

# Persistent Appreciations and Overshooting: A Normative Analysis

Ricardo J. Caballero and Guido Lorenzoni\*

MIT and NBER

December 9, 2006

## Abstract

Most economies experience episodes of persistent real exchange rate appreciation, when the question arises whether there is a need for intervention to protect the export sector. In this paper we present a model of irreversible export destruction where exchange rate stabilization may be justified if the export sector is financially constrained. However the criterion for intervention is not whether there are bankruptcies or not, but whether these can cause a large exchange rate overshooting (and real wage decline) once the factors behind the appreciation subside. The optimal policy often involves a mild initial intervention followed by an increasingly aggressive stabilization as the appreciation persists and the financial resources of the export sector dwindle. In some instances, the policy also involves an exacerbation (but shortening) of the initial overshooting during the depreciation phase. On the methodological front, the solution approach should be useful in other optimal dynamic taxation problems with financially constrained agents.

**JEL Codes:** E0, E2, F0, F4, H2.

**Keywords:** Appreciations, overshooting, financial frictions, irreversible investment, pecuniary externality, real wages, optimal policy, exports, Dutch disease.

---

\*Respectively, caball@mit.edu and glorenzo@mit.edu. We are grateful to Arvind Krishnamurthy, Enrique Mendoza and seminar participants at MIT, IFM-NBER, WEL-MIT and the World Bank for their comments, and to Nicolas Arregui and, especially, Pablo Kurlat for excellent research assistance. Caballero thanks the NSF for financial support. First draft: September 2006. This paper circulated previously as “Persistent Appreciations, Overshooting, and Optimal Exchange Rate Interventions.”

# 1 Introduction

Most economies experience episodes of large real exchange rate appreciations. There are many factors with the potential to fuel these appreciations. For example, they stem from domestic policies aimed at taming a stubborn inflationary episode, from the absorption of large capital inflows caused by domestic and external factors, from exchange rate interventions in trading partners, from a sharp rise in terms of trade in commodity producing economies or, in its most extreme form, from the discovery of large natural resources wealth (the so-called Dutch disease).

While there are idiosyncrasies in each of these instances, the common policy element is that, when persistent enough, the question arises whether there is a need for intervention to protect the export sector (“competitiveness” policies). This widespread concern goes beyond the purely distributional aspects associated to real appreciations. The fear is that somehow the medium and long run health of the economy is compromised by these episodes. If this concern is justified, should policymakers intervene and stabilize the exchange rate before is too late? At which stage of the appreciation cycle should a potential intervention take place? More generally, how does the optimal policy look like?

In this paper we propose a framework to address this common policy element. We present a dynamic model of entry and exit in the export sector where entrepreneurs face financial constraints and exchange rate stabilization may be justified. We argue that not all appreciations are equal and hence intervention should be selective. On one extreme, if the appreciation is short lived the export sector can handle it with its own resources and hedges. On the other extreme, if the appreciation is expected to be long lived, it is optimal to let the export sector contract and reallocate resources toward nontradables. Thus neither of these extremes justifies intervention. The problem lies in the medium run, with temporary but persistent appreciations.<sup>1</sup> These can drain private sector resources and hence lead to bankruptcies and, most importantly, hamper the economy’s ability to deal with the often hard phase that follows a sustained appreciation.

We show that when financial constraints damage the export sector’s ability to recover, the economy experiences a large exchange rate overshooting (real wage decline) once the factors behind the appreciation subside and nontradable demand contracts. The overshooting results

---

<sup>1</sup>Goldfajn and Valdes (1999), using a large panel of countries, document that the average appreciation episode (defined as a real exchange rate that exceeds its long-run “equilibrium” value by at least 15 percent) lasts between two and three years.

from the export sector's inability to absorb the resources (labor) freed from the contraction in nontradables demand. This overshooting amplifies the fall in the price of non-tradables and real wages (both measured in terms of tradables), which is costly to consumers and workers.<sup>2</sup> There is a connection between the severity of the overshooting and the extent of the contraction in the export sector during the preceding appreciation phase. If consumers were to reduce their demand for non-tradables in this phase, then there would be less destruction ex-ante and a faster recovery ex-post. However, rational atomistic consumers ignore the effect of their individual decisions during the appreciation phase on the extent of the overshooting during the depreciation phase. It is this pecuniary externality that justifies and informs policy intervention in our framework.

Our economy starts by transiting into an appreciation phase, which it exits into an absorbing depreciation phase with a Poisson probability. In the first and main part of the paper we assume that agents face complete markets. For any given expected duration, this simple setup allow us to characterize several regions of interest, indexed by the financial resources of export sector entrepreneurs at the onset of the appreciation. When financial resources are plentiful, the economy reaches the first best as real exchange rates (and real wages) are pinned down by purely technological free entry and exit conditions, and hence are orthogonal to consumers' actions. At lower levels of financial resources, financial constraints may become binding during the appreciation phase, the depreciation phase, or both. If they are only binding during the appreciation phase, then the economy experiences bankruptcies but the recovery of the export sector is swift once the depreciation phase starts and the exchange rate is again pinned down by purely technological factors. In contrast, if the financial constraint is binding during the depreciation phase, the recovery of the export is slow and the real exchange rate depreciation overshoots the long run depreciation.<sup>3</sup>

For the optimal policy analysis we adopt the perspective of a social planner that seeks to maximize consumers' (workers') welfare, subject to not worsening entrepreneur' welfare and to their financial constraints. We rule out direct transfers across groups (as these are limited by a series of informational factors in practice) and focus instead on real exchange rate interventions. By the latter we mean interventions that change consumers' choices

---

<sup>2</sup>In practice the drop in the (real) price of non-tradables and real wages often takes the form of a sharp nominal depreciation which is not matched by a substantial rise in the nominal price of nontradables and wages. See Goldfajn and Valdes (1999) and Burnstein et al (2005).

<sup>3</sup>Alternatively, a financial constraint during the appreciation phase combined with a technological friction on the speed of recovery during the depreciation phase, yields similar (but not identical) policy insights. We return to this issue in the conclusion.

and affect the entrepreneurial sector through their effect on equilibrium prices. From this perspective, we show that consumers gain from stabilizing the appreciation whenever this leads to a faster recovery of the export sector once the appreciation subsides. The gain derives from the increase in real wages associated to a faster reconstruction of the export sector. It follows from this observation that no policy intervention is justified if there is no overshooting in equilibrium. Even if the economy experiences bankruptcies during the appreciation, collectively consumers would not be willing to distort their decisions during the appreciation if such action has no impact on their wages during the appreciation phase.

Even when overshooting is expected, there are limits on how much intervention is desirable during the appreciation phase as this comes at the cost of a consumption distortion.<sup>4</sup> This trade-off means that in some instances the optimal policy carries over to the depreciation phase, leading to the somewhat paradoxical result that the social planner may want to exacerbate the extent of the initial overshooting. It does so because by causing a sharp depreciation, the policy accelerates the recovery of the export sector and hence shortens the duration of the overshooting phase. Of course, there are limits to this over-overshooting policy result as well, especially if the nontradables sector also experiences difficulties during the depreciation phase (as in the sudden stops literature for emerging markets), which we discuss in the extensions section of the paper.

Finally, we also study the incomplete markets case, although here most of the results are numerical. The main substantive issue that arises from this realistic extension is an actual (as opposed to expected) time-dimension within the appreciation phase. As time goes by, the financial resources of the export sector dwindle and the reasons for intervention during the appreciation phase rise since the expected recovery becomes harder. Thus the optimal policy often involves a mild initial intervention, followed by an increasingly aggressive stabilization as the appreciation persists. There is also a time dimension in the decomposition of the intervention, between appreciation and depreciation phases: As time goes by, the optimal policy tilts intervention toward the former.

Our paper belongs to an extensive literature on consumption and investment booms in open economies, as well as on the role of financial factors in generating inefficiencies in these booms (see, e.g. Aghion et al 2003, Gourinchas et al 2001, Caballero and Krishnamurthy 2001). The pecuniary externality that justifies intervention in our framework is similar in

---

<sup>4</sup>At least this is the case when consumers' decision are rational rather than driven by myopia or other behavioral traits, which we postpone until later in the paper.

kind to those discussed in Geanakoplos and Polemarchakis (1996), Caballero and Krishnamurthy (2001, 2004), Lorenzoni (2006), and Farhi et al (2006). Aside from its specific context, our paper differs from these in that we characterize the dynamic aspects of the problem. These dynamics are central to the issues we discuss.

In terms of its mechanism, the paper also belongs to the literature on Dutch disease. There, intervention is justified by the presence of dynamic technological externalities through learning-by-doing (see, e.g. van Wijnbergen 1984, Corden 1984, and Krugman 1987). In contrast, our paper highlights financial frictions and the pecuniary externalities that stem from these. The policy implications between these two approaches is also different: While learning-by-doing offers a justification for industrial policies as a development strategy, the financial frictions we highlight have intertemporal reallocation implications of the sort that matter for business cycle policies. In this sense, the problem we highlight is relevant not only to industrializing economies but to developed economies as well.

The method of analysis of the optimal policy in our paper resembles that of the literature on dynamic optimal taxation. In this dimension, the main methodological innovation of the paper is in our treatment of the policy restrictions implied by the financial constraints faced by a subset of the agents. Our solution method should be useful outside our particular application.

Section 2 presents a stylized complete markets model of creative destruction over appreciation and depreciation cycles. Section 3 characterizes optimal exchange rate intervention in such setup. Section 4 analyzes the additional dynamics introduced by incomplete markets, while Section 5 discusses the effect of “frictions” in consumption decisions and in the nontradable sector. Section 6 concludes and is followed by an extensive appendix.

