{E_0}{P_0} I_0$, the initial consumption is indeterminate if $E_0 I_0 > 0$. Assuming $E_0 I_0 = 0$ avoids this indeterminacy without affecting the characterization of equilibria from period one on¹⁰.

3. Equilibria with commitment

In this section, we consider full commitment policies under the assumption that the government chooses the policy $(fM_{t+1}g; fd_{t+1}g; fg_tg)$ that maximizes its preferences for revenues (or transfers). More precisely, we assume that the government's problem is to maximize $\sum_{t=0}^{\infty} \beta^t G(g_t)$ (where, for standard reasons, the function G is assumed to be increasing and strictly concave), subject to (2.10). Thus, given that in equilibrium the gross real interest rate is constant and equal to β^{-1} , the government will always choose a constant sequence of transfers. Therefore, the value to the government of different allocations can be measured in terms of g :

Since the government can fully commit to a future path of interest rates $fI_{t+1}g$; an optimal program satisfies $\frac{M_0}{P_0} = 0$. The choices of interest rates are likely to be constrained by the presence of currency substitutes. To see this, notice that without inside money competition, revenues from the inflation tax are given by $f(I) = (I - 1)m(I)$. Our standard assumptions on u imply that $f(1) = 0$; $f'(1) > 0$. We assume that the Laffer curve is concave, $f''(I) < 0$ for $I \geq 1$. It follows that an optimal plan is stationary $I_{t+1} = I$; $t \geq 0$. Furthermore, whenever

⁹The formal treatment of the existence and non-uniqueness of monetary equilibria in economies with inside money only is, to our knowledge, best performed by Drèze and Polemarchakis (2000).

¹⁰This cashless economy is not the limit of economies with well defined currency demands, as in Woodford (1998) and, therefore, it is not possible to determine the initial price level in our cashless equilibrium as the limit of a sequence of equilibria with valued currency.

$f^0(1 + \mu) > 0$; the unconstrained choice of the interest rate is above $1 + \mu$; so inside money competition prompts the government to choose $I = 1 + \mu$. Given that there is competition among financial intermediaries, condition (2.8) implies that $I_{t+1}^f = 1$. More formally,

Proposition 1 Assume $f^0(I) < 0$ for $I \geq 1$ and $f^0(1 + \mu) \geq 0$; i.e., revenues from the inflation tax are increasing at $1 + \mu$. Then, the commitment solution for the revenue maximizing government is $I_{t+1} = 1 + \mu$; resulting in $I_{t+1}^f = 1$ for $t \geq 0$:

It follows as a corollary that as the intermediation costs are reduced (i.e., $\mu \rightarrow 0$),—for example, because the suppliers of currency substitutes become more efficient—the commitment equilibrium approaches the Friedman rule, where the rents to the monopolist supplier of currency are zero.

The commitment solution is time inconsistent. In this solution, at time zero, the government runs a big open market operation holding real bonds issued by the private sector that are exchanged for currency so that the real value of the outstanding money stock is zero. In addition, monetary policy from time one on is such that the gross nominal interest rate is constant over time and set at $1 + \mu$. At time t , the government has outstanding liabilities $\frac{M_t}{P_t} + (1 + \mu)^{-1} d_t$; and is tempted to revise the plan by setting $\frac{M_t}{P_t} = 0$; thereby attempting another big open market operation.

So, the interest rate plan $I_t = 1 + \mu$ will not be optimal for a government that can decide sequentially. Therefore, we turn our attention to an economy without a fully committed monetary authority.

4. Equilibria without commitment

In this section, we define and characterize equilibria when the government makes choices sequentially. These decisions depend on the history of the economy, which is given by

$$h_0 = \left(M_0; R_0 d_0; R_0 b_0^h; R_0 b_0^e; I_0^f; E_0 \right) \text{ and } h_{t+1}^1 = \left(f h_t; M_{t+1}; d_{t+1}; g_t g; \right) \text{ for } t \geq 0:$$

$$\text{and } h_{t+1} = \left(h_{t+1}^1; b_{t+1}^h; b_{t+1}^e; E_{t+1} \right) \text{ for } t \geq 0:$$

Given a history h_t at the beginning of period t , the government moves ...rst and chooses the policy for the period $(g_t; M_{t+1}; d_{t+1})$.¹¹ Thus, h_{t+1}^1 is known within period t , at the time households make their choices.

A sequential policy for the government is a sequence of functions $\gamma = \{\gamma_t\}_{t=0}^\infty$, where $\gamma_t(h_t)$ specifies the choice of a government action $(g_t; M_{t+1}; d_{t+1})$ as a function of the history h_t . As in the commitment case, the government takes the competitive behavior of the private sector as a given when choosing a policy. An allocation rule for the private sector λ is a sequence of functions $\lambda_t(h_t^1)$; where $\lambda_t(h_{t+1}^1)$ specifies a one-period allocation for households and financial intermediaries $(c_{1t}; c_{2t}; n_t; M_{t+1}^d; E_{t+1}; b_{t+1}^h; b_{t+1}^e)$ as a function of the history h_{t+1}^1 . If $\gamma^t(h_t)$ denotes the continuation of γ from h_t , sequential rationality implies that for each $(t; h_t)$; $\gamma^t(h_t)$ is optimal (i.e., maximizes transfer revenues subject to (2.10)) given the allocation rules of the households.

