CONFIDENCE CRISES, CREDIBILITY AND THE INFLATION TARGET LEVEL

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March-2007

Abstract

Considering the inflation targeting regime, we appraise how the target level should be set in the presence of uncertainty about the commitment strength. First, based on a concise framework, where confidence crises and imperfect information are neglected, we conclude that a higher target for inflation increases the credibility in the precommitment. Optimal target is higher than the one obtained using the Cukierman-Liviatan [5] model, where the increasing credibility effect is not considered. Second, extending the model to make confidence crises possible, multiple equilibria solutions become possible too. In this case, higher targets for inflation may increase the probability of above the target inflation. On the other hand, multiple (bad) equilibria may be avoided. The optimal target depends on the likelihood of each equilibrium to be selected and on the policymaker's willingness to avoid a confidence crisis.

Finally, uniqueness is ensured when common knowledge is perturbed, even considering speculative attacks, as in Morris-Shin[6]. The first result is also recovered, i.e. a higher target for inflation increases the credibility in the precommitment. Adding a *precise* public signal, self-fulfilling actions and equilibrium multiplicity may still exist for some lack of common knowledge (as in Angeletos and Werning[1]). In this case, higher targets for inflation may increase the probability of above the target inflation again, reducing the policymaker's credibility. From another aspect, multiple (bad) equilibria may be avoided. The optimal target depends on the likelihood of each equilibrium to be selected and on the policymaker's willingness to avoid a confidence crisis. Results also indicate that more precise public information may open the door to bad equilibrium, contrary to the conventional wisdom that more central bank transparency is always good when an inflation targeting regime is considered.

*Fundação Getulio Vargas and Instituto de Matematica Pura e Aplicada.

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1 Introduction

The framework used as a starting point to our analysis is based on the Cukierman-Liviatan model [5] and it is similar to the one presented in Barro [2]. It has some features that consider the agent's uncertainty about the commitment enforcement and suggest that the optimal target should be decreasing in the policymaker credibility[1]. During this section, we gradually extend this framework to compute not only the effect of the credibility on the target, but also the effect of the target choice on the credibility. We also present different approaches for uncertainty. First, the Cukierman-Liviatan model considers two policymaker types: the "strong" one which always adheres to the announced policy and the "weak" one which does it only as an *ex-post* expedient. It does not consider the possibility of a more sophisticated policymaker's decision, namely: to fulfill the target depending on the intensity of an occasional shock observed after the target announcement. Second, when supposing that the cost of not fulfilling the target is increasing in the credibility, credibility itself may affect the policymaker's decision about respecting (or not) its commitment. Finally, individuals may have different information sets (as in Angeletos and Werning [1]) and may disagree about their guesses for future inflation. One possible reason is the presence of imperfect information, what may be relevant to policy prescriptions.

2 Basic Model

There are two types of agents: the policymaker and private agents (public). Actions are taken in three stages: the policymaker announces the target for inflation (π_a), expectations are formed (π_e) by the public and actual inflation is chosen (π). There are two policymaker types $i \in \{1, 2\}$ with different abilities to precommit. The first type ("strong") always fulfills its commitment while the second one ("weak") does it only if it is *ex post* expedient. Their objective function is positively related to surprise

[1] We will define "credibility" as the extent to which agents believe that policymakers will carry out their announced plans (target).

inflation and negatively related to actual inflation, as follows[2]:

$$v^i(\pi_e, \pi_a) = \max_{\pi \geq 0} A[\pi - \pi_e] - \frac{\pi^2}{2} - c^i(\pi_a, \pi) \text{ where } c^i(\pi_a, \pi) = \left\{ {}^{k^i,\, \pi_a \neq \pi}_{0,\, \pi_a = \pi} \right\}, i\epsilon \{1, 2\}$$

$$k^1 = A^2,\ k^2 = 0,\ A > 0,\ \pi_a \geq 0, \text{ and } \pi_e \geq 0$$

Note that the policymaker best response for actual inflation (π^*) is either the target level (π_a) pre-announced or the discretionary inflation level rate (A). We can compute the welfare gain (w_a^i) of type (i) keeping the target (π_a) as follows[3]:

$$w_a^i = k^i + f(A, \pi_a), \text{ where } f(A, \pi_a) \equiv A(\pi_a - A) - \frac{\pi_a^2}{2} + \frac{A^2}{2}$$

Since the goal of the inflation target is to coordinate expectations from the discretionary inflation level rate (A) to socially optimal level (0), it is easy to check that $(w_a^1 > 0)$ and $(w_a^2 \leq 0)$ for any $\pi_a \in [0, A]$, and both types of ability are justified for any possible target level.

There is a continuum of private agents without a strategic behavior. Their role is to process information, to form beliefs concerning the policymaker's type and to compute the expected inflation. First, it is assumed that the private expectation about the policymaker type is formed based on the exogenous probability (α) of the type being strong $(i = 1)$, which is the same for all private agents. This probability measures the policymaker's credibility. The expected inflation is given by:

$$E\left[\pi | \alpha, \pi_a\right] = \pi_e = \alpha \pi_a + (1 - \alpha) A$$

Based on this framework Cukierman and Liviatan [5] answered the following question: *"what should be the optimal announcement π_a^* for each type (i)?"*. For $(\alpha = 1)$, the target and the expected inflation are the same. Then, the policymaker type 1 promises and delivers zero inflation rate. If we consider $\alpha \in (0, 1)$, the policymaker 1 promises and delivers $A(1 - \alpha)$ inflation rate. As (α) tends to zero the announcement effect on the expectations vanishes and the policymaker 1, who always

[2]We add the "cost of not fulfilling the target" function $c^i(\pi_a, \pi)$ to the original Cukierman-Liviatan model [5] to formalize that the strong type always fulfills the pre-announced target while the weak type is not concerned about the previous announcement.