## **2 A Simple Model of a Destructive Appreciation and Overshooting**

In this section we present a model of an economy experiencing a temporary, but persistent, real appreciation. If the export sector has large sunk creation costs, then it limits the extent of its desired contraction in order to keep capital operational and preserve the option to produce once the appreciation is over. However, this waiting strategy generates losses that require financing. If this financing is limited, the export sector experiences a larger contraction than desired. From the point of view of the economy as a whole, these excessive

contractions may compromise the recovery of the export sector once the appreciation is over, leading to a prolonged period of deep real depreciation and low wages.

## 2.1 The Environment

There is a unit mass of each of two groups of agents within the domestic economy: consumers and entrepreneurs (exporters). There are two goods: a tradable and a nontradable. Each period the consumer receives an endowment of  $e$  units of tradable and one unit of labor. The latter can be used as an input for the production of tradables or nontradables. In both cases one unit of labor produces one unit of output. In addition, the production of tradables requires setting up an export production unit, which means investing  $f$  units of tradable goods. After an export production unit has been set up, it needs to be maintained in operation, otherwise it is irreversibly shut down.<sup>5</sup> The entrepreneurs are the only agents that have access to the technology to run and maintain export units, and hence are the owners of all export units. The markets for tradables, nontradables and labor are competitive. Furthermore, there is a competitive market where entrepreneurs can trade export units among themselves. At date 0 they begin with  $n_{-1}$  open production units.

Entrepreneurs are risk neutral and only consume tradable goods:

$$\mathbb{E} \sum \beta^t c_t^{T,e}.$$

Consumers have log-separable instantaneous utility on tradables and nontradables:

$$\mathbb{E} \sum \beta^t \theta_t (u(c_t^T) + u(c_t^N))$$

where  $u(c) = \log c$  and  $\theta_t$  is a taste shock.

The taste shock is the only source of uncertainty and depends on a simple Markov process, described by the state  $s_t$ , which can take two values in  $\mathcal{S} = \{A, D\}$ . The economy begins with a transition into the “appreciation” state  $s_t = A$ , with  $\theta_t = \theta_A$ . Each period, with probability  $\pi(D|A) = \delta$ , the economy switches back to the “depreciation” state  $s_t = D$ , with  $\theta_t = \theta_D$ . Once the latter transition takes place,  $D$  becomes an absorbing state,  $\pi(A|D) = 0$ . We assume that:

$$\theta_A > \theta_D = 1.$$

---

<sup>5</sup>These assumptions capture the fact that export oriented firms often have more specific (sunk) capital and operations than less complex firms producing primarily for domestic markets. Of course there are important exceptions to this generalization. Later in the paper we discuss briefly the effect of constraints in the nontradables sector on our conclusions.

Thus, in the appreciation state the taste shock drives up consumers' demand for both tradable and nontradables, putting upward pressure on the real exchange rate (since the supply of tradables is fully elastic while that of nontradables is not – see below). In reality, the main sources of appreciations are sharp improvements in terms of trade and capital inflows. The taste shock is a convenient device to capture the increase in consumption demand that derives from these primitive shocks, without having to add additional intertemporal frictions. We will return to this issue later in the paper, once we have developed our main points.

Both groups have access to the international capital market, where they can trade a full set of state contingent securities. On each date  $t$ , agents trade one-period state-contingent securities that pay one unit of tradable good in period  $t + 1$  if state  $s_{t+1}$  is realized. The entrepreneurs holdings of securities are denoted by  $a(s_{t+1}|s^t)$ , where  $s^t = \langle s_t, s_{t-1}, \dots \rangle$  is the history of the economy up to date  $t$ . Note that our simple Markov chain yields histories that are limited to a block of periods in  $A$ , followed by  $D$ 's (there are no alternations).

The initial financial positions of entrepreneurs is equal to  $a_0$ . For consumers we set it to zero without loss of generality. The consumers face no financial constraint. The entrepreneurs face the financial constraint

$$a(s_{t+1}|s^t) \geq 0. \tag{1}$$

That is, entrepreneurs cannot commit to make any positive repayment at future dates. This is a simple form of financial markets imperfection, which captures the idea that entrepreneurs have limited access to external finance. This is the only friction we introduce in the model.

The rest of the world is captured by a representative consumer with linear preferences  $E \sum \beta^t c_t^{T,*}$ . Therefore asset pricing is risk neutral: at date  $t$ , the price of a security paying one unit of tradable in state  $s_{t+1}$  is  $\beta \pi(s_{t+1}|s_t)$ .

## 2.2 Decisions and Equilibrium

Let  $p_t$  denote the price of the nontradable good in terms of units of tradable (the numeraire), or the real exchange rate (defined à la IMF). Given the linear technology in the nontradable sector the equilibrium wage must be equal to this price. Consumers and entrepreneurs take the real exchange rate as given. Entrepreneurs also take as given the price of an open export unit,  $q_t$ .

Notice that equilibrium prices and quantities are, in general, function of the whole history  $s^t$ . To save on notation, whenever confusion is not possible we omit this dependence and only use the time index  $t$ .

### 2.2.1 Consumers

Since markets are complete, the optimal consumption of tradables and nontradables for the consumers are proportional to the taste shock  $\theta_t$  and take the form:

$$\begin{aligned} c_t^T &= \kappa \theta_t, \\ c_t^N &= \kappa \frac{\theta_t}{p_t}. \end{aligned}$$

Since wages are equal to  $p_t$  the present value of the consumers' income is  $E \sum_t \beta^t (e + p_t)$ , and the constant  $\kappa$  takes the form:

$$\kappa = \frac{E \sum_t \beta^t (e + p_t)}{2 \cdot E \sum_t \beta^t \theta_t}, \quad (2)$$

ensuring that the consumers' budget constraint is satisfied.

There are two important features from the consumption block. First, during  $A$  periods the demand curve for nontradables shifts upward. This is the source of the appreciation. Second, note that  $\kappa$  is endogenous and rises with each real exchange rate. This will be important later on, as it makes clear that, from the consumers' point of view, stabilizing an appreciation only can be justified if it yields higher wages (real exchange rates) in the future.

### 2.2.2 Exporters and Equilibrium

Even though consumption volatility is not the result of any friction, it may create problems for *both* firms and consumers if the export sector has limited financial resources. Before discussing this issue in detail, we need to understand exporters' decisions.

To describe the entrepreneurs' problem it is useful to separate their decisions regarding consumption and investment in physical and financial assets, from the problem of creating new units. A simple way of doing this is to assume that there is a competitive "adjustment" sector, that creates and destroys export units and makes zero profits. Equilibrium in this sector requires that:

$$\begin{aligned} q_t &\in [0, f], \\ n_t &> n_{t-1} \text{ implies } q_t = f, \\ n_t &< n_{t-1} \text{ implies } q_t = 0. \end{aligned}$$

That is, if units are being created the price of an export unit must be equal to the creation cost  $f$ , while if they are being destroyed its price must be zero.

The entrepreneur's problem, then, is to maximize his expected utility at each node  $s^t$  subject to the financial constraint (1) and the flow of funds constraint:

$$c_t^{T,e} + q_t (n_t - n_{t-1}) + \beta \sum_{s_{t+1} \in \mathcal{S}} \pi(s_{t+1}|s_t) a(s_{t+1}|s^t) \leq (1 - p_t) n_t + a_t, \quad (3)$$

Each period, the entrepreneur uses his current profits,  $(1 - p_t) n_t$ , and his financial wealth,  $a_t = a(s_t|s^{t-1})$ , to finance consumption, investment in physical assets (new production units), and investment in financial assets (state contingent securities). Notice that our timing assumption is that the production units created at date  $t$  are immediately productive (i.e., they immediately generate unitary profits of  $(1 - p_t)$ ).

In the Appendix we describe a recursive representation of the representative entrepreneur's problem, and we show that his only individual state variable at date  $t$  is his initial total assets:  $a_t + q_t n_{t-1}$ . Moreover, given the linearity of the entrepreneur's problem, his expected utility at date  $t$  can be written as:

$$\phi(s^t) \cdot (a_t + q_t n_{t-1}), \quad (4)$$

where  $\phi(s^t)$  represents the marginal return on a dollar of entrepreneurs' wealth (see Lemma 1 in the Appendix). Here we skip the steps that give (4) and proceed directly to discuss the optimality conditions of the entrepreneur, as they are intuitive enough.

In each period, the entrepreneur chooses how many production units to operate, whether and how much to consume, and how many contingent claims to purchase for states  $A$  and  $D$  in the next period. The first order conditions with respect to these variables are

$$(-q(s^t) + (1 - p(s^t))) \phi(s^t) + \beta \sum_{s_{t+1} \in \mathcal{S}} \pi(s_{t+1}|s_t) \phi(s^{t+1}) q(s^{t+1}) = 0, \quad (5)$$

$$1 - \phi(s^t) \leq 0, \quad c^{T,e}(s^t) \geq 0 \quad (6)$$

$$-\phi(s^t) + \phi(s^{t+1}) \leq 0, \quad a(s_{t+1}|s^t) \geq 0, \text{ for all } s_{t+1} \in \mathcal{S}. \quad (7)$$

where  $s^{t+1} = \langle s^t, s_{t+1} \rangle$ . Note that next to the conditions in (6) and (7), we have written the respective complementary slackness conditions.

The first equation states that the opportunity cost of the resources used in keeping the marginal unit in operation must be equal to the expected value of that unit tomorrow.

The cost of keeping a unit in operation correspond to the price of acquiring that unit,  $q_t$ , minus the current profits that the unit generates,  $(1 - p_t)$ . Remember that an open unit must remain active, so when  $p_t > 1$  it incur in losses. The second equation states that if the entrepreneur is consuming, the marginal value of a unit of wealth is one, otherwise it exceeds one. The third equation says that the marginal value of a unit of wealth must be equal across all states with positive holdings of financial wealth.