A Sustainable Equilibrium (SE) is a pair $(\gamma; \lambda)$ such that: i) $(\gamma; \lambda)$ defines a competitive equilibrium, with corresponding prices and interest rates $\{P_t; r_t; R_{t+1}; I_{t+1}^f\}_{t=0}^\infty$; and ii) for each $(t; h_t)$; $\gamma^t(h_t)$ is optimal given λ .

In order to characterize the set of sustainable equilibrium values, we ...rst need to ...nd the worst one.

Proposition 2. The value of a competitive equilibrium where currency is not held and deposits are used for transactions is the value of the worst sustainable equilibrium.

Proof. Let λ be the allocation rule for the private sector corresponding to an equilibrium where only inside money is valued (as defined in Section 2). Let the strategy of the government γ be given by $M_{t+1} = (1 + \theta)M_t$; where $(1 + \theta)^{-i-1} > 1 + \mu$; $d_t = d_0$; $g_t = \beta(1 - \beta)^{-i-1}d_0$; for all t : Since currency is not valued, such a policy is sequentially rational for the government. Currency is dominated and is not held in the equilibrium defined by $(\gamma; \lambda)$. It follows that the value of this equilibrium outcome is

$$V^{WSE}(d_0) = \beta(1 - \beta)^{-i-1}d_0 \quad (4.1)$$

where WSE stands for worst sustainable equilibrium. Also note that since we assume away the possibility of default in any competitive equilibrium, then the value

¹¹The implementation of the policy has to obey the timing of transactions spelled out in Section 2 where good markets open ...rst and asset markets open at the end of the period, taken from Svensson (1986).

of the government is bounded below by $V^{WSE}(d_0)$ in any competitive equilibrium. As a sustainable equilibrium must be a competitive equilibrium, it is clear that there cannot be a sustainable equilibrium with a value lower than $V^{WSE}(d_0)$ ■

In line with Chari and Kehoe(1990), we apply Abreu (1988)'s optimal penal codes and use the reversion to the worst sustainable equilibrium as the means of supporting equilibrium outcomes. As mentioned above, we will concentrate on stationary equilibria, except for the initial big open market operation.

More explicitly, consider the following government strategy γ : $M_{t+1} = (1 + \pi^0)M_t$, $d_{t+1} = d_t - m(I^0)$; and $g_t = g = \pi^0 m(I^0) - d_t(\pi^0 - 1)$; for all t ; as long as $M_{s+1} = (1 + \pi^0)M_s$, for $0 \leq s \leq t-1$; where $I^0 = (1 + \pi^0)^{-1} \cdot 1 + \mu$. While if $M_{s+1} \neq (1 + \pi^0)M_s$ for at least one $s \leq t-1$, then, for $t > s$, $M_{t+1} = (1 + \theta)M_t$; $d_{t+1} = d_s$ and $g_t = g_s = \theta d_s(\pi^0 - 1)$; where $(1 + \theta)^{-1} > 1 + \mu$. For example, for $I^0 = 1 + \mu$, this strategy corresponds to the policy achieved under full commitment (Proposition 1), as long as such a full commitment path is followed, while a deviation to an inside money equilibrium -without cash, as in Proposition 2- follows if a deviation from the full commitment path is observed.

Notice that the policy following a deviation is optimal if private agents lose confidence in the currency and it ceases to be valued, whenever $M_{s+1} \neq (1 + \pi^0)M_s$ for at least one $s \leq t-1$. As we have seen (Proof of Proposition 1), the value for the government after a deviation is $V^{WSE}(d_t)$. Therefore, it is not profitable for the government to deviate from the path of constant growth of the money supply π^0 if $V(I^0; d_t) \geq V^{WSE}(d_t)$; where

$$\begin{aligned} V(I^0; d_t) &= - (I^0 - 1)m(I^0) - (1 - \pi^0)^3 m(I^0) + d_t(\pi^0 - 1) \\ &= - (I^0 - 1)m(I^0) - (1 - \pi^0)d_t(\pi^0 - 1) \\ &= - (I^0 - 1)m(I^0) + V^{WSE}(d_t) \end{aligned}$$

That is, $V(I^0; d_t) \geq V^{WSE}(d_t)$ only if $-(I^0 - 1) = \pi^0 \geq 0$; where π^0 is the constant-inflation rate when I^0 is the gross nominal interest rate.

So far, we have shown that sustainability requires the inflation rate to be nonnegative. In addition, competition from currency substitutes requires $I^0 = (1 + \pi^0)^{-1} \cdot 1 + \mu$. Thus, the set of (from period one) stationary sustainable competitive outcomes with valued currency is characterized by

$$0 \leq \pi^0 \leq -(1 + \mu) - 1 \quad (4.2)$$

In the absence of competition from currency substitutes, the set of sustainable

equilibria is a very large one that includes the commitment solution, i.e. the stationary inflation rate that allows achieving the maximum of the Laffer curve. As long as that value is positive, the equilibrium is sustainable. The punishment is autarchy, but from the perspective of a revenue maximizing government, autarchy has the same value as the deposits-only equilibrium.

Competition reduces the set of sustainable inflation rates by imposing an upper bound on equilibrium inflation when currency is valued. Thus, competition from currency substitutes allows reducing the maximum level of the inflation rate in a sustainable equilibrium.