[3]From now on, the welfare gain from keeping actual inflation on the target will be denoted by (w_a).

keeps its promises tends to preannounce the discretionary inflation level rate. Although type 2 ends up inflating at the discretionary rate, it has an interest to keep itself indistinguishable at the announcement stage in order to stimulate lower expectations $(\pi_e < A)$. It follows that $\pi_a^{*i} = A(1-\alpha)$ for both types (i). Accordingly, full credibility $(\alpha = 1)$ is not required for inflation targeting to be implemented. In the absence of pre commitment, the result leads to an inflationary bias (A) that can be reduced whenever policymakers are able to precommit with some credibility $(\alpha > 0)$. This bias reduction improves welfare. To totally eliminate the inflationary bias and to achieve the socially optimal inflation rate (zero), the ability to commit must not be only present but must also be undoubtedly recognized by the public. Otherwise, a lower inflationary bias reappears.

Next, we gradually extend this framework to argue that there are some other reasons for the inflation target to be higher than the socially optimal level.

3 Endogenous Credibility

Less ambitious (*higher*) targets are attained more often when monetary policy is subordinated to fiscal financing requirements and the economy is subject to shocks that can make more inflation tolerable. In this sense, (α) should not be an exogenous variable because when (π_a) is selected (α) should be affected. Therefore, we consider the uncertainty about the ability to precommit coming not from some private suspicion related to the central bank type. Perhaps, the cost of being above the target varies as from the announcement stage if the economy is hit by an adverse shock and *the uncertainty about the future inflation may be present in the policymaker's office too*.

The model considered here is the same as the previous one, but with only one type of policymaker, which is common knowledge. Instead of being a real number, the cost of not fulfilling the target (k) is now uniformly distributed on the support $\left[\underline{K}, \overline{K}\right]$ and it is drawn after the public's expectations have been formed. Actual inflation is chosen at the end of the period. A low realization of k can be viewed as a shock that decreases the value of keeping the commitment without using inflation short run effects. If we set $\left(\overline{K}=0\right)$ or $\left(\underline{K}=A^2\right)$ the equilibria can be computed as follows: with $(\alpha^*=0)$ and discretionary inflation rate, or with $(\alpha^*=1)$ and zero inflation rate, respectively. To keep attention on the intermediate case where $\alpha^* \in (0,1)$, we assume that k is drawn from $U[0,B]$, with $B>0$[4]. Depending on the values of k and π_a the commitment is realized or not. The credibility is given by:

$$\begin{aligned}\alpha(\pi_a) &= prob\left(w_a(k,\pi_a)>0\right)\\ \alpha(\pi_a) &= \max\left\{1-\frac{1}{B}\left[A\left(A-\pi_a\right)-\frac{A^2}{2}+\frac{\pi_a^2}{2}\right];0\right\}\end{aligned}$$

When choosing the target, the policymaker understands that the higher is its level, the more credible its policy tends to be. In particular, only A-inflation commitment is fully credible.

As in the previous model, because the possibility of the cost of not fulfilling the target being positive, the commitment is heard. Thus, commitment drives expectations and adds value to the economy. But now we have a different answer to the following question: *"what should be the optimal target* π_a^* *?"*. On the one hand, for a given (α), the closer to zero the target announcement is, the lesser is the expected inflation since (π_a) drives it. This fact increases welfare to any fixed $(\alpha \neq 0)$. But on the other hand, the closer the target announcement to zero is, the closer to the zero (or equal) the credibility (α) is.

With this background in mind we define the economy with only one type of policymaker; given by the parameter (A) and the common knowledge distribution of k~U $[0,B]$; with two positive parameters (A,B). The following proposition characterizes the equilibrium:

Proposition 1 *For any economy* $(A>0, B>0)$*, the equilibrium target* (π_a^*) *exists, it is unique, and it is in the interior of the set* $[0,A]$. *If we solve the original Cukierman-Liviatan model for* $\alpha^*\left(\pi_a^*\right)$, *its solution* (π_a^{**}) *is lower than* (π_a^*) *.Proof: Appendix.*

A less ambitious target improves credibility in the announcement and induces positive welfare effect for economies where the policymaker is not able to set "fully-credible" commitment. Then,

[4] $\underline{K}=0$ and $\overline{K}=B>0$.

when setting targets, the policymaker must be aware that the announcement effect on expectations is reduced by credibility **and** that the announcement itself affects credibility. We present in Figure 1 the optimal target announced when considering the credibility effect versus the one announced when this effect is neglected. Note that, the weaker the ability in pre commitment (lesser B) is, the higher the difference between the two announcement values is[5].

3.1 Self-fulfilling Inflation

One possible reason to assume some cost for not fulfilling the target may come from the fact that the public may use the policymaker's decisions to learn and to compute expectations. In this sense, to fail to fulfill the target is punished by credibility loss in the subsequent target announcement. With such assumptions, credibility (α) may also affect the policymaker's incentive to defend the target. To show this we use the previous framework but considering infinite periods, discount factor equals to $\beta \in (0,1)$ and $B = 0$. In the first period, the policymaker decides one fixed target for inflation (π_a). Given the target, the *initial* public expectations (α) are formed. Then, actual inflation is picked-up by the policymaker at the end of each period. We also assume that the next period's credibility (α) is the same if the inflation matches the target, or falls permanently from (α) to (0) otherwise. This fall is the penalty for not fulfilling the target and an explicit positive cost (k) is not required for the target to be fulfilled.