Finally, we are in a position to characterize the competitive equilibrium. The consumers' side is fully characterized by the constant  $\kappa$ , while the entrepreneurs' side determines the number of open export units and the dynamics of  $a$  and  $n$ . The real exchange rate must then satisfy the labor market clearing condition:

$$n_t + \kappa \frac{\theta_t}{p_t} = 1,$$

and we only need to check that the value of  $\kappa$  is consistent with the equilibrium exchange rate dynamics.

## 2.3 The Appreciation and Depreciation Phases

Recall that our economy starts with a stock of export units,  $n_{-1}$ , and has just entered state  $A$ . The situation that concerns us is one in which  $n_{-1}$  exceeds the units that the export sector wants to keep in operation during the appreciation, and where the latter units are less than the units the export sector wants to operate during the following depreciation phase. As a result, there is destruction of units during the appreciation, and creation during the depreciation. Moreover, we also wish to focus on a scenario where the option to wait is sufficiently positive that it is not optimal to destroy all export units during the appreciation. The export sector has financial resources  $a_0$  to finance the losses during the appreciation phase.

### 2.3.1 An Efficient Benchmark

As a benchmark, let us first study a case where  $a_0$  is sufficiently large that financial constraints are never binding. With some abuse of notation, we replace the subindex  $t$  with the corresponding exogenous state  $s_t$ . Similarly, there is no need to keep track of history in this case since, as we will see, all equilibrium prices and quantities are stationary, both in the  $A$  and in the  $D$  phase.

In the absence of financial constraints,

$$\phi_A = \phi_D = 1,$$

and since destruction takes place in  $A$  and creation in  $D$ , we also have that:

$$q_A = 0; \quad q_D = f.$$

The first order condition for  $n$  in the  $A$  and  $D$  regions, respectively, reduce to:

$$\begin{aligned} (1 - p_A) + \beta\delta f &= 0 \\ -f + (1 - p_D) + \beta f &= 0 \end{aligned}$$

which fully determine the real exchange rate in each region:

$$p_A^{fb} = 1 + \beta\delta f \tag{8}$$

$$p_D^{fb} = 1 - (1 - \beta)f. \tag{9}$$

We will assume from now on that  $(1 - \beta)f < 1$ , ensuring that creation is profitable in the  $D$  phase.

Given these supply determined prices, we can find the consumption of tradables and nontradables in each state:

$$\begin{aligned} c_A^{T,fb} &= \kappa^{fb}\theta_A, & c_A^{N,fb} &= \kappa^{fb}\frac{\theta_A}{1 + \beta\delta f}, \\ c_D^{T,fb} &= \kappa^{fb}, & c_D^{N,fb} &= \kappa^{fb}\frac{1}{1 - (1 - \beta)f}, \end{aligned}$$

and the number of units open in each state:

$$n_A^{fb} = 1 - \kappa^{fb}\frac{\theta_A}{1 + \beta\delta f}, \quad n_D^{fb} = 1 - \kappa^{fb}\frac{1}{1 - (1 - \beta)f}. \tag{10}$$

Note that in this case  $\kappa^{fb}$  is equal to:

$$\kappa^{fb} = (1 + e)\frac{1 - \beta(1 - \delta)}{2((1 - \beta)\theta_A + \beta\delta)}.$$

Later on we will show that this  $\kappa^{fb}$  is an upper bound for the constrained economy's  $\kappa$ .

We need two assumptions on the model parameters to ensure that: (i) there is destruction when the economy enters state  $A$  at date 0, and (ii) there is positive creation when the

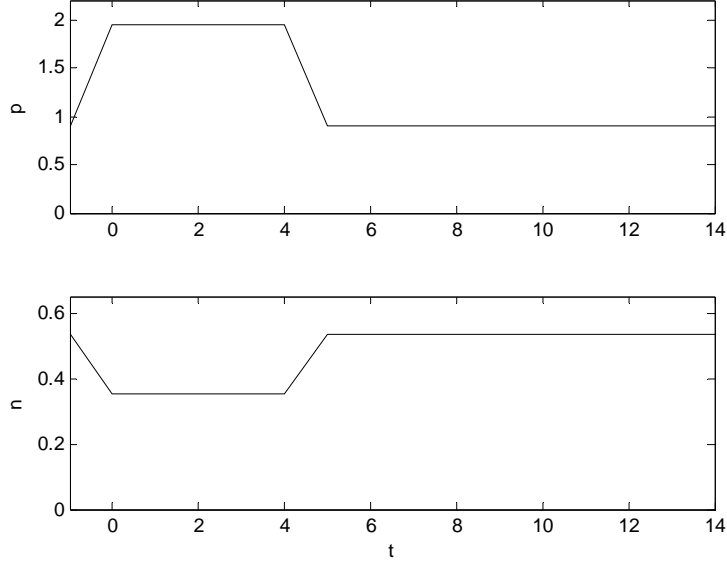


Figure 1: First Best

economy shifts from the  $A$  to the  $D$  state. The following two conditions guarantee that this is the case, and we will assume they hold from now on:

$$n_{-1} > 1 - \kappa^{fb} \frac{\theta_A}{1 + \beta \delta f}, \quad (\text{A1})$$

$$\theta_A > \frac{1 + \beta \delta f}{1 - (1 - \beta)f}. \quad (\text{A2})$$

Figure 1 summarizes the benchmark economy, assuming for simplicity that  $n_{-1} = n_D^{fb}$  and  $p_{-1} = p_D^{fb}$ .<sup>6</sup> The figure illustrates the equilibrium dynamics of the real exchange rate and of the number of firms in a case where the appreciation phase lasts three periods. During the  $A$  phase, it is optimal for the economy to accommodate the increased demand for nontradables by contracting the export sector temporarily. However, since shutting down units wastes creation costs, it is also optimal for the export sector to keep  $n_A^{fb} > 0$  units in operation, with each of them incurring flow losses of  $\beta \delta f$  due to the appreciated exchange rate.

The following Proposition summarizes the case of high entrepreneurial wealth. The explicit expression for the cutoff  $\hat{a}^{fb}$  is in the Appendix.

---

<sup>6</sup>See Appendix B for the algorithms and parameters used in all the figures. Whether the particular combination  $n_{-1} = n_D^{fb}$  and  $p_{-1} = p_D^{fb}$  is feasible or not depends on assumptions about a period we do not model in the paper. This is not important for now, as the qualitative features of the picture are fairly robust.

**Proposition 1** (*First best*) *There is a cutoff  $\hat{a}^{fb}$  such that if the entrepreneurs' initial wealth satisfies*

$$a_0 \geq \hat{a}^{fb},$$

*then the equilibrium real exchange rate and the number of firms are constant within the  $A$  and  $D$  phases, and correspond to expressions (8), (9), and (10). The marginal value of entrepreneurial wealth,  $\phi_s$ , is constant and equal to 1.*

### 2.3.2 The Constrained Economy and Overshooting

Suppose now that  $a_0$  is not enough to implement the benchmark path. There are two margins through which this can happen. First, the export sector may not have enough resources to finance the flow of losses  $(p_A^{fb} - 1)n_A^{fb}$  during the appreciation. Second, even if it can, financial resources may be so depleted by the end of the appreciation phase that the financial constraint binds during the recovery in  $D$ , slowing down the reconstruction of the export sector. Either way, a constrained exports sector lowers real wages and hence consumers' income.

Relative to the benchmark case, we now need to keep track not only of the current exogenous state  $s_t$ , but also of the number of periods since the  $D$  phase started. The reason for this is that, as we mentioned earlier, in this case there is transition phase in the  $D$  region while the export sector rebuilds and is constrained by limited financial resources. Let  $D, j$  index these transition dates.

In this case we still have  $q_A = 0$  and  $q_D = f$ , but now the  $\phi$ 's exceed one until the time when the export sector has been fully reconstructed in the  $D$  phase.

Note also that due to complete markets the  $A$  phase is stationary. Thus we can write the first order conditions during this phase as:

$$(1 - p_A)\phi_A + \beta\delta f\phi_{D,0} = 0, \tag{11}$$

$$1 - \phi_A < 0, \tag{12}$$

$$-\phi_A + \phi_{D,0} \leq 0, \quad (a_{D,0} \geq 0) \tag{13}$$

and the budget constraint:

$$(1 - \delta)\beta a_0 + \delta\beta a_{D,0} = (1 - p_A)n_A + a_0,$$

This equation can be re-written as:

$$(1 - (1 - \delta)\beta)a_0 = (p_A - 1)n_A + \delta\beta a_{D,0}$$

which says that the flow generated by initial resources  $a_0$  can be used to finance the operational losses of production units that remain open during the appreciation, and to transfer financial resources to the recovery phase in  $D$ , to facilitate rebuilding the export sector.

The third equation above distinguishes between a case where the two margins are active,  $a_{D,0} > 0$  and  $\phi_A = \phi_{D,0}$ , and one in which only the former is active,  $a_{D,0} = 0$  and  $\phi_A > \phi_{D,0}$ . For given parameters, one can show that the latter case occurs for low levels of  $a_0$ , where the firm struggles to finance the losses during the appreciation, while the former case takes place as  $a_0$  rises. Henceforth we focus on the low  $a_0$  case, so that all resources are used to save production units during the appreciation phase and  $a_{D,0} = 0$ .

The first order condition for  $n_A$  (first equation above) yields an expression for the real exchange rate in the  $A$  region:

$$p_A = 1 + \beta\delta f \frac{\phi_{D,0}}{\phi_A} \leq p_A^{fb}$$

where the inequality comes from (13). We see that, as in the benchmark case, the appreciation is such that production units incur in losses ( $p_A > 1$ ). These losses are attenuated by a smaller appreciation than in the benchmark ( $p_A < p_A^{fb}$ ). However far from being good news, the smaller appreciation reflects the fact that financially constrained firms are unable to keep open as many production units as they would wish and hence are forced to reduce production and labor demand. The following proposition sums up the previous discussion.