Because competition from currency substitutes reduces the future gains from issuing currency, it is more difficult to sustain equilibria where currency has value. If the supply of currency substitutes is very efficient, the set of sustainable equilibria with valued currency may be empty. That would be the case if competition from currency substitutes drove the inflation rates into negative numbers, which would happen if $\mu < \frac{1}{1+i}$. In that case, there would be no sustainable equilibrium with valued currency but there would still be a sustainable equilibrium outcome with deposits being used for transactions. In this equilibrium, the cost of transactions is given by the real intermediation cost μ . It follows that, with limited commitment, relatively less efficient competitors can drive currency out of circulation.

Proposition 3. The policy with full commitment (of Proposition 1) is a sustainable equilibrium path if intermediation cost μ satisfies $\mu \leq \frac{1}{1+i}$; i.e., if the equilibrium inflation rate is non-negative. If $\mu < \frac{1}{1+i}$, there is no sustainable equilibrium with valued currency, but there is a sustainable equilibrium with (only) inside money.

Thus, when the policy with full commitment is sustainable, the set of sustainable equilibria can be relatively large although, as (4.2) shows, it shrinks with competition.

5. The case of a representative government

In this section, we show how the results we have obtained so far are modified when we assume that the government maximizes welfare. As in the standard Ramsey problem, we assume exogenous -per period- government expenditures, g . As the ability to collect seigniorage will be limited by the efficiency of the financial

intermediaries, we allow the government to levy consumption taxes, τ_t , to ensure that expenditures can be financed.

As in the previous section, the timing of events is as in Svensson. Nicolini (1998) shows that with this timing and a benevolent government, the time inconsistency problem is of a different nature than in the classic papers of Calvo (1978) and Lucas and Stokey (1983), since there are costs of unanticipated inflation. The two main differences that this timing introduces are that the optimal deviation is always finite and that for the government to be willing to deviate from the Ramsey policy and inflate at a higher rate than promised, the consumption elasticity has to be larger than one. We will assume that this is indeed the case.

The consumer's problem is the same as before, except for the presence of a tax on consumption. We simplify the analysis in this section by assuming preferences of the form (2.1), but with the additional restriction of constant relative risk aversion (CRRA), i. e., $\frac{u^0(c_t)c_t}{u^0(c_t)} = \frac{1}{\sigma}$, where $\frac{1}{\sigma} > 1$ is the price elasticity of c_t . For the reasons stated above, we assume that $\frac{1}{\sigma} > 1$: If $\frac{1}{\sigma} = 1$ for $t \geq 0$; $t \geq 0$; the budget and cash-in-advance constraints are

$$M_{t+1} + P_t b_{t+1}^h + E_{t+1} \leq P_t n_t + (1 + \tau_t)P_t(c_t^1 + c_t^2) + M_t + P_t R_t^b b_t^h + I_t^f E_t \quad (5.1)$$

$$(1 + \tau_t)P_t c_t^1 \leq M_t + E_t \quad (5.2)$$

for $t \geq 0$; M_0, b_0, E_0 given. The tax on consumption imposes a distortion between consumption and leisure, translated in the following first order conditions for $t \geq 0$:

$$1 + \tau_t = \frac{u^0(c_t^2)}{u^0(c_t^1)} \quad (5.3)$$

Let the function $c^2(\tau)$ be given by $u^0(c^2) = u^0(c^1) \frac{1}{1 + \tau}$: As before, when inside money has value the marginal rate of substitution between consumptions is given by

$$I_{t+1} = \frac{u^0(c_{t+1}^1)}{u^0(c_{t+1}^2)} \quad t \geq 0 \quad (5.4)$$

Let the function $c^1(I; \tau)$ be given by $u^0(c^1) = I u^0(c^2(\tau))$: However, when only inside money circulates consumptions satisfy

$$\frac{u^0(c_{t+1}^1)}{u^0(c_{t+1}^2)} = 1 + I_{t+1} \text{ if } I_{t+1}^f = 1 + \mu \quad t \geq 0 \quad (5.5)$$

where the second equality in (5.5) follows from the zero profit condition in financial intermediation. Let the function $c_\mu^1(\tau)$ be given by $u^0(c^1) = (1 + \mu)u^0(c^2(\tau))$:

5.1. Optimal policy under commitment

The optimal policy under commitment is the solution of a dynamic Ramsey problem, as in Lucas and Stokey (1983); like they did, we follow the primal approach. The objective of the government is to maximize the welfare of the representative household, subject to feasibility and competitive equilibrium constraints. In the primal approach, these competitive equilibrium constraints are consolidated in an implementability condition.

We first consider the case where $\pi_t \geq 0$ and only currency is used for transactions. The following implementability condition is a necessary condition for the optimal solution to be decentralized as a competitive equilibrium with taxes¹²:

$$u^0(c_0^2)c_0^2 - n_0 - d_0^{-1} + \sum_{t=1}^{\infty} -\pi_t [u^0(c_t^1)c_t^1 + u^0(c_t^2)c_t^2 - n_t] = 0 \quad (5.6)$$

We can now define the Ramsey problem as the maximization of the utility function subject to (5.6) and

$$c_t^1 + c_t^2 + g \leq n_t, \quad 0, \quad t \geq 0. \quad (5.7)$$

In the following proposition we characterize the Ramsey solution.

Proposition 4. Assume CRRA. In the Ramsey solution, $I_{t+1} = 1$ and $\lambda_t^c = \lambda^R$, with $t \geq 0$. If $\frac{1}{2} < 1$ $c_0^1 = c_0^R < c_0^2 = c_{t+1}^2 = c_{t+1}^1 = c^R$; $t \geq 0$.