The initial payoff (W_o) and the payoff for the deviation (D) from the target, given (π_a, α), equals to:

$$W_o(\pi_a, \alpha) = \max_{\{\pi_t\}_t} \sum_{t=0}^{\infty} \beta^t \left\{ A\left[\pi_t - \pi_{t,e}(\alpha, \pi_{t-1}, \pi_a)\right] - \frac{\pi_t^2}{2} \right\}$$
$$\pi_{t,e}(\alpha, \pi_{t-1}, \pi_a) \equiv \begin{cases} A & ;\ \text{if}\ \pi_{t-1} \neq \pi_a \\ \alpha\pi_a + (1-\alpha)A; & \text{otherwise.} \end{cases}$$
$$D(\pi_a, \alpha) = A\alpha\left[A - \pi_a\right] - \frac{A^2}{2(1-\beta)}$$

The payoff of reaching the target is given by:

[5] On the other hand, as (B) increases the credibility (α^*) becomes closer to one and the difference between π_a^* and π_a^{**} is smaller.

$$W(\pi_a, \alpha) = \frac{\beta(1-\alpha)D - A(1-\alpha)\left[A - \pi_a\right] - \frac{\pi_a^2}{2}}{[1-\beta\alpha]}$$

It follows that:

$$w_a(\pi_a, \alpha) = \frac{A}{(1-\beta\alpha)}\left\{\frac{A}{2} - \frac{\pi_a^2}{2A} - (A - \pi_a)(1-\beta\alpha)\right\}$$

It is easy to check that if the policymaker is sufficiently concerned about the future $\left(\beta > \frac{1}{2}\right)$, then $w_a(\pi_a, 1)$ is not negative and $w_a(\pi_a, 0)$ is negative for any $\pi_a \in [0; A)$. In other words, if this economy is initially populated by very optimistic agents ($\alpha = 1$), the policymaker announces and implements the zero inflation rate. If this economy is initially populated by very pessimistic agents ($\alpha = 0$), the unique equilibrium for inflation is the discretionary one. Commitment enforcement will depend on the credibility *selected.*

If we consider some exogenous distribution for the initial credibility (α), the zero target would not be fulfilled for any α smaller than $\frac{1}{2\beta}$. Considering some $\alpha-$value for the exogenous credibility, the optimal target obtained, i.e. $A(1-\alpha)(1-\beta\alpha)$, is also decreasing in the exogenous credibility as in the Cukierman-Liviatan model. As β goes to zero the optimal target goes to $A(1-\alpha)$, which is the optimal target attained using the Cukierman-Liviatan model.

Although we have made a strong hypothesis that the policymaker cannot improve the credibility path and that "not fulfilling" is punished with zero credibility, this very simple framework shows that the value of respecting the target may be increasing in the current credibility whenever benefits from "keeping the target decisions" are computed in a much slower way than the credibility loss associated with "not keeping the target decisions". This feature may open the door for confidence crises and self-fulfilling inflations.

Back to the three-stage framework, we define the economy (ξ) with the following set of parameters $\{(A > 0); (\pi_a \in [0, A]); (\epsilon \geq 0); (n \in \mathbb{R})\}$ plus the increasing function $h(\alpha)$, which are all common knowledge. The policymaker type is unique and is the same as in the first-proposition model except for the cost of "not keeping the target" function, now defined as follows: $c(\pi \neq \pi_a) \equiv h(\alpha) + k$. k is a random variable distributed according to $U\left[n-\epsilon, n\right]$, (α) is the endogenous credibility that solves $\alpha = prob(\pi = \pi_a|\alpha)$, and $h\left(.\right)$[6] measures how much the cost of not keeping the target depends on the public expectations. The timing of actions is the same: the target is announced, expectations are formed, uncertainty (k) is solved and actual inflation is implemented. With such assumptions, both fundamental and expectations shocks may be important to compute the policymaker's incentives in the choosing actual inflation, since the welfare gain (w_a) from keeping the target can be computed as follows:

$$w_a(k,\alpha) = k - x + h(\alpha)$$
$$x \equiv A(A-\pi_a) + \frac{\pi_a^2}{2} - \frac{A^2}{2}$$

It is always possible to reach an equilibrium for any economy (ξ) and it may be possible to reach more than one. When the uncertainty about the future policymaker's incentives is high, i.e. $(\epsilon > h\left(1\right) - h\left(0\right))$, we denote the economy (ξ) as (ξ^u), otherwise we denote the economy (ξ) as (ξ^m). The following proposition characterizes the equilibrium:

Proposition 2 *The economy ξ always admits an equilibrium. For ξ^u type economy it is possible that: (i) $x \leq (n-\epsilon+h\left(1\right))$ and only perfect commitment ($\alpha = 1$) equilibrium is possible, (ii) $x \in (n-\epsilon+h\left(1\right), n+h\left(0\right))$ and only imperfect commitment is possible ($\alpha \in$(0,1)), (iii) $x \geq n+h\left(0\right)$ and only discretionary ($\alpha = 0$) equilibria is possible. For (ξ^m) type economy it is possible that: $(i) x \leq (n-\epsilon+h\left(1\right))$ and perfect commitment equilibrium is possible, $(ii) x \in (n+h\left(0\right), n-\epsilon+h\left(1\right))$ and perfect commitment, imperfect commitment and discretionary equilibria are possible, (iii) $x \geq n+h\left(0\right)$, and discretionary equilibria is possible.*
Proof: Appendix.

According to this proposition, if there is too much uncertainty concerning future policymaker's incentives $(\epsilon > h\left(1\right) - h\left(0\right))$, then the equilibrium is unique: perfect commitment for very strong

[6] In order to reach a simple characterization of the equilibrium we also assume that $h:[0,1] \to \mathbb{R}$ is linear.

policymaker, discretionary for very weak policymaker and imperfect commitment otherwise. When ϵ is sufficiently large, there is no room for self-fulfilling inflation.