**Proposition 2** (*Constrained appreciation phase*) *There is a cutoff  $\hat{a}^A < \hat{a}^{fb}$  such that if  $a_0 > \hat{a}^A$  the real exchange rate in the  $A$  phase is  $p_A^{fb}$  and  $a_{D,0} > 0$ , while if  $a_0 < \hat{a}^A$  the real exchange rate in the  $A$  phase is  $p_A < p_A^{fb}$  and  $a_{D,0} = 0$ .*

To determine  $n_A$ , note that from the consumption side and labor market equilibrium, we have that

$$p_A = \frac{c_A^T}{1 - n_A}.$$

Replacing this expression back into the budget constraint pins down the number of production units that are kept during the appreciation:

$$a_0(1 - (1 - \delta)\beta) = \left( \frac{c_A^T}{1 - n_A} - 1 \right) n_A$$

As is to be expected, for a given consumption level  $c_A^T$ , lower financial resources  $a_0$  lower the number of production units that are kept open during the appreciation. In general equilibrium,  $c_A^T$  falls as well and hence the final effect on  $n_A$  is ambiguous. What is unambiguous (see the Appendix), is that  $n_A/\bar{n}_D$  drops as  $a_0$  declines, where  $\bar{n}_D$  represents the stationary size of the export sector after the recovery phase of  $D$  is completed. This is important, because a lower  $n_A/\bar{n}_D$  means that the reconstruction effort needed during the  $D$  phase rises with the tightening of  $a_0$ . We turn to the  $D$  region next.

Starting backwards, once the recovery phase is completed, entrepreneurs consume and  $\phi_{\bar{D}} = 1$ . Thus, from the first order conditions, we have that in the stationary phase of  $D$ :

$$p_{\bar{D}} = 1 - (1 - \beta)f = p_D^{fb}$$

Eventually, the real exchange rate converges to the benchmark level.

It follows from the equilibrium condition in the labor market and the fact, yet to be shown, that the level of consumption is lower in the constrained than in the benchmark case, that:

$$n_{\bar{D}} = 1 - \frac{c_D^T}{1 - (1 - \beta)f} > 1 - \frac{c_D^{T,fb}}{1 - (1 - \beta)f} = n_D^{fb}$$

Since in the constrained economy not only entrepreneurs but also consumers are poorer than in the benchmark economy, demand is depressed and hence the export sector eventually expands to absorb the labor freed by the smaller nontradable sector. However, this stationary state is not reached instantly, as financial constraints also hamper the recovery phase. The first order conditions for this transition phase are:

$$(-f + 1 - p_{D,j})\phi_{D,j} + \beta f\phi_{D,j+1} = 0, \quad (14)$$

$$\phi_{D,j} > 1, \quad (15)$$

$$\phi_{D,j} > \phi_{D,j+1}, \quad (16)$$

$$f(n_{D,j} - n_{D,j-1}) = (1 - p_{D,j})n_{D,j} \quad (17)$$

for  $j = 0, \dots, J$ , where  $J$  is the last period of the transition phase in  $D$  and, with abuse of notation,  $n_{D,-1} = n_A$ .

The last condition states that during the recovery phase, firms use all their profits to rebuild the sector. The next to last condition reflects that financial constraints are tightest early on in the recovery and gradually decline, and hence there is no reason to accumulate

“cash” or to consume (the second first order condition). Reorganizing the first equation, we obtain an expression for the real exchange rate during the transition:

$$p_{D,j} = 1 - f \left( 1 - \beta \frac{\phi_{D,j+1}}{\phi_{D,j}} \right) < 1 - f(1 - \beta) = p_{\overline{D}} = p_D^{fb}$$

That is, during the recovery phase the depreciation is deeper when the economy is constrained. We refer to this deeper depreciation as the *overshooting* implication of financial constraints.

The presence of overshooting means that wages are not only lower than in the benchmark case during the appreciation phase, but also during the transition phase of  $D$ . This observation closes our argument, as it explains why the consumption level is lower in the constrained case. Recall that the consumption level is indexed by  $\kappa$ :

$$\kappa = \frac{\mathbb{E} \sum_t \beta^t (e + p_t)}{2 \cdot \mathbb{E} \sum_t \beta^t \theta_t}.$$

and we have that history by history  $p_t$  is greater (with some strictly greater) in the benchmark than in the constrained case.

Figure 2 summarizes the constrained economy, assuming for simplicity that  $n_{-1} = n_{\overline{D}}$  and  $p_{-1} = p_{\overline{D}}$ . The first column shows a case of very low  $a_0$  while the second one shows a case of significantly higher (but still constrained) level of financial resources. Since for now we are only interested in the shape of these figures, we also adjust  $e$  across both scenarios so that  $\kappa$  is identical (i.e., for now we suppress the income effect of the financial constraint, which we discuss extensively in the policy section). As in the benchmark economy (which is represented with dashes in each panel), the exchange rate appreciates in the  $A$  phase, more so when the financial constraint is not so tight, and it experiences a large and protracted overshooting in the depreciation phase, especially when the financial constraint is tight. The export sector contracts during the  $A$  phase, and this contraction is larger when the financial constraint is tighter. Unlike the benchmark economy, the recovery is only gradual during the  $D$  phase. The bottom panels show the paths of the marginal value of a unit of wealth, which are highest in the  $A$  region, drop sharply upon the transition into  $D$ , and gradually decline within the  $D$  region. Also note that the gap between  $\phi_A$  and  $\phi_{D,0}$  shrinks as financial resources rise. This observation will play an important role in the policy section.

Let us conclude with a summary proposition:

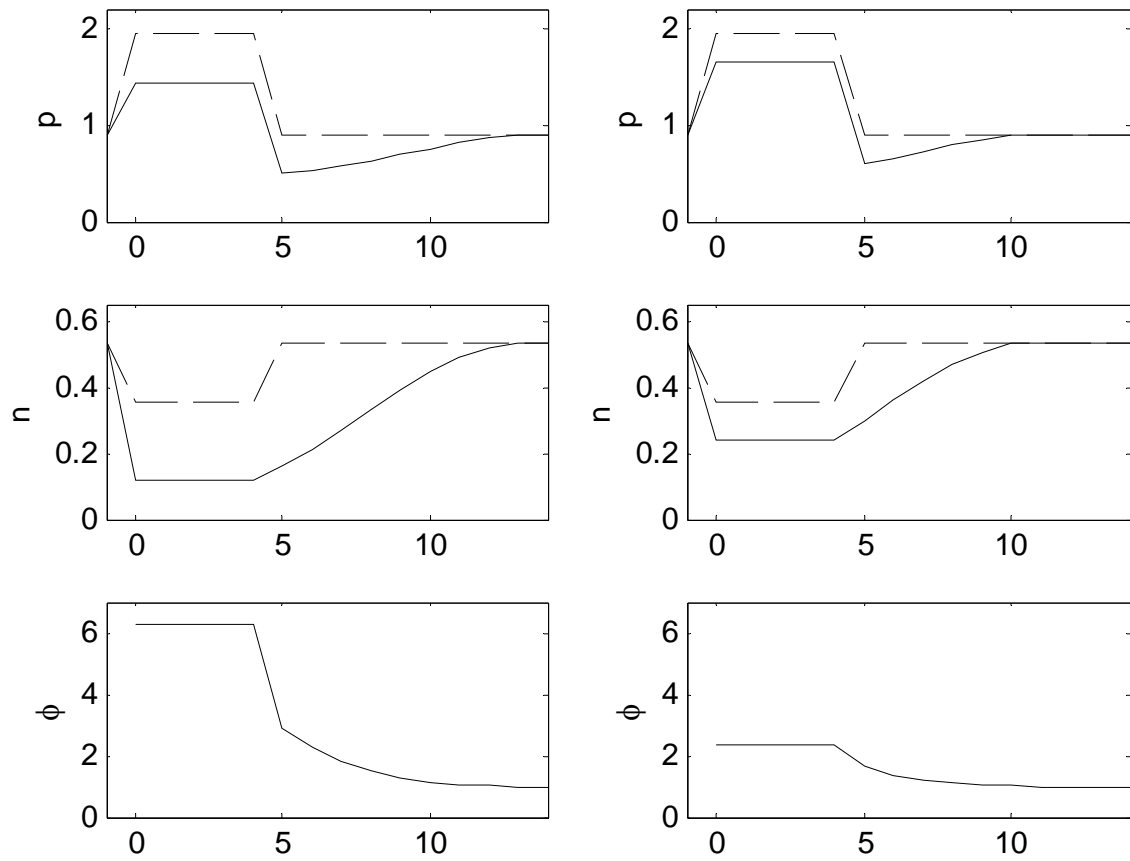


Figure 2: Constrained Economy

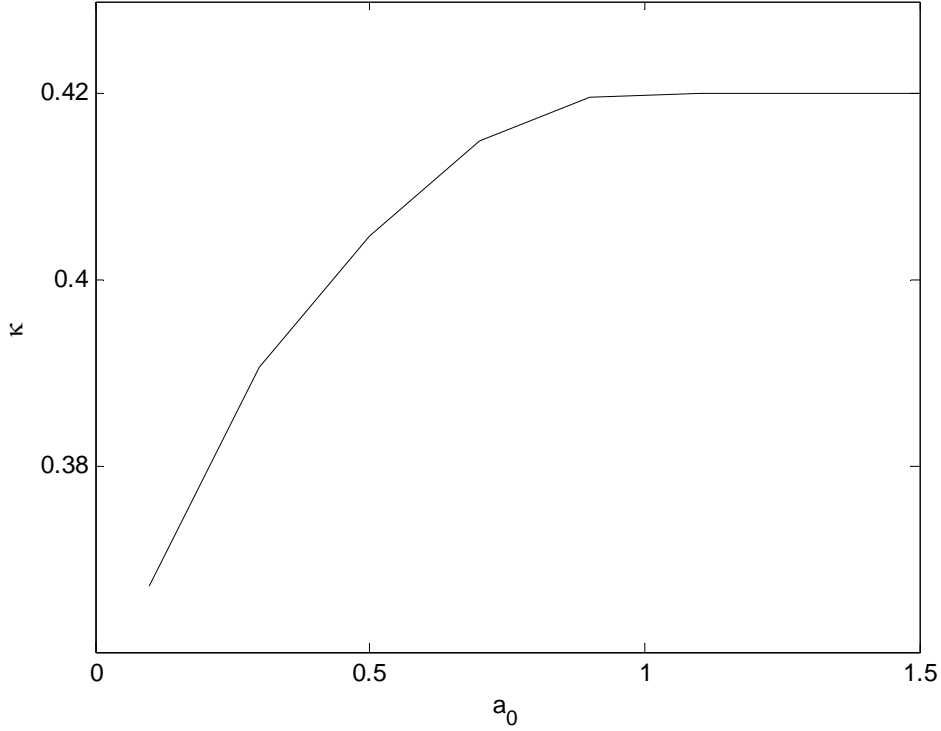


Figure 3: The Income Effect of the Financial Constraint

**Proposition 3** (*Constrained depreciation phase and overshooting*) *There is a cutoff  $\hat{a}^D < \hat{a}^{fb}$  such that if  $a_0 > \hat{a}^D$  the real exchange rate throughout the  $D$  phase is  $p_D^{fb}$ , while if  $a_0 < \hat{a}^D$  the real exchange rate depreciation in the  $D$  phase overshoots its long run value early on in the transition. That is  $p_{D,j} < p_D^{fb}$  for  $j = 0, 1, \dots, J$ . (Note that the cutoff  $\hat{a}^D$  may be greater or smaller than  $\hat{a}^A$ , depending on the model's parameters).*

## 2.4 General Equilibrium Feedback

Our discussion above highlights the export firms problem given a consumption demand. However, firms' actions affect households' income through labor demand. The tighter is the financial constraint on firms, the lower is labor demand and income. This feedback is summarized by  $\kappa$ , which is maximized when  $a_0 \geq \bar{a}^{fb}$ . Figure 3 plots the value of  $\kappa$  as a function of  $a_0$ : It rises until it reaches its maximum for  $a_0 \geq \hat{a}^{fb}$ .

Note that it is this general equilibrium feedback that generates some counterintuitive

results. For example, the model has a sort of sclerosis as  $a_0$  declines. Even though export firms are more financially constrained in this case, in the long run they absorb a larger share of  $n$ . To see this, recall that

$$n_{\overline{D}} = 1 - \frac{\kappa}{1 - (1 - \beta)f}.$$

which rises as  $\kappa$  drops. This simply says that an economy with poorer consumers allocates a larger share of its resources to satisfy foreign than domestic demand. This sclerosis also affects the short run, which is one of the reasons we chose to report most of our results in terms of their implications for the real exchange rate rather than in terms of quantities.

The next section shows how, for a given  $a_0 < \bar{a}^{fb}$ , in some cases the social planner can raise  $\kappa$  by reorganizing  $c_t^N$  (and hence  $n_t$ ) over time.

### 3 Overshooting and Optimal Exchange Rate Intervention

In the previous section we showed that when the export sector has limited financial resources, the depreciation phase following a persistent appreciation may come with a protracted exchange rate overshooting (a sharp real wage decline) while the export sector rebuilds. In practice, it is this overshooting phase that primarily concerns policymakers and leads to a debate on whether intervention should take place during the appreciation phase to protect the export sector. In particular, the concern is whether by overly-stressing the export sector during the appreciation, the economy may be exposing itself to a costly recovery phase once the factors behind the appreciation subside. In this section we study this policy problem and conclude that if an overshooting is expected, there is indeed scope for policy intervention. The reason for such intervention is that the competitive equilibrium is not constrained efficient, as consumers ignore the effect of their individual decisions during the appreciation phase on the extent of the overshooting during the depreciation phase.

The form of the optimal policy is subtle. There are instances where all that is required is an exchange rate stabilization during the appreciation phase. There are others where the scope for appreciation stabilization is limited and the policy still leaves the economy with an expected overshooting in the depreciation phase. In such case, one interesting feature of the optimal policy is that it frontloads the overshooting of the exchange rate, even to the point of generating a larger initial overshooting than in the competitive equilibrium (but with a faster recovery).

Throughout the section we assume the social planner adopts the perspective of consumers but is constrained to consider only policies that do not make entrepreneurs worse off. Moreover, we rule out direct transfers between consumers and entrepreneurs, as in practice both micro-policies and aggregate transfers do take place but are limited by a host of informational and political economy impediments which we do not model here. In this context, the question the social planner effectively asks is whether by modifying the demand for non-tradables, and hence the real exchange rate, it may be able to increase consumers' welfare without reducing entrepreneurs' welfare. The reason such policy may be effective is that the current wage cut associated to the exchange rate intervention, by relaxing the entrepreneurs financial constraint, raises future wages.

### 3.1 Pecuniary Externality and Policy Perturbation

Before characterizing the optimal policy, let us set up the policy problem and identify the pecuniary externality by studying the impact of small policy interventions around the competitive equilibrium.

The planner's objective is to maximize the consumer's utility, which can be written as:

$$\theta_A (u(c_A^T) + u(c_A^N)) + \delta\beta \left( \frac{1}{1-\beta} u(c_D^T) + \sum_{j=0}^{\infty} \beta^j u(c_{D,j}^N) \right),$$

where we have normalized expected utility by the factor  $(1 - \beta(1 - \delta))$ .<sup>7</sup> Let the planner choose directly the consumption paths for tradables and nontradables. In terms of implementation, we can think of a planner that intervenes by taxing the consumption of nontradables and returns the tax revenue to consumers as a lump sum transfer. As usual in optimal taxation problems, it is simpler to characterize the problem directly in terms of equilibrium quantities rather than in terms of the underlying tax rates.

Since we ruled out transfers between consumers and entrepreneurs, the consumers' budget

---

<sup>7</sup>Lemma ... in the Appendix shows that the second best allocation takes the following form: the consumption of tradables is constant and equal to  $c_A^T$  and  $c_D^T$ , respectively, in the  $A$  phase and in the  $D$  phase, and the consumption of non-tradables is constant and equal to  $c_A^N$  in the  $A$  phase.

constraint is (also multiplying through by  $(1 - \beta(1 - \delta))$ ):

$$c_A^T + p_A c_A^N + \delta\beta \left( \frac{1}{1 - \beta} c_D^T + \sum_{j=0}^{\infty} \beta^j p_{D,j} c_{D,j}^N \right) \leq \frac{1 - \beta(1 - \delta)}{1 - \beta} e + p_A + \delta\beta \sum_{j=0}^{\infty} \beta^j p_{D,j}.$$

Relative to individual consumers, the social planner's problem is different in that it takes into account the effect of consumer's decisions on  $p_A$  and  $\{p_{D,j}\}$ . The consumers' decisions affect the equilibrium prices by changing the demand for nontradables and, thus, equilibrium wages. Using the entrepreneurs' optimality condition and market clearing in the labor market we can derive a relation between the quantities chosen by the planner and the prices  $p_A$  and  $\{p_{D,j}\}$ . In the appendix we show that entrepreneurs' optimality determines a mapping between the labor allocations  $n_A, \{n_{D,j}\}$  and the price sequences  $p_A, \{p_{D,j}\}$ . Therefore, the planner chooses the consumption of nontradables, market clearing gives the labor allocations  $n_A = 1 - c_A^N$  and  $n_{D,j} = 1 - c_{D,j}^N$ , and the mapping above yields the associated equilibrium prices.

In short, the planner's problem is effectively one of allocating labor between the production of nontradables and exports at each point in time, trying to increase consumers' income without distorting too much their consumption decisions.

Finally, the constraint that entrepreneurs cannot be made worse off is (also multiplying through by  $(1 - \beta(1 - \delta))$ ):

$$c_A^{T,e} + \delta\beta \sum_{j=0}^{\infty} \beta^j c_{D,j}^{T,e} \geq (1 - \beta(1 - \delta)) \bar{U}^e,$$

where  $\bar{U}^e$  denotes the entrepreneurs' welfare in the competitive equilibrium.

Let us study the effect of stabilizing the appreciation phase, starting from the competitive equilibrium studied in Section 2. Specifically, consider the effect of reducing  $c_A^N$  or, equivalently, increasing  $n_A$ , while keeping the  $n_{D,j}$ 's unchanged.

The following expression captures the marginal effect of a change in  $n_A$  in the planner's problem:

$$\begin{aligned} & -\theta_A u'(1 - n_A) + p_A \lambda \\ & + \lambda \left( \frac{\partial p_A}{\partial n_A} n_A + \beta \delta \frac{\partial p_{D,0}}{\partial n_A} n_{D,0} \right) + \\ & + \mu \beta \frac{\partial c_{D,0}^{T,e}}{\partial n_A}, \end{aligned} \tag{18}$$

where  $\lambda$  represents the Lagrange multiplier of the consumers' budget constraint and  $\mu$  that of the entrepreneurs' participation constraint.

The first row captures the direct effect of the policy and is equivalent to the consumers' first order condition in the competitive economy. The second row captures the impact of the policy on consumers' net (of consumption) income.<sup>8</sup> Since we keep all the  $n_{D,j}$ 's constant, this policy only affects the prices  $p_A$  and  $p_{D,0}$ , and the entrepreneurs' consumption at date  $t_{D,0}$ .