Proof. The result follows directly from the first order conditions of the Ramsey problem. Let $\pi \geq 0$ be the Lagrange multiplier associated with condition (5.6). Then, $\frac{u^0(c_0^1)}{u^0(c_0^2)} = 1 + \pi(1 - \frac{1}{2})$, while for $t \geq 0$: $I_{t+1} = \frac{u^0(c_{t+1}^1)}{u^0(c_{t+1}^2)} = \frac{1 + \pi(1 - \frac{1}{2})}{1 + \pi(1 - \frac{1}{2})} = 1$ and $1 + \lambda_t = \frac{u^0(c_t^2)}{\pi} = \frac{1 + \pi}{1 + \pi(1 - \frac{1}{2})}$.

In order to set up the Ramsey problem we have assumed that the optimal policy results in $I_{t+1}^f = 1$. Suppose, instead, that $I_{t+1}^f > 1$; in that case, by (5.5), $\frac{u^0(c_{t+1}^1)}{u^0(c_{t+1}^2)} = 1 + \mu$; for $t \geq 0$. However, that marginal rate of substitution was a feasible solution to the Ramsey problem by setting $I_{t+1} = 1 + \mu$ (resulting in $I_{t+1}^f = 1$). That (suboptimal) policy increases revenues and saves on intermediation costs. It

¹²Building the implementability constraint by replacing first order conditions on the life-time budget constraint of the households is standard practice in the primal approach. For details, see the appendix of the working paper version, available from the authors upon request.

follows that the Ramsey solution is such that $I_{t+1}^f = 1$, $t \geq 0$, so that currency substitutes will not be held, and only currency will be used for transactions. ■

This proposition states that under commitment, a benevolent government will follow the Friedman rule, $I_{t+1} = 1$. The Friedman rule means that both cash and credit goods are taxed at the same rate. This is the optimal taxation solution since the utility function is homothetic in the two goods and separable in leisure. These are the conditions for uniform taxation of Atkinson and Stiglitz (1972), as highlighted by Lucas and Stokey (1983) and Chari, Christiano and Kehoe (1993). It also follows from standard optimal taxation principles that, since the price elasticity is greater than one ($\eta < 1$), the consumption in period 0 of the cash good is lower than the consumption from period 1 on. That is, there is a higher tax on the initial cash good (with a price elasticity of one)¹³.

Characterizing the policies that support the Ramsey allocation is straightforward. First, the Friedman rule implies that the nominal money supply should be set such that $M_{t+1} = M_t$ for $t \geq 0$. Second, τ^R must solve the households' optimal condition (5.3) when consumption is equal to the Ramsey solution c^R . The implementability constraint implies that the Ramsey allocation and taxes must satisfy

$$\tau^R c^R + g + \frac{1}{1+i} [2\tau^R c^R + g] = c_0^R + d_0^{R-1} \quad (5.8)$$

Since the Ramsey solution is stationary from period one on, the value of the debt will also be. In fact, computing the value for $d_t = d_1^R$ all t is straightforward, since an implementability condition at time one can be written as

$$(1 + \tau^R) c^R + (2c^R + g) + \sum_{t=1}^{\infty} \tau^R [2(1 + \tau^R) c^R + (2c^R + g)] = d_1^{R-1}$$

Rearranging, we obtain

$$\tau^R c^R + g + \frac{1}{1+i} [2\tau^R c^R + g] = c^R + d_1^{R-1} \quad (5.9)$$

This equation, together with (5.8), implies that

$$c^R + d_1^{R-1} = c_0^R + d_0^{R-1}$$

¹³See Nicolini (1998) for a further discussion of this.

or

$$d_1^R = c_0^R - c_1^R + d_0 \quad (5.10)$$

Since $c_0^R < c_1^R$, it follows that $d_1^R < d_0$. The intuition is straightforward: the revenues raised by the initial money injection are used to reduce the debt and lower consumption taxes forever.

The Ramsey problem in the cashless economy Note that in the cashless economy, the government has lost the inflation tax as an instrument. This is reflected in the fact that the intra-period marginal rate of substitution between the two consumption goods is given by the technology of the banking sector, as (5.5) shows. The Ramsey problem in this case is to maximize the utility function, subject to feasibility, implementability, and the additional constraint (5.5). The feasibility condition is the same as in the cash economy. The implementability condition is also the same, as long as the initial holdings of electronic money are zero, as we assume. Thus, the solution will be given by a consumption tax that is constant over time and satisfies the government's budget constraint. Let $V^{RD}(d; \mu)$ be the value of the Ramsey problem in the cashless economy with initial real debt d and intermediation costs μ . Note that $V^{RD}(d; \mu)$ is a continuous and decreasing function of μ ; since as μ becomes larger, the distortion between the two goods is larger, and the consumption of the cash good is lower. Thus, the tax rate required to finance the same level of expenditures is higher, as is the distortion of the consumption tax. In addition, a larger amount of resources is spent on intermediation.

5.2. Optimal policy without commitment

As we have seen, the Ramsey solution is time inconsistent. If, however, deviating from such a path is costly enough for the benevolent government, then it may be a sustainable equilibrium. In order to now consider economies with taxes and exogenous government expenditures, we must modify the notation of Section 4. Histories are defined as before, except that g_t must be replaced by ζ_t . Strategies and allocation rules $(\gamma; \pi)$ are similarly defined. A government action, given history h_t , now takes the form $\gamma_t(h_t) = (\zeta_t; M_{t+1}; d_{t+1})$. A Sustainable Equilibrium is, as before, a $(\gamma; \pi)$ that defines a competitive equilibrium with the property that γ maximizes the utility of households subject to feasibility and private agents' competitive behavior based on expectations of exchange rates that are fulfilled in equilibrium for any history and any period.