On the other hand, if the region for the possible policymaker's incentives shrinks (ϵ decreases), the uniqueness remains only for a very strong or a very weak policymaker. The intuition is that some economies may be subject to multiple equilibria when the decision about respecting the target or not depends much on the credibility (α) before the k's assortment. In such case, to relax (to increase) the target for inflation may have two welfare effects. First, and the new one, it is possible that only perfect commitment equilibrium remains when the target is increased. Second, as long as $h\prime(.) > \epsilon$, the *critical* k^*[7] tends to be greater and the state region for good expectations shrinks when the target is increased. Then, the announcement credibility may be increasing in the target or not.

As we have shown in the Figure 2, when $h(\alpha) \equiv \rho\alpha$ and multiple equilibria are possible, to increase the target *may* be a good deal if it avoids multiplicity. But this decision also depends on the policymaker's willingness to avoid a confidence crisis and on the probability of each equilibrium to be selected. Because of the coordination failure, any of them could be the one and, unfortunately, this model can not help us compute their likelihood.

Such difficulty is usually avoided by the definition of an arbitrary sunspot variable[8] that would allow us to compute expected welfare for each target. Obviously, the policy recommendations would be very different depending on the assumptions about the sunspot distribution and depending on the policymaker's willingness to avoid crises.

In the following section, we consider the public as a strategic player. Up to this point its role has been to process information, to form beliefs concerning the policymaker's incentive and to compute inflationary expectations. Adding strategies and payoff structure to private agents (public) and assuming an exogenous information structure, we can appraise the coordination aspect in a different way. The coordination motive arises from the strategic complementarity in public actions.

[7] k^* solves: $k = x - h\left(\frac{n-k}{\epsilon}\right)$.

[8] See Cole and Kehoe [4] for coordination failure in the public debt market.

4 Self-Fulfilling Inflation with Imperfect Information

The economy (ξ) is defined as a one-shot game with two stages, the function $h(\alpha) = \alpha\rho$ and the following set of parameters $\{(A > 0), (\rho > 0), (\pi_a \in [0, A]), (c > 0), (\sigma > 0), (\sigma_p > 0)\}$. The last two of those define information structure as we describe next. In the last stage of the game the policymaker chooses the actual inflation after observing the speculative actions $(1 - \alpha)$. The policymaker keeps the inflation equal to the target (π_a) if and only if $(w_a \equiv k + \rho\alpha - x(\pi_a) \geq 0)$ [9]. Otherwise it inflates at level A. k is drawn in the beginning of the game from the support of the improper uniform (over the entire real line), but its value is not observed directly by the public (speculators).

The population of speculators is continuous and normalized to unit. Each speculator (j) may set α^j equal to one or zero. If he sets α^j equal to zero he believes that the target will probably[10] not be reached. With some cost, he speculates based on his beliefs (buying foreign currency, for example). If he sets α^j equal to one he believes that the target will probably be reached. In this case, he does not bet against the policymaker (keeping savings denominated in local currency, for example). Then, the size of the attack (α) is given by $prob\,(\alpha^j = 1)$. Each (j) payoff is defined as being equal to $(1 - \alpha^j)(g_s - c)$. The speculative gain g_s depends on the policymaker's response. If the target is sustained, then $g_s = g_a$, otherwise $g_s = g_A$, where $(g_A > c > g_a)$. With this payoff structure, to speculate is a good deal only when the target is abandoned since $(g_A - c > 0)$ and $(g_a - c < 0)$. The incentive to attack tends to be increasing in the size of the attack. As we have argued, in some economies, the target may be abandoned when credibility is low. Note that, the greater the size of the attack is, the lower the credibility is.

To keep our framework as close as possible to the one proposed in Angeletos and Werning [1] we define $g_A \equiv 1$ and $g_a \equiv 0$ and consider that the strength of the status-quo k is not common knowledge. Instead of observing the realization of the k-value, each player (j) observes the public signal (s^p) and

[9] $x = A(A - \pi_a) + \frac{\pi_a^2}{2} - \frac{A^2}{2}$

[10] With probability higher than (c).

the private signal (s^j),

$$s^j = k + \sigma\varepsilon_j \,;\, \sigma > 0 \text{ and } \varepsilon_j \sim N(0,1)$$
$$s^p = k + \sigma_p\varepsilon_p \,;\, \sigma_p > 0 \text{ and } \varepsilon_p \sim N(0,1)$$

(ε_j) is assumed to be independent of (k) and $(\varepsilon_{j\prime})$ for all $j\prime \neq j$. (ε_p) is also assumed to be independent of (k) and (ε_j). $N(0,1)$ denotes the standard normal distribution.

4.1 The Equilibrium

Results are based on monotone equilibria defined as perfect Bayesian. For each public signal, the agent (j) attacks if and only if its private signal (s^j) is less than some threshold $s^*(s^p)$. The mass of agents that ends up attacking is given by:

$$prob(s^j < s^*(s^p, \pi_a) \,|s^p, k, \pi_a) = \Phi(\frac{s^*(s^p, \pi_a) - k}{\sigma}) = 1 - \alpha$$

where $\Phi(.)$ denotes the cumulative distribution function for the standard normal. The policymaker will sustain the target if and only if k is greater than k^*, which is given by:

$$k^*(s^p, \pi_a) = x + \rho.\Phi(\frac{s^*(s^p, \pi_a) - k^*(s^p, \pi_a)}{\sigma}) - \rho$$

The expected payoff from attacking must be equal to zero whenever $s^j = s^*(s^p, \pi_a)$[11], which implies the following indifference condition:

$$\sqrt{\tau}.\Phi^{-1}(c) = k^*(s^p, \pi_a) - \frac{\tau s^*(s^p, \pi_a)}{\sigma^2} - \frac{\tau s^p}{\sigma_p^2}, \text{ where } \tau = \frac{\sigma_p^2\sigma^2}{\sigma_p^2 + \sigma^2}$$

Which after replacing $s^*(k^*)$ becomes:

$$\Phi^{-1}\left(\frac{k^* + \rho - x(\pi_a)}{\rho}\right) = \frac{\sigma}{\sigma_p^2}[k^* - s^p] + \frac{\sigma}{\sqrt{\tau}}\Phi^{-1}(1-c)$$

It is always possible to find at least one $k^* \in [x - \rho, x]$ that solves this equation and this solution will be unique for every public signal (s^p) if and only if $\sigma \in (0, \frac{\sigma_p^2\sqrt{2\pi}}{\rho}]$.