We consider two cases. First, suppose the competitive equilibrium displays  $p_A < p_A^{fb}$  and  $p_{D,0} < p_D^{fb}$ .

Let us start with the effect of a unit increase in  $n_A$  on  $p_A$ . If the planner wants entrepreneurs to carry an extra unit of  $n_A$ , then  $p_A$  needs to drop for the firm to be able to finance the extra losses from that unit. Recall that the firm's budget constraint in phase  $A$  is

$$(1 - (1 - \delta)\beta) a_0 = (p_A - 1) n_A,$$

from which we obtain:

$$\frac{\partial p_A}{\partial n_A} = -\frac{p_A - 1}{n_A}.$$

Then, turn to the effects of  $n_A$  on  $p_{D,0}$ . Since  $p_{D,0} < p_D^{fb}$ , i.e., there is equilibrium overshooting, the entrepreneur budget constraint at date  $t_{D,0}$  is

$$f(n_{D,0} - n_A) = (1 - p_{D,0}) n_{D,0}.$$

The entrepreneur's financial constraint is binding and he uses all his current profits to invest in new units. In this case, a unit increase in  $n_A$  affects  $p_{D,0}$  since it reduces by  $f$  the investment required to rebuild to  $n_{D,0}$ . Wages must rise to compensate for this fall in investment expenditure, so as to keep the financial constraint exactly binding at  $n_{D,0}$ . Thus:

$$\frac{\partial p_{D,0}}{\partial n_A} = \frac{f}{n_{D,0}}.$$

Finally, in this case  $c_{D,0}^{T,e} = 0$  so the third line of (18) is zero.

Consumers are hurt by the decline in their wage (real exchange rate) during the  $A$  phase, but gain from the rise in their wage in the first period of the  $D$  phase. Which

---

<sup>8</sup>Note that the consumer's labor income is  $p_A$  while its expenditure on nontradables is  $p_A(1 - n_A)$ . Thus net income is  $p_A n_A$ .

effect dominates? Replacing the price derivatives in the expression in the second row of the first order condition we have:

$$\frac{\partial p_A}{\partial n_A} n_A + \delta \beta \frac{\partial p_{D,0}}{\partial n_A} n_{D,0} = 1 - p_A + \beta \delta f > 0.$$

The inequality follows from the fact that  $p_A < p_A^{fb} = 1 + \beta \delta f$ . That is, in the planner's problem there is an extra term capturing the marginal benefit of increasing  $n_A$  on the expected present value of wages. The planner has an incentive to reduce nontradables consumption, so as to reduce the appreciation (i.e. reduce  $p_A$ ) and allow firms to keep a larger number of units open, which in turn raises wages at  $t_{D,0}$ . Because  $\phi_A > \phi_{D,0}$ , reducing wages in  $A$  generates an excess return in export firms that is transferred back to workers in the form of substantially higher wages at  $t_{D,0}$ .

The planner is choosing the consumption of nontradables  $c_A^N$  as if the real exchange rate was higher than the market price. How much higher? It is easy to show from the marginal benefit of increasing  $n_A$  that the planner would like to pick  $c_A^N$  as if the price was at its first best level:

$$\begin{aligned} & -\theta_A u'(1 - n_A) + \lambda p_A + \lambda(1 - p_A + \beta \delta f) = \\ & -\theta_A u'(1 - n_A) + \lambda p_A^{fb}. \end{aligned}$$

In a competitive equilibrium, consumers increase their demand for nontradables in response to the taste shock, which leads to an appreciation of the real exchange rate. However, due to the firms' financial constraint, the appreciation is smaller than it would be in the first best. This price gap implies that consumers further increase their consumption of nontradables, which hurts firms by increasing the cost of keeping units open. The planner taxes consumption of nontradables enough to offset this additional effect, and in so doing stabilizes the real exchange rate and allows firms to maintain a larger number of production units open.

Consider now a second case, where  $p_A < p_A^{fb}$  and  $p_{D,0} = p_D^{fb}$ . That is, consider a case where the equilibrium displays no overshooting in the  $D$  phase. In this case, entrepreneurs are unconstrained at  $t_{D,0}$  and  $c_{D,0}^{T,e} > 0$ , which implies

$$\frac{\partial p_{D,0}}{\partial n_A} = 0$$

and

$$\frac{\partial c_{D,0}^{T,e}}{\partial n_A} = f.$$

Replacing these terms in (18) we obtain

$$-\theta_A u'(1 - n_A) + \lambda p_A + \lambda(1 - p_A) + \mu \beta f.$$

Suppose initially that the participation constraint was slack ( $\mu = 0$ ), then  $\theta_A u'(1 - n_A) = \lambda < \lambda p_A$ . The planner would want to increase  $c_A^N$  and reduce  $n_A$ . The reason is that it makes no sense for consumers to cut their wage today if this action does not raise wages in the future, which it will not when there is no overshooting to remedy and  $p_{D,0}$  is pinned down by the free entry condition. Instead, the planner, representing the consumers, would like to exercise its “monopoly” power during the appreciation phase and raise wages by increasing their demand for nontradables. However this increase would reduce the consumption of entrepreneurs and violate their participation constraint. Therefore  $\mu = 0$  is not possible. In fact, when there is no expected overshooting, no intervention is optimal and  $\mu$  can be chosen so that

$$-\theta_A u'(1 - n_A) + \lambda p_A + \lambda(1 - p_A) + \mu \beta f = 0,$$

and the planner first order condition holds at the competitive equilibrium allocation. This discussion gives us an important reference result:

**Proposition 4** (*Constrained efficiency*) *If  $a_0 > \hat{a}^D$  (no overshooting), then the competitive equilibrium is constrained efficient and it is optimal not to stabilize the appreciation, even if firms are financially constrained and the export sector contracts more than in the first best (i.e., even if  $a_0 < \hat{a}^A$ ).*

Put differently, if there is no overshooting, there is no intertemporal pecuniary externality for the consumers, so they cannot trade-off a wage reduction today for a wage increase in the recovery phase. The flip side of this argument is that it is the presence of overshooting that makes individual consumers underestimate the social cost of their increased demand during the appreciation phase.

Henceforth we shall focus on the case where firms are financially constrained during the  $D$  phase and the competitive equilibrium experiences overshooting. We show that the optimal policy may involve not only a stabilization of the exchange rate during the appreciation but also intervention during the depreciation phase.

The analysis of policies that affect the  $n_{D,j}$ 's is similar to the discussion in the  $A$  phase. On one extreme, when  $p_{D,j}$  has already converged to  $p_D^{fb}$ , there is no scope for policy. On the other, when entrepreneurs are constrained and the economy is in the overshooting phase, it

is optimal to reduce  $c_{D,j}^N$  further and *exacerbate* the depreciation. By doing so, consumers accelerate the recovery of the export sector and real wages. Let us study this interior (overshooting) region in more detail and postpone discussion of other regions to the optimal policy analysis.

In this interior region, the first order conditions are similar to that for  $n_A$ . In particular, since these derivatives take future  $n$ 's as given, a change in  $n_{D,j}$  only affects the current and next period's prices. As before, the latter are affected so that the financial constraint is not relaxed by the increase in initial  $n$ . Moreover,  $c_{D,j+1}^{T,e} = c_{D,j}^{T,e} = 0$ , thus

$$\begin{aligned} & -u'(1 - n_{D,j}) + p_{D,j}\lambda \\ & + \lambda \left( \frac{\partial p_{D,j}}{\partial n_{D,j}} n_{D,j} + \beta \frac{\partial p_{D,j+1}}{\partial n_{D,j}} n_{D,j+1} \right) = 0 \end{aligned} \quad (19)$$

and

$$\frac{\partial p_{D,j}}{\partial n_{D,j}} n_{D,j} + \beta \frac{\partial p_{D,j+1}}{\partial n_{D,j}} n_{D,j+1} = -(f - (1 - p_{D,j})) + \beta f > 0.$$

Replacing this expression back into the first order condition we find that the social planner's first order conditions use the first best rather than the overly-depreciated competitive equilibrium exchange rate for its allocation decisions:

$$\begin{aligned} u'(1 - n_{D,j}) &= \lambda p_{D,j} + \lambda \left( \frac{\partial p_{D,j}}{\partial n_{D,j}} n_{D,j} + \beta \frac{\partial p_{D,j+1}}{\partial n_{D,j}} n_{D,j+1} \right) \\ &= \lambda p_{D,j} + \lambda (-(f - (1 - p_{D,j})) + \beta f) \\ &= \lambda (1 - (1 - \beta)f) \\ &= \lambda p_D^{fb} \end{aligned}$$

In the competitive equilibrium, the overshooting caused by the firms' financial constraints induces consumers to demand more nontradables, which complicates the export sector's recovery. The social planner offsets the consumers' reaction to the overshooting and reduces  $c_{D,j}^N$  (it taxes nontradable consumption). Note that as a result of this reduction in  $c_{D,j}^N$  the overshooting is exacerbated, but this is precisely what allows financially constrained firms to increase the pace of investment as profits rise. The trade-off is between a deeper overshooting and lower wages today in exchange for a faster recovery in wages. Again, because  $\phi_{D,j} > \phi_{D,j+1}$ , reducing wages at  $t_{D,j}$  generates an excess return in export firms that is transferred back to workers in the form of substantially higher wages at  $t_{D,j+1}$ .

### 3.2 Optimal Stabilization and (Over) Overshooting

We learned from the perturbation analysis above that if there is no expected overshooting the competitive equilibrium is constrained efficient, even if firms are financially constrained during the appreciation. On the other hand, if there is an expected overshooting, then the competitive equilibrium is constrained inefficient and there is scope for policy. Let us focus on the latter scenario.