As with the revenue maximizer government, a strategy with a fast growth of the money supply, $M_{t+1} = (1 + \theta)M_t$ such that $(1 + \theta)^{-i-1} > 1 + \mu$, results in an equilibrium where only inside money is held, and characterizes the worst sustainable equilibrium with a value to the government given by $V^{RD}(d; \mu)^{14}$:

Proposition 5. The value of a competitive equilibrium where currency is not held, deposits are used for transactions, and taxes are constant over time is the value of the worst sustainable equilibrium.

To find conditions such that the Ramsey allocation can be supported as a SE we characterize trigger strategies, as in Section 4. In particular, the government γ is given by the values of the Ramsey policies, $M_{t+1} = \bar{M}_t$; $\zeta_t = \zeta^R$; $d_{t+1} = d_1^R$, for all $t \geq 0$ as long as $M_s = \bar{M}_{s-1}$ for $0 \leq s \leq t$. While if there is a deviation such that $M_{s+1} \neq \bar{M}_s$ for at least one $s \leq t-1$, then for $t \geq s$, $M_{t+1} = (1 + \theta)M_t$; $d_{t+1}^R = d_1^R$ and $\zeta_t = \zeta^d$; where $(1 + \theta)^{-i-1} > 1 + \mu$ and ζ^d satisfies the budget constraint after the deviation

$$\zeta^d c^2(\zeta^d) i - g + \frac{-}{1+i} [\zeta^d (c^2(\zeta^d) + c_\mu^1(\zeta^d)) i - g] = d_1^{R-i-1} \quad (5.11)$$

There are two differences between this equation and (5:9): First, in the period of the deviation, consumption of the cash-good, and therefore the value of real money, is zero, so only the value of the debt is on the right-hand side. Second, after the deviation, consumption of the cash-good in the cashless economy is given by the function $c_\mu^1(\zeta)$ instead of $c^1(l; \zeta)$:

Given the strategy of the government, the allocation rules are described by the first order conditions of the maximum problem of the households. To show that this is indeed an SE, we must show that the government has no incentives to deviate from the described strategy.

Note that, contrary to the case of the revenue maximizer government, in this case the allocation is stationary only from period one on. Consider a government who deviates at period $s \geq 1$ ¹⁵: As before, from the next period on, the government will pursue a policy such that only deposits are used in equilibrium and the

¹⁴The proof of Proposition 5 is similar to that of Proposition 2; see, Marimon, Nicolini and Teles (1999a) for details.

¹⁵We will further show that for the cases of $\bar{\gamma}$ close to one that we will consider, if the government does not want to deviate at $s \geq 1$; then it will not have incentives to deviate at time zero.

resulting path is the Ramsey outcome of the cashless economy. The value for the government after a deviation satisfies

$$(1 - \beta)V^{RD}(d_1^R; \mu) = (1 - \beta)fv(0) + v(c^2(\tau^d)) - \beta g + \\ - fv(c^2(\tau^d)) + v(c_\mu^1(\tau^d)) - \beta(c_\mu^1(\tau^d)\mu + g)g$$

where $v(x) = u(x) - \beta x$ and the tax rate τ^d satisfies the government budget constraint (5.11)¹⁶. On the contrary, if the government continues with the Ramsey policy, the value would be

$$(1 - \beta)V^{RC}(d_1^R) = 2v(c^R) - \beta g$$

where the tax rate satisfies the budget constraint (5.8)¹⁷.

There are three key differences between $V^{RD}(d_1^R; \mu)$ and $V^{RC}(d_1^R)$. The first two can be seen in the budget constraints. First, (5.8) implies that, along the Ramsey path, both the value of the bonds and the value of the outstanding money stock must be paid out with future surpluses. However, after a deviation, only the value of the bonds must be paid out since the value of the outstanding money stock is zero. This effect decreases τ^d relative to τ^R and is the benefit of the deviation. Second, following a deviation, the base of the consumption tax is lower for a given tax rate, since the cash good is more expensive in equilibrium due to the intermediation cost μ : Therefore, to raise the same revenue after a deviation, the tax rate must be higher. This effect increases τ^d relative to τ^R ; consequently, the distortion will be higher. In addition to this indirect cost, there is the direct cost μ of the inside money technology. Notice that these intermediation costs decrease to zero as μ decreases to zero.

Thirdly, note that consumption of the cash good immediately after a deviation is zero, due to the destruction of money balances. During that same period, agents build deposits that can be used to buy cash goods the following period. Thus,

¹⁶If the government deviates at time zero, the value of the tax will have to satisfy a budget constraint similar to (5.11), where d_0 shows up in the right hand side instead of d_1 : As $d_1 < d_0$, the consumption tax after a deviation at time zero must be higher than after a deviation at $s = 1$. It follows that the value of deviating at time zero is lower than $V^{RD}(d_1; \mu)$:

¹⁷Note that $V^{CR}(d_0)$, the value of the continuation of the Ramsey, is larger than the value of the Ramsey $V^R(d_0)$, since in the Ramsey allocation consumption of the cash good in the first period is lower than in all other periods. Note also that as the only difference is given by the first period consumption, $V^R(d_0) < V^{CR}(d_0)$ as $\beta < 1$:

instantaneous utility in the deviation period is very low due to what can be called a liquidity shortage.