[11] $s^*(s^p, \pi_a) = \sigma.\Phi^{-1}\left(\frac{k^*(s^p,\pi_a)+\rho-x}{\rho}\right) + k^*(s^p, \pi_a)$

For any (positive) doubt related to the public signal, σ_p, uniqueness is ensured by a sufficiently small (positive) doubt related to the private signal, σ. That is the first proposition from Angeletos-Werning [1] and states that the multiplicity may vanish when the common knowledge is perturbed, as in Morris and Shin [6]. This result always holds for some exogenous information structure because precise private information anchors individual behavior and makes it difficult to predict the actions of others. Under the reasonable assumption that the improvement in the private signal implies improvement in the public signal, it is possible that public information becomes more precise faster than the private one, and so multiplicity may still exist even for very small common knowledge perturbation ($\sigma \to 0$). In this case, the public signal drives individual behavior more than the private signal, motivating mass movements.

Keeping the exogenous information structure it is possible to set multiple-equilibria economies (ξ^m) assuming that $\sigma > \frac{\sigma_p^2\sqrt{2\pi}}{\rho}$ and unique-equilibrium economies (ξ^u) assuming that $\sigma \in (0, \frac{\sigma_p^2\sqrt{2\pi}}{\rho}]$.

Proposition 3 *For (ξ^u) type economy, higher target increases the commitment credibility. For (ξ^m) type economy, higher target may turn the commitment more credible or not. The effect on the credibility will depend on the likelihood of each equilibrium $k^*(s^p)$ to be selected. Proof: Appendix.*

The key intuition is that when the target is increased, the shock required for the commitment to be abandoned becomes greater (smaller $k-$realization), for a given (α). This fact inhibits attacks and adds credibility. On the other hand, as the policymaker sets a higher target for inflation, new attack-strategies (or beliefs) are settled and this fact may increase the attack mass (decrease the credibility). In this case, a higher target gives more room for above the target inflation. The first effect is always preponderant for (ξ^u) type economy.

For (ξ^m) type economy, the first effect tends to be preponderant when extreme equilibria are selected (the $k^*(s^p)$ closest to x (or to $x - \rho$)). Note that, when strategies are too optimistic or too pessimistic, the size of an attack is closer to zero or to one, respectively. For more pondered strategies, based on the not-extreme equilibrium $k^*(s^p)$, the attack-mass and the no-attack-mass are both significant. Then, enlargement in the attack size induced by more aggressive strategies is relevant and critical k becomes greater for higher targets (see Figure 3). If not-extreme equilibrium tends to be the selected one in (ξ^m) type economy, to relax the target in order to get more credibility may be a good idea only if multiplicity is avoided in many states (s^p). Otherwise, the result would be more strength to speculative movement and the commitment enforcement would be reduced.

4.2 Central Bank Transparency

A lower (σ_p or ϵ) value may be viewed as more central bank transparency. However, according to our results, more precise public information may open the door for bad equilibrium, contrary to the conventional wisdom that more transparency is always good in an inflation targeting framework. Some other papers have argued in the same direction, but based on different models. In Metz [3], more precise public information increases the likelihood of currency crises in case of bad fundamentals. Morris and Shin ([7] and [8]) have pointed out that welfare effect of increased public disclosures is ambiguous and that there is a dilemma between managing market prices and learning from market prices. They also conclude that when a Central Bank cannot actually control inflation, the inflation targeting regime could fail and undermine credibility. In this sense, it would be better for the central bank to simply forecast inflation and point out the extent to which its forecasts are contingent on fiscal policy. Our results suggest that inflation targeting may be a good set-up whenever the policymaker can actually control *some* level of inflation in *some* states of nature.

5 Concluding Remarks

Using different theoretical approaches, we appraise how the target level for inflation should be set in the presence of uncertainty about the ability in precommiting.

First, ruling out confidence crises, we conclude that less ambitious target for inflation increases credibility in precommitment. This effect makes optimal target bigger than the target level selected when considering the same economy, except for the fact that the credibility effect is not computed[12].

When confidence crises are possible, multiple equilibria may come up. In this case, to set higher targets for inflation may stimulate above the target inflation and reduce the policymaker credibility. On the other hand, multiple (bad) equilibria may be avoided. The optimal target will depend on the likelihood of each equilibrium to be selected and on the policymaker's willingness to avoid a confidence crisis.

We restore uniqueness breaking common knowledge with exogenous information structure. In this case, it is possible to ensure again that a higher target for inflation increases credibility in precommitment.

Finally, adding a *precise* public signal, confidence crises and equilibrium multiplicity may still exist even for a small lack of common knowledge, as in Angeletos and Werning ([1]). Thus, more precise public information may open the door for bad equilibrium, contrary to the conventional wisdom that more central bank transparency is always good news when considering the inflation targeting regime. In multiple equilibria case, to set higher targets for inflation may stimulate above the target inflation and reduce the policymaker credibility. On the other hand, multiple (bad) equilibria may be avoided. Depending on the characterization of the *target strength* uncertainty, it may be optimal to have an *ideal* status-quo (socially optimal target for inflation) or a *more defensible* one (higher target). The optimal target will also depend on the policymaker's willingness to avoid a confidence crisis.