Figure 4 plots the real exchange rate in the competitive equilibrium and in the “second best” allocation for three prototypical cases. In the first panel, the planner depreciates the exchange rate relative to the first best and the competitive equilibrium only in the appreciation phase. This early depreciation (reduction in appreciation) is sufficient to fully stabilize the exchange rate in the  $D$  phase as well, by achieving the first best exchange rate at  $t_{D,0}$ . In the other two panels, the second best allocation includes a depreciation at  $t_{D,0}$  that overshoots the long run depreciation (and first best depreciation). Importantly, however, if it is optimal to have an overshooting in the  $D$  phase, this must take place only in the first period.<sup>9</sup> In some cases (panel c), the optimal overshooting at  $t_{D,0}$  is so large, that it exceeds that of the competitive equilibrium.

Let us explore these optimal paths in more detail, by analyzing first the intervention in the  $A$  phase (i.e.,  $n_A$ ) and then the intervention in the  $D$  phase (i.e., the  $n_{D,j}$ 's). In all the three cases depicted in Figure 4 there is some degree of exchange rate stabilization during the  $A$  phase. This feature is most easily understood in the cases illustrated in the second and third panel where the optimal policy involves  $p_A < p_A^{fb}$  and  $p_{D,0} < p_D^{fb}$ . In such cases we can proceed as we did above and write the planner's first order condition for  $n_A$  as:

$$\theta_A u'(1 - n_A) = \lambda p_A^{fb} = \lambda(1 + \beta \delta f). \quad (20)$$

The social planner allocates  $n_A$  as if prices were at first best. Since firms are financially constrained, this allocation can be achieved only by depressing the consumption of nontradables and hence lowering the exchange rate in equilibrium.

However, this first order condition also reveals that the optimal depreciation during the  $A$  phase is not unlimited. Once quantities correspond to those implied by the consumers' first order condition when facing first best prices, the intervention stops. This happens because the planner has to balance the benefits of the depreciation (reduction in appreciation), given

---

<sup>9</sup>If the real exchange rate reaches 0 in the first period, then the optimal depreciation will continue to a second period, and so on.

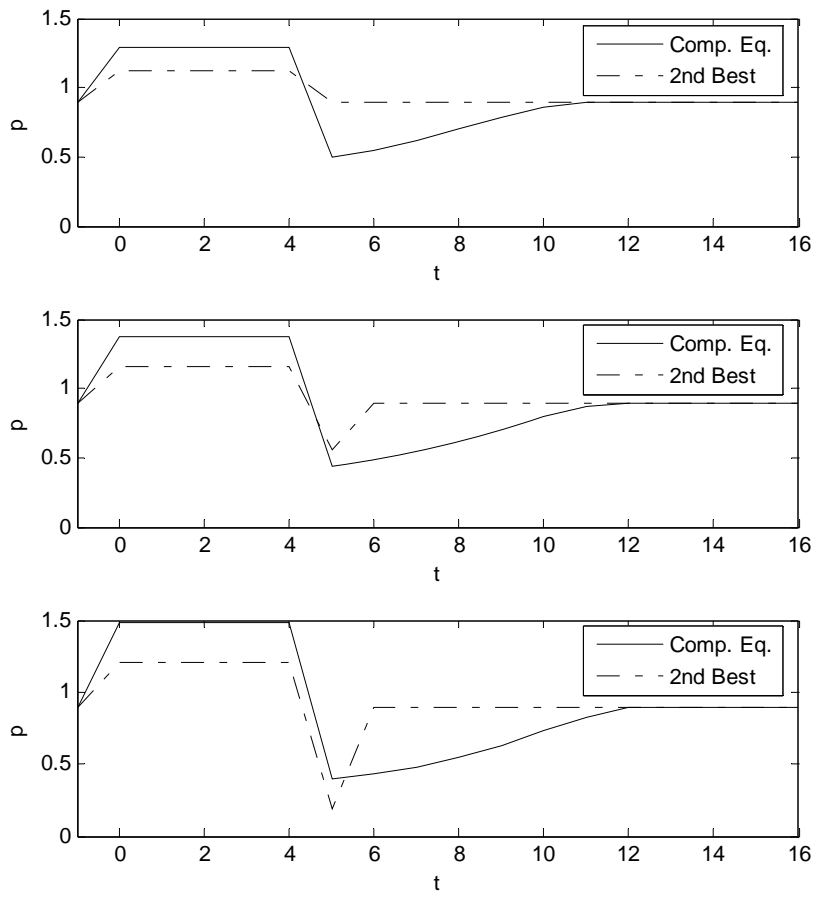


Figure 4: Optimal Exchange Rate Intervention

by a higher present value of wages, with its costs in terms of distorted consumption of nontradables.

Let us now turn to the case illustrated in the first panel of Figure 4. A depreciation in  $A$ , by increasing  $n_A$ , tends to increase  $p_{D,0}$ . Therefore, it is possible that before reaching the level of  $n_A$  that satisfies (20),  $p_{D,0}$  reaches its first best level  $p_D^{fb}$ . At this point there is no gain for the consumer from cutting wages further during the  $A$  phase, since this has no effect on wages in the  $D$  phase. Remember that (20) was derived under the assumption that  $p_A < p_A^{fb}$  and  $p_{D,0} < p_D^{fb}$ . Once  $n_A$  reaches the level such that  $p_{D,0} = p_D^{fb}$ , the Lagrangian for the planner problem has a kink and the first order condition for  $n_A$  takes the form

$$\theta_A u'(1 - n_A) = \lambda p_A + \lambda [1 - p_A + \xi \beta \delta f] \quad (21)$$

where  $\xi \in [0, 1]$ .<sup>10</sup> In this case, the optimal  $n_A$  is the one that makes the entrepreneurs financial constraint at date  $t_{D,0}$  exactly binding and  $\xi$  adjusts so as to ensure that (21) is satisfied.

Next, let us turn to the  $D$  phase and to the optimal determination of  $n_{D,j}$ . Now the easiest case to consider is that depicted in panel (a) of Figure 4, where the planner achieves full stabilization of the exchange rate in the  $D$  phase. Let us start with this case. To interpret the optimal path of  $p_{D,j}$ , note that along the recovery path the entrepreneurs' financial constraint is exactly binding:

$$f(n_{D,j} - n_{D,j-1}) = (1 - p_{D,j}) n_{D,j},$$

until the point where  $n_{D,j}$  reaches its undistorted level. I.e., the value  $\bar{n}_D$  that satisfies

$$u'(1 - \bar{n}_D) = \lambda p_D^{fb}.$$

Entrepreneurs use all their profits for investment and delay their consumption until they have reached  $\bar{n}_D$ .

The distortion in the consumption of nontradables is concentrated in the early periods of the  $D$  phase. In all these periods the following inequality holds,

$$u'(1 - n_{D,j}) < \lambda p_D^{fb},$$

---

<sup>10</sup>At this point the map from  $n_A, \{n_{D,j}\}$  to  $p_A, \{p_{D,j}\}$  has a kink, and, in terms of generalized derivatives we have

$$\frac{\partial p_{D,0}}{\partial n_A} = \xi \frac{\delta f}{n_{D,0}},$$

with  $\xi \in [0, 1]$ .

and the planner would like to decrease the consumption of nontradables (i.e., increase  $n_{D,j}$ ) so as to smooth nontradable consumption. However, since the entrepreneurs financial constraint is exactly binding, increasing  $n_{D,j}$  in any of these periods would reduce the current wages,  $p_{D,j}$ , below their first best level. This has no advantages in terms of future wages, given that  $p_{D,j+1}$  is already at its maximum level  $p_D^{fb}$ . The potential cost in terms of current wages exactly compensates for the distortion in nontradables consumption.

In the scenarios depicted in (b) and (c) a similar reasoning applies. However, in those cases the price  $p_{D,0}$  is below its first best level. Why does the planner accept this early wage loss? Just because it has no alternative. In order to increase  $p_{D,0}$  the planner would have to reduce  $n_{D,0}$ . But then, since the entrepreneurs are exactly constrained at  $t_{D,1}$ , this would imply a wage loss at that date. For the consumers, the net effect of this reduction in  $n_{D,0}$  would be

$$\begin{aligned} u'(1 - n_{D,0}) - \lambda p_{D,0} - \lambda [1 - f - p_{D,0} + \beta f] &= \\ u'(1 - n_{D,0}) - \lambda p_D^{fb} &< 0. \end{aligned}$$

On the other hand, if the planner tried to increase  $n_{D,0}$ , the current wage would drop even further, but there would be no gain in terms of future wages.

Notice that this reasoning applies only because  $p_{D,j} = p_D^{fb}$  for all the periods following  $t_{D,0}$ . This is key since it explains why the overshooting can only happen in the first period of the recovery. If we had  $p_{D,j} < p_D^{fb}$  in some other period, then in the previous period it would be optimal to increase  $n_{D,j-1}$  and accelerate the adjustment toward  $\bar{n}_D$ . Essentially, the optimal path requires that if the planner wants to allow for some depreciation in the  $D$  phase to speed up the recovery, it completely frontloads this depreciation.

Figure 5 shows the paths of  $n$  in the competitive equilibrium and second best corresponding to this scenario. As we explained above, in the second best entrepreneurs continue to be constrained even as the real exchange rate has reached its first best level (period 6 in the figure). The social planner, representing the consumers, has no incentive to relax their constraint further.