The Ramsey policy is sustainable as long as $V^{RC}(d_1^R) > V^{RD}(d_1^R; \mu)$ or

$$0 \leq (1 - \beta) f[v(0) + v(c^2(\bar{c}))] - 2v(c^R)g + \beta f[v(c^2(\bar{c})) + v(c_\mu^1(\bar{c}))] - \mu c_\mu^1(\bar{c}) - 2v(c^R)g \quad (5.12)$$

The relative strength of the three effects discussed above determines whether the Ramsey is sustainable. Note that the first term of the right hand side of (5.12) is likely to be negative due to the liquidity shortage effect. Whenever this liquidity effect is large enough, the Ramsey solution is sustainable for any degree of inside money competition.

To consider the effect of the presence of inside money on the set of sustainable equilibria, it is more interesting to abstract from the liquidity shortage effect. Formally, we analyze the case in which $u(0)$ is bounded (as in the CRRA case, with $\frac{1}{2} < 1$, considered in this Section) and β close to 1; which means that the time period required for the representative agent to attend the financial intermediaries and accumulate deposits is relatively short. In this case, the second term on the right hand side of (5.12) determines the sustainability of the Ramsey path. That is, only the intermediation costs may deter the representative government from exploiting the benefits of a deviation.

As equations (5.9) and (5.11) show, the gains from deviating are given by the reduction in government liabilities, in an amount equal to the value of real money in the Ramsey allocation $c^R > 0$. These benefits are bounded away from zero, while the intermediation costs can be very small if the financial intermediation sector is sufficiently efficient. Thus, for values of μ that are small enough, the Ramsey outcome may fail to be sustainable.

Note also that when $\mu = \beta^{-1} - 1$; the household's optimal conditions imply that $u^0(c_{t+1}^1) = \beta^{-1} u^0(c_{t+1}^2)$; which is the same first order condition of the outside money equilibrium with a stationary inflation rate of zero. In addition at zero inflation, the government is not paying real interest on money, so the value of real money does not show up in the right hand side of the government budget constraint. Thus, this - suboptimal - monetary policy of zero inflation can result in the same¹⁸ allocation that would prevail after a deviation (with only inside money) if $\mu = \beta^{-1} - 1$, except that labor in the cashless economy will be

¹⁸Provided that both start with the same level of debt.

higher due to the intermediation costs. This suggests that (even without liquidity shortage effects) the Ramsey outcome may be sustainable even for modest values of the intermediation costs, since it appears that the value of a deviation may be lower than the value of a suboptimal stationary outside money equilibrium, and therefore lower than the Ramsey allocation. The following proposition makes these claims precise; its proof uses the arguments that we have just described.

Proposition 5.1. There is a $\bar{\tau} < 1$ such that for every $\tau \geq 2(\bar{\tau}; 1)$ there exists a μ^- satisfying $0 < \mu^- < \tau^{-1} \leq 1$ such that $V^{RD}(d_1^R; \mu) \leq V^{RC}(d_1^R)$ for all $\mu \geq \mu^-$ and $V^{RD}(d_1^R; \mu) > V^{RC}(d_1^R)$ for all $0 < \mu < \mu^-$.

Proof. First we show that if $\mu = 0$ and τ is high enough, the inequality (5.12) cannot be satisfied. Notice that $v(c^2(\tilde{z}))$ is a strictly decreasing function of $\tilde{z}$; for $\tilde{z} \geq 0$; while $V^{RD}(d_1^R; \mu)$ is a continuous and decreasing function of μ . Furthermore, (5.8) and (5.11) imply $\tilde{z}^d < \tilde{z}^R$ if $\mu = 0$, while the first order conditions of the households imply that $c^2(\tilde{z}^d) = c_{\mu=0}^1(\tilde{z}^d)$. Thus, $v(c^2(\tilde{z}^d)) = v(c_{\mu=0}^1(\tilde{z}^d)) > v(c^R)$. Therefore (5.12), together with the boundness on $u(0)$, implies that there exists a $\bar{\tau}_1 < 1$ such that for every $\tau \geq 2(\bar{\tau}_1; 1)$, $V^{RD}(d_1^R; 0) > V^{RC}(d_1^R)$.

Second, we now show that when $\mu = \tau^{-1} \leq 1$; the Ramsey solution is sustainable. Consider first the case in which $\tilde{z}^d \geq \tilde{z}^R$: In such a case, $c^2(\tilde{z}^d) \leq c^R$: It follows from (5.12) that the Ramsey is sustainable. Thus, assume $\tilde{z}^d < \tilde{z}^R$; so $c^2(\tilde{z}^d) > c^R$: Consider a path in which currency is valued, initial consumption of the cash good is given by an arbitrary value e_0^1 ; initial debt is d ; and inflation is constant and equal to $\frac{1}{4}$: Assume also that consumption taxes $\tilde{z}(\frac{1}{4})$ are constant. The notation makes explicit the fact that the consumption tax rate that will satisfy the budget constraint depends on the inflation tax. In this path, the solution to the problem of the households implies that $c_t^2 = c^2(\tilde{z}(\frac{1}{4}))$ for all $t \geq 0$; and $c_t^1 = c^1(\tau^{-1}(1 + \frac{1}{4}); \tilde{z}(\frac{1}{4}))$ for $t \geq 1$: Thus, we can let $W(d; e_0^1; \frac{1}{4})$ denote the value to the government of such a path and $W^C(d; e_0^1; \frac{1}{4})$ denote the value of that path starting at any period different from zero¹⁹. The budget constraint for the government when $\frac{1}{4} = 0$ and initial debt is d_0 ; is given by

$$\begin{aligned} & \tilde{z}(0)c^2(\tilde{z}(0)) + g + \frac{\tau}{1+\tau}[\tilde{z}(0)(c^2(\tilde{z}(0)) + c^1(\tau^{-1}; \tilde{z}(0))) + g] \\ & = d_0\tau^{-1} + e_0^1 + (1 + \tilde{z}(0))c^1(\tau^{-1}; \tilde{z}(0)) \end{aligned}$$

¹⁹Recall that the path is stationary from period one on.