[12] As in Cukierman and Liviatan ([5]).

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6 Figures

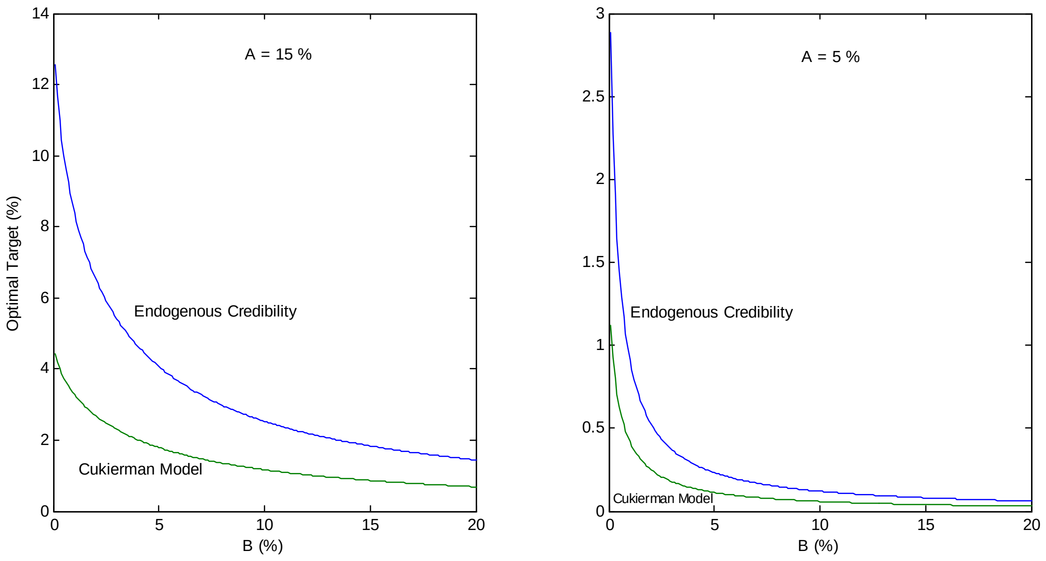

Figure 1: Optimal Targets

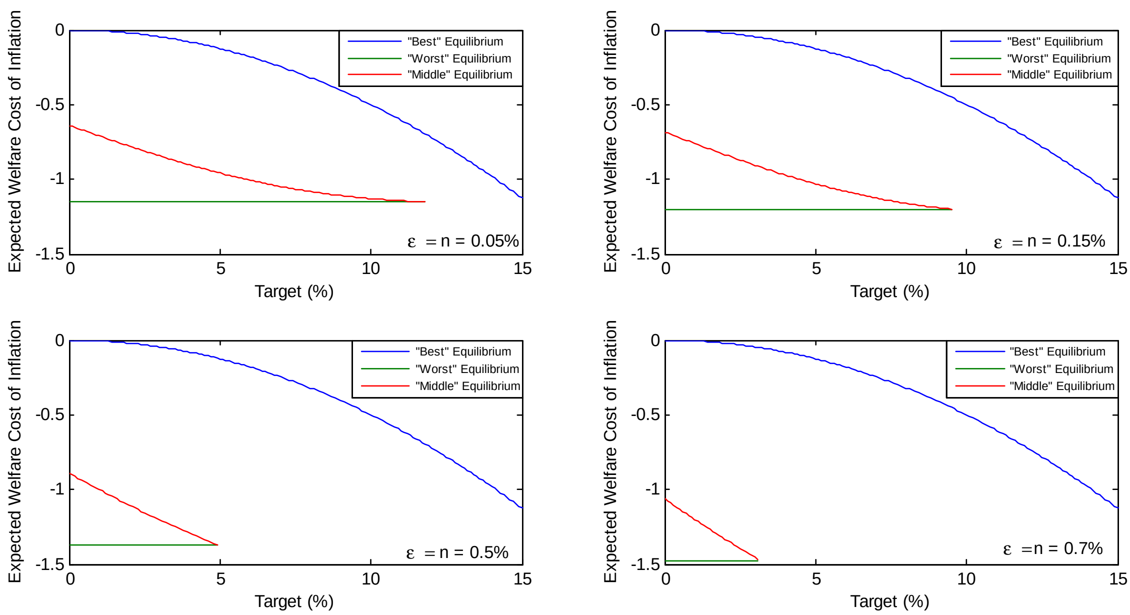

Figure 2: Self-fullfiling Equilibria (A = 15% ; $\rho = 2.5\%$)

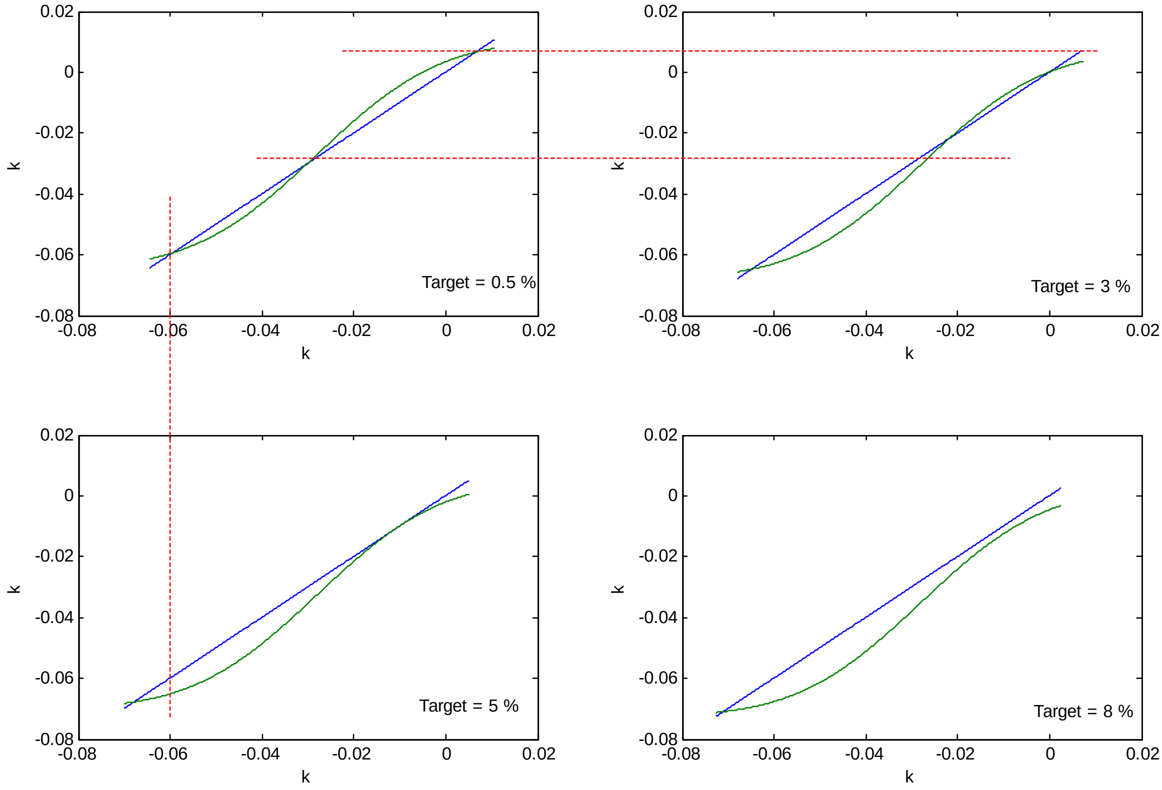

Figure 3: No-Common Knowledge Equilibrium $k^*(s^p = 0)$ $\left(A = 15\%, \frac{\sigma}{\sigma_p} = 15, c = .5, \rho = 7.5\%,\right)$

7 Appendix

Proof. Proof of proposition 1: The policymaker from economy (A, B) solves the following problem:

$$\pi_a^* = \arg\max_{\pi_a \epsilon [0,A]} E\ [v(\pi_a, k)]\,;\, k \sim U[0, B]$$
$$v(\pi_a, k) = \max_{\pi \geq 0} \left[A(\pi - \pi_e(\pi_a)) - \frac{\pi^2}{2} - c(\pi_a, \pi, k) \right]$$
$$c(\pi_a, \pi, k) = \{ {}^{0 \text{ if } \pi_a = \pi}_{k \text{ if } \pi_a \neq \pi}$$

and it is easy to check that,

$$\pi_e = \alpha \pi_a + (1 - \alpha) A$$
$$\alpha(\pi_a) = \max \left\{ 1 - \frac{1}{B} \left[A(A - \pi_a) - \frac{A^2}{2} + \frac{\pi_a^2}{2} \right]; 0 \right\}$$
$$E[k | \pi_a \neq \pi] = \frac{A(A - \pi_a)}{2} + \frac{\pi_a^2}{4} - \frac{A^2}{4}$$
$$\alpha(\pi_a^*) > 0$$

It follows that:

$$\pi_a^* = \arg\max_{\pi_a} \frac{1}{B} \left[B + A\pi_a - \frac{\pi_a^2}{2} - \frac{A^2}{2} \right] \left(\frac{3A^2}{4} - \frac{A\pi_a}{2} - \frac{\pi_a^2}{4} \right) - \frac{3A^2}{4} - \frac{\pi_a^2}{4} + \frac{A\pi_a}{2}$$

The equilibrium, π_a^*, must solve:

$$u(\pi_a^*) = v(\pi_a^*)$$
$$u(\pi_a) \equiv \frac{B}{2}(A - \pi_a)$$
$$v(\pi_a) \equiv \left(\frac{A + \pi_a}{2} \right) \left(B + A\pi_a - \frac{\pi_a^2}{2} - \frac{A^2}{2} \right) - \left(\frac{3A^2}{4} - \frac{A\pi_a}{2} - \frac{\pi_a^2}{4} \right)(A - \pi_a)$$

Since $\left[\frac{AB}{2} - A^3 = v(0) < u(0) = \frac{AB}{2}\right], [AB = v(A) > u(A) = 0]$ and $[u\prime(.) < 0;$ and $v\prime(.) > 0\, \forall (\pi_a \leq A)]$, then the equilibrium $\pi_a^* \in (0, A)$ exists and is unique for any $(A > 0, B > 0)$. Now, defining $D(\pi_a^*) \equiv \pi_a^* - A(1 - \alpha(\pi_a^*))$, it is easy to check that $sign(D(\pi_a^*)) = sign\left(2B\pi_a^* + 2A^2\pi_a^* - A^3 - A(\pi_a^*)^2\right)$. $D(.)$ is positive for $\pi_a = A$ and negative for $\pi_a = 0$. Next we will show that $sign(D(\pi_a^*))$ is always positive, since