Note that if e_0^1 is chosen as to satisfy

$$d_0^{-i-1} + e_0^1 (1 + \lambda(0))c^1(-i-1; \lambda(0)) = d_1^{R-i-1};$$

the budget constraint will be the same as (5.11), given that $\mu = -i-1$. Thus, the allocation of consumption²⁰ and taxes will be the same as the allocation after a deviation, except that first period consumption of the cash good will be e_0^1 instead of zero. But (5.8) implies that $d_0^{-i-1} - d_1^{R-i-1} = c^R - c_0^R$; so we obtain $e_0^1 = (1 + \lambda^d)c^1(-i-1; \lambda^d) - c^R + c_0^R$. If $\bar{\mu}$ is close to one, $c^1(-i-1; \lambda^d)$ is close to $c^2(\lambda^d) > c^R$, so there is a $\bar{\mu}_2 < 1$ such that for every $\bar{\mu} \geq (\bar{\mu}_2, 1)$, $e_0^1 > 0$ and, therefore, $W(d_0; e_0^1; 0) > V^{RD}(d_1^R; -i-1; 1)$. The allocation is feasible for a Ramsey government, so $W(d_0; e_0^1; 0) > V^R(d_0)$. In summary,

$$V^{RC}(d_1^R) > V^R(d_0) \implies W(d_0; e_0^1; 0) > V^{RD}(d_1^R; -i-1; 1)$$

If we let $\bar{\mu} = \max\{\bar{\mu}_1, \bar{\mu}_2\}$; the result follows from the fact that $V^{RD}(d_1^R; \mu)$ is a continuous and decreasing function of μ . ■

The above result provides conditions under which the government has no incentives to deviate from the Ramsey path for $t \geq 1$. Given that $\bar{\mu}$ is close to one, the value of the Ramsey allocation at time zero is close to $V^{RC}(d_1^R)$; while the value of the deviation at time zero $V^{RD}(d_0; -i-1; 1)$ does not converge to $V^{RD}(d_1^R; -i-1; 1)$ as $\bar{\mu}$ converges to 1; since $V^{RD}(d_0; \mu)$ is strictly decreasing on d ²¹. Thus, for high enough values of $\bar{\mu}$; the government does not want to deviate at time zero and the Ramsey allocation is sustainable.

What about other stationary - from period one on - SE in which cash circulates? Assume there is no inside money competition and consider the set of competitive equilibria with valued currency, initial government debt d ; stationary inflation rates λ , and stationary taxes. These stationary policies support stationary allocations for all t ; except for the value of consumption of the cash good at time zero, c_0^1 . For any given value of c_0^1 ; which in turn determines the value for the debt from period one on, the tax rate $\lambda(\lambda)$ is the lowest one that satisfies the budget constraint of the government. For the CRRA preferences that we assumed in this section, the marginal utility of consumption goes to infinity as consumption goes to zero. Thus, assume that government expenditures are not too high such that it is feasible to satisfy the government budget constraint by imposing a consumption tax to the credit good only. Then, given a value for c_0^1 ;

²⁰Labor will actually be higher after the deviation, due to the intermediation costs.

²¹Recall footnotes 14 and 15.

the set of stationary competitive equilibria described above can be indexed by the corresponding $\frac{1}{4} \in [-1; 1]$: Welfare at any of these equilibria is given by the function $W(d; c_0^1; \frac{1}{4})$; as defined in the proof of proposition 6. Given a value for d ; the set of equilibria is indexed by the pair $(c_0^1; \frac{1}{4})$: We are interested in describing the set of stationary inflation rates that can be sustainable equilibria, so we will only focus on the allocations such that c_0^1 is equal to the Ramsey²² value c_0^R ; as defined in the proof of proposition 4.

Thus, let $W^s(d_0; \frac{1}{4}) = W(d_0; c_0^R; \frac{1}{4})$: Clearly, $W^s(d_0; -1) = V^R(d_0)$; and it is a straightforward exercise to show that $W^s(d_0; \frac{1}{4})$ is decreasing on $\frac{1}{4}$, so $W^s(d_0; \frac{1}{4}) \leq V^R(d_0)$ for $\frac{1}{4} \leq -1$: In addition, if we let $d_1^s(\frac{1}{4})$ be the debt from period one on for the stationary equilibrium indexed by $\frac{1}{4}$; an equation similar to (5:10) implies that $d_1^s = c_0^R + c^1((1 + \frac{1}{4})^{-1}; \frac{1}{4}) + d_0$: However, since $c^1((1 + \frac{1}{4})^{-1}; \frac{1}{4}) < c^R$; equation (5:10) implies that $d_1^s > d_1^R$: Then, $W^{sC}(d_1^s; \frac{1}{4}) \leq V^{RC}(d_1^R)$; where $W^{sC}(d_1^s; \frac{1}{4})$ is the continuation value of $W^s(d_0; \frac{1}{4})$:

We want to characterize the subset of those stationary competitive equilibria that are SE given a value for μ : First, consider values such that $\mu < \mu^-$: Proposition 6 implies that there exists no SE with valued currency since $W^{sC}(d_1^s; \frac{1}{4}) \leq V^{RC}(d_1^R) < V^{RD}(d_1^R; \mu)$ for all $0 \leq \mu < \mu^-$:

Second, assume that $\mu \geq \mu^-$: In this case, there are two reasons why high inflation rates may not be sustainable. The first one is competition from financial intermediaries. If we let $\frac{1}{4}_1(\mu)$ be defined by $\frac{1}{4}_1 = -(1 + \mu)^{-1}$; any inflation higher than $\frac{1}{4}_1(\mu)$ is ruled out by inside money competition. The second reason is that for high inflation rates, the cashless allocation may be better. If we let $\frac{1}{4}_2(\mu)$ be implicitly defined by $W^{sC}(d_1^s; \frac{1}{4}_2) = V^{RD}(d_1^R; \mu)$, it is clear that no inflation rate higher than $\frac{1}{4}_2(\mu)$ can be sustained, since $W^s(d; \frac{1}{4})$ is decreasing on $\frac{1}{4}$ ²³.

The preceding discussion shows that the set of stationary inflation rates that are SE with valued currency is given by $[-1; \min(\frac{1}{4}_1(\mu); \frac{1}{4}_2(\mu))]$; provided that $\mu \geq \mu^-$: As both $\frac{1}{4}_1(\mu)$ and $\frac{1}{4}_2(\mu)$ are decreasing with μ ; the set of sustainable outcomes shrinks as the banking sector becomes more efficient. If $\mu < \mu^-$; then there is no SE with valued currency.

In summary, when governments are benevolent, competition from currency substitutes reduces the punishment of a deviation and therefore makes it harder

²²We do not think that indexing equilibria according to the first period consumption is a relevant economic question. Focusing on the ones that satisfy $c_0^1 = c_0^{1R}$ is convenient, since welfare converges to the Ramsey value when the stationary inflation rate approaches the Friedman rule.

²³It can be shown that the condition $W^{sC}(d_1^s; \frac{1}{4}) \leq V^{RD}(d_1^R)$ restricts sustainable inflation rates more than $W^s(d_0; \frac{1}{4}) \leq V^R(d_0^R)$; so we do not need to impose this last condition.

to sustain monetary outcomes. However, the punishment may be enough of a deterrence to sustain the Ramsey allocation. This depends essentially on how efficient the financial intermediaries are. In addition, inside money competition imposes a ceiling on sustainable inflation rates²⁴.

6. Concluding remarks

In this paper we have seen that the implications for monetary policy of competition from inside money depend, in general, on the objectives of the government, its ability to commit, and the relative efficiency of financial intermediaries. With full commitment, the benevolent government already implements the Friedman rule - of zero nominal interest rates - as part of the Ramsey solution and, therefore, there is no role for competition. Instead, inside money competition is a disciplinary mechanism when there is a revenue maximizing government who can commit. In fact, if financial intermediaries' operating costs go to zero, the equilibrium converges to the efficient Friedman rule allocation.

Without full commitment, there is an interplay between reputation and competition. Interestingly enough, though, the impact this interplay has on the set of equilibria will not depend critically on the preferences of the government. In the case of a revenue maximizing government, inside money competition sets a ceiling on sustainable inflation rates. This ceiling is determined by the relative efficiency of the financial intermediaries; as long as it determines a non-negative inflation rate, the ceiling is the same as in the case of full commitment. As financial intermediaries become more efficient, the ceiling is reduced, such that the worst outcomes are no longer sustainable. In the limit, when intermediation costs are driven down to zero, the monetary authority can not credibly set a monetary policy resulting in deflation. Therefore, when inside money can circulate at a very low cost it drives outside money out of circulation.

If the government is benevolent and there is no inside money competition, then the deterrence path is one (the worst sustainable equilibrium) where no money circulates. In contrast, with inside money competition, only inside money circulates in the deterrence path. The more efficient financial intermediaries are, the less of a punishment this prospect is for a government that maximizes welfare.

²⁴ Imposing restrictions on beliefs, as in Marimon, Nicolini and Teles (1999b), will result in the Ramsey outcome being the unique (regular) sustainable equilibrium, whenever $\mu \geq \mu^-$: Inside money competition guarantees that inflation can not be much higher, even when beliefs are unrestricted.

Thus, by increasing the value of the deterrence path, the efficiency of the financial intermediaries affects the values that can be sustained. Without currency competition, there is a continuum of sustainable equilibria, and some of them have inflation rates so large that the value attained is very low. By increasing the value of a deviation, inside money competition sets a ceiling on how high inflation can be in a sustainable equilibrium. This ceiling decreases as the financial sector becomes more efficient. It is also the case that, if the financial intermediation costs—as well as the rate of time preference—are low enough, the Ramsey solution is not sustainable and the only sustainable equilibrium is one where only inside money circulates. Thus, regardless of the preferences of the government, as financial intermediation becomes very efficient, inside money drives outside money out.

In summary, inside money competition tends to reduce inflation rates in general. Such a result may help to explain the reduction of inflation rates in most advanced economies in a period (the last two decades) where currency substitutes have been supplied with increased efficiency. Only an empirical exploration can elucidate the role of inside money competition in bringing down the inflation rates of the mid 1970s.

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