π_a^* is lower bounded by some positive value and $D\prime(.)$ is positive for $\pi_a \epsilon\, [0, A]$. From $sign\,(u(\pi_a) - v(\pi_a)) = sign\left((\pi_a)^3 + 2A^3 - \pi_a\,(3A^2 + 2B)\right)$ we conclude that π_a^* solves $(\pi_a^*)^3 + 2A^3 - \pi_a^*\,(3A^2 + 2B) = 0$. Then, we can set $\pi_a^* = \frac{2A^3 + (\pi_a^*)^3}{(3A^2+2B)}$. Since $D(\pi_a^*)$ is positive whenever $\pi_a^* \geq \frac{A^3}{2B+A^2}$, we must check if $\left(\frac{2A^3+(\pi_a^*)^3}{(3A^2+2B)} \geq \frac{A^3}{2B+A^2}\right)$ holds. It is easy to check that it holds for any $\left(B \geq \frac{A^2}{2}\right)$. Now, if we set $\pi_a = \frac{A^3}{B}$, we have $sign\,(u(\pi_a) - v(\pi_a)) = sign\left(\frac{A^4}{B^2} - 3\right)$. We conclude that $\frac{A^3}{B}$ is a lower bound to π_a^* whenever $\left(B < \frac{A^2}{\sqrt{3}}\right)$. In this case, $\pi_a^* \geq \frac{A^3}{B} \geq \frac{A^3}{2B+A^2}$ and $D(\pi_a^*) > 0$ again. Since $\left(\frac{A^2}{\sqrt{3}} > \frac{A^2}{2}\right)$, we conclude that $\pi_a^* \geq A\,(1 - \alpha\,(\pi_a^*))$ for any $(A > 0, B > 0)$.

■

Proof. Proof of proposition 2: The target is fulfilled whenever $w_a = k + h\,(\alpha) - x \geq 0$, with $x\,(\pi_a) \equiv \left[A(A - \pi_a) + \frac{\pi_a^2}{2} - \frac{A^2}{2}\right]$. The region for which the target (π_a) may induce multiple equilibria expectations is given by the interval $[K^d, K^u]$, where:

$$
\begin{aligned}
K^u(\pi_a, \underline{\alpha}) &= \inf\left\{k \in \mathbb{R} |\,(-x + k + h\,(\underline{\alpha})) \geq 0\right\} = x(\pi_a) - h\,(\underline{\alpha}) \\
K^d(\pi_a, \overline{\alpha}) &= \sup\left\{k \in \mathbb{R} |\,(-x + k + h\,(\overline{\alpha})) \leq 0\right\} = x(\pi_a) - h\,(\overline{\alpha}) \\
\overline{\alpha} &= \min\left\{\frac{n - K^d}{\epsilon}; 1\right\} \text{ if } [K^d, K^u] \cap [n - \epsilon, n] \neq \phi \\
\underline{\alpha} &= \max\left\{\frac{n - K^u}{\epsilon}; 0\right\} \text{ if } [K^d, K^u] \cap [n - \epsilon, n] \neq \phi \\
\overline{\alpha} &= \underline{\alpha} = 0 \text{ if } K^d > n \\
\overline{\alpha} &= \underline{\alpha} = 1 \text{ if } K^u < n - \epsilon
\end{aligned}
$$

There are five possible cases for the "$[K^d, K^u]$-position" related to the support $[n - \epsilon, n]$, as follows:

$Case$	$Exist \Leftrightarrow$	K^d	K^u	$Equilibrium$
1	$x \in [n + h(0), n - \epsilon + h(1)]$	$\in [n - \epsilon, n]$ and $K^u = K^d$		$\alpha \in [0, 1]$
2	$x \in [n + h(0), n - \epsilon + h(1)]$	$\in [n - \epsilon, n]$	$> n$	$\alpha \in [0, 1]$
3	$x \in [n + h(0), n - \epsilon + h(1)]$	$< n - \epsilon$	$\in [n - \epsilon, n]$	$\alpha \in [0, 1]$
4	$n - \epsilon + h(1) > x$	$K^u < n - \epsilon$		$\alpha = 1$
5	$n + h(0) < x$	$K^d > n$		$\alpha = 0$

,

considering $(h(1) - h(0) \geq \epsilon)$. Otherwise, cases 2 and 3 do not exist and for cases 1, 4 and 5 we set $x \in [n - \epsilon + h(1), n + h(0)]$ instead of setting $x \in [n + h(0), n - \epsilon + h(1)]$. ■

Proof. Proof of proposition 3:

Since $\left(\frac{dx}{d\pi_a}\right) = \pi_a - A$, to relax (to increase) target means to reduce x. From $\Psi(k^*, x) \equiv \Phi^{-1}\left(\frac{k^*+\rho-x}{\rho}\right) - \frac{\sigma}{\sigma_p^2}[k^*] = -\frac{\sigma}{\sigma_p^2}s^p + \Phi^{-1}(1-c)\frac{\sigma}{\sqrt{\tau}}$ we conclude that $\Psi(., x)$ is increasing in k^* for every s^p if $\frac{\sigma\rho}{\sigma_p^2} \leq \sqrt{2\pi}$. Reduction in x must be compensated by reduction in k^* in order to keep $\left[-\frac{\sigma}{\sigma_p^2}s^p + \Phi^{-1}(1-c)\frac{\sigma}{\sqrt{\tau}} = \Psi(k^*, x)\right]$ valid. The region over the $\widetilde{k}-$support where the target is fulfilled increases for all (s^p) and the size of attack decreases with the decreasing in $s^*(s^p)$.

$\Psi(., x)$ will be decreasing in k^* for *some* possible equilibrium $\overline{k}^*(s^p)$ whenever $\frac{\sigma\rho}{\sigma_p^2} > \sqrt{2\pi}$. In this case, reduction in x must be compensated by an increasing in $\overline{k}^*$ in order to keep $\left[-\frac{\sigma}{\sigma_p^2}s^p + \Phi^{-1}(1-c)\frac{\sigma}{\sqrt{\tau}} = \Psi(\overline{k}^*, x)\right]$ valid. So, an increase in the target may imply an increasing in $\overline{k}^*$, $s^*\left(\overline{k}^*\right)$, and an increase in the size of attack. ■