Let us summarize the main results of this section with a proposition:

**Proposition 5** (*Optimal policy*) *If  $a_0 < \hat{a}^D$ , then the competitive equilibrium is constrained inefficient. Depending on parameters, the optimal policy may involve some depreciation (relative to the competitive equilibrium) of the exchange rate in A, some further depreciation*

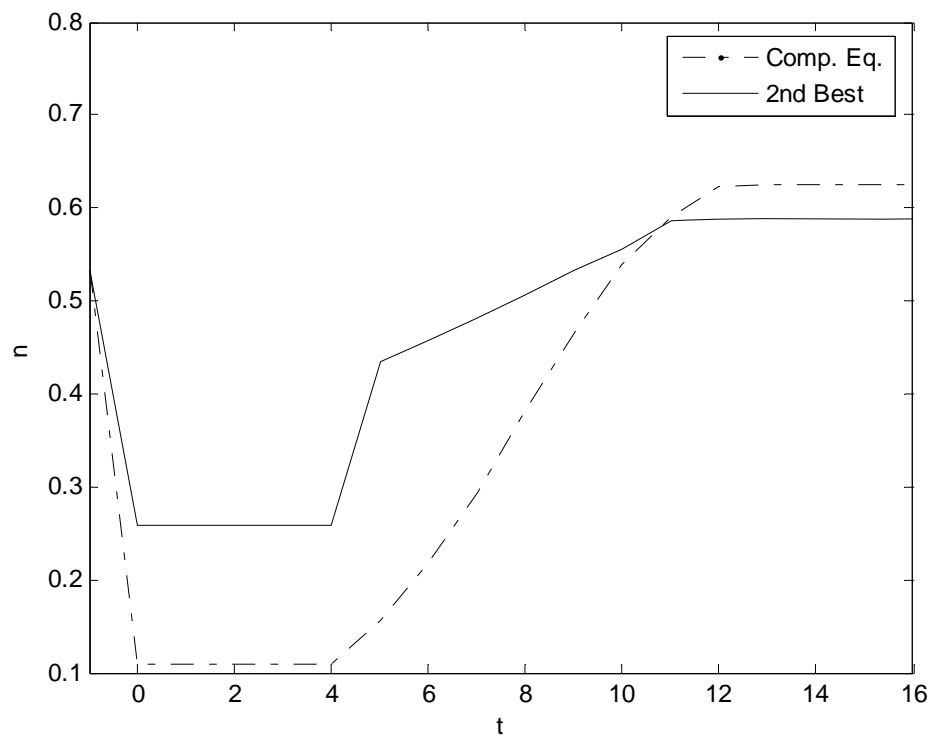


Figure 5: The Path of Export Units in the Second Best

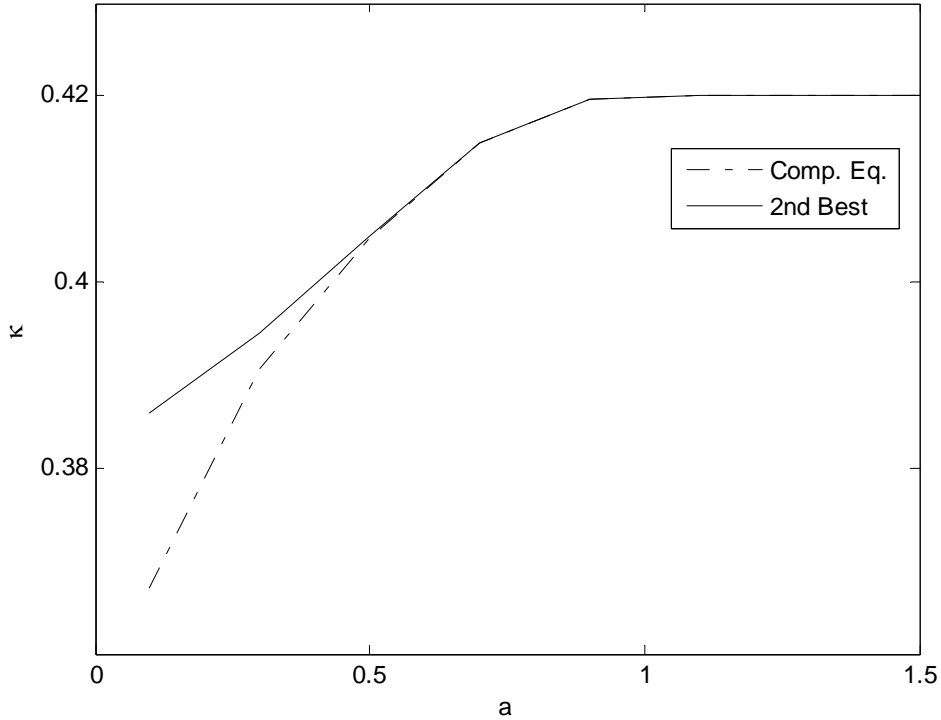


Figure 6: Income Effect of Optimal Intervention

*in the first period of the D phase, or some combination of both. In all of the above the overshooting phase in the second best lasts at most one period.*

For completeness, note that in terms of quantities the unambiguous results are that the optimal policy reduces fluctuations in  $n$  and the long run size of the export sector. The reason for the latter is that consumers are richer in the “second best,” and hence use a larger share of their labor resources to produce nontradables. To illustrate this wealth effect, Figure 6 compares the value of  $\kappa$  in the competitive equilibrium and the “second best,” for different levels of financial resources in the export sector,  $a_0$ . As expected, for high values of  $a_0$  the competitive equilibrium is close to the second best (and it becomes equal for  $a_0 \geq \hat{a}^D$ ). However, for low levels of  $a_0$ , the pecuniary externality is significant and the second best income is substantially higher than that of the competitive equilibrium.

## 4 Appreciation Persistence and Intervention

In the introduction we stated that the appreciations that concern policymakers are medium run ones. We develop this idea in two steps. First, we continue with the complete markets environment, where persistence only matters in an ex-ante sense. That is, since complete markets effectively provide appreciation-duration insurance, persistence matters only in that it affects the price of this insurance. Second, we study the incomplete markets version of our model, where realized duration matters. In this context, the export sector resources dwindle as the appreciation progresses, and hence the nature and scope for policy change as well.

### 4.1 Ex-ante persistence (Complete markets)

Ex-ante persistence, captured by the parameter  $\delta$ , matters for the positive and normative conclusions we have reached up to now. On one extreme, if  $\delta$  is close to one (very short lived appreciations) then the losses to be financed are not much and entrepreneurs' internal resources may suffice. On the other extreme, if  $\delta$  is very close to zero (very persistent appreciations), then the option value of keeping units is low, and there is no reason to protect the export sector either. It is for intermediate  $\delta$ 's that policy intervention may be needed.

Figure 7 illustrates this by showing the region where policy intervention is called for in the  $(\delta, a_0)$  space. The shaded region corresponds to the case where the equilibrium is constrained inefficient and exchange rate intervention is warranted. Note that there are many general equilibrium effects hidden in this figure. For example, as  $\delta$  changes, so does  $\kappa$ . Also, when  $\delta$  rises, firms reluctance to destroy during the appreciation rises. This reluctance exacerbates the (now shorter lived) appreciation, and hence the resources required to survive each period of appreciation. However, none of these effects is strong enough to change the qualitative shape of the figure and the conclusion that follows from it: that the medium run appreciations are the ones that are most likely to justify intervention.

### 4.2 Ex-post persistence (Incomplete markets)

In this section we relax the complete markets assumption and study the polar opposite case, where export firms only have access to a riskless bond (in order to isolate the effect that concerns us here, consumers still face complete markets). In this context, the export sector resources dwindle as the appreciation progresses, and hence the nature and scope for policy

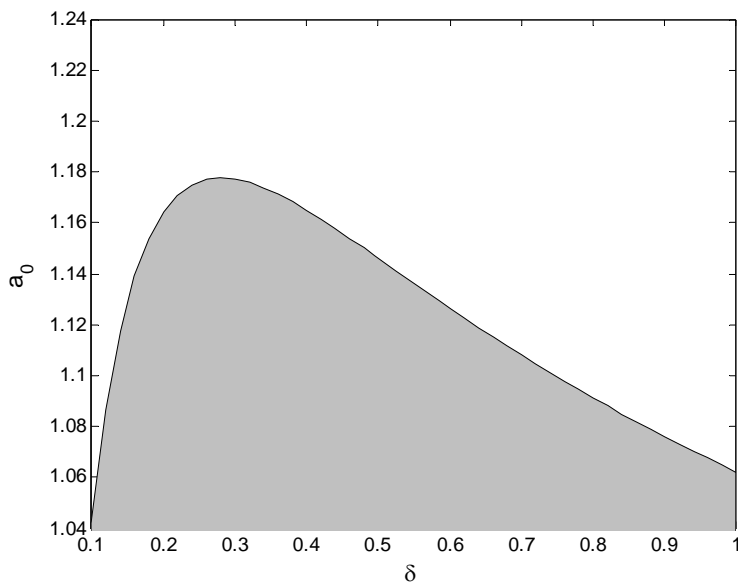


Figure 7: Persistence, Initial Wealth, and Intervention

change as well.

The main policy implication that follows from this modification is the timing of the exchange rate stabilization in the appreciation phase. Early on in the appreciation, the optimal policy is to postpone much of the intervention to the  $D$  phase. However, as the appreciation continues and the export sector's resources dwindle, the optimal policy not only increases in size but also starts shifting a larger share of the intervention to the appreciation phase.

#### 4.2.1 Incomplete Markets

The model in this section is identical to the one discussed earlier, except that firms hoard resources in a riskless bond. Thus their budget constraint when not consuming is:

$$\beta a_{t+1} - a_t = (1 - p_t)n_t. \quad (22)$$

Also, there are no longer first order conditions relating the marginal value of wealth state by state. These are replaced by a single first order condition:

$$-\phi_t + E\phi_{t+1} \leq 0, \quad (a_{t+1} \geq 0). \quad (23)$$

The consumer side of the problem remains unchanged (recall that consumers still trade in complete markets), except that the real exchange rate is no longer constant in the  $A$  phase, and hence neither is  $c_A^N$ .

The presence of dynamics within the  $A$  phase changes our notation slightly: First, we need to keep track of the number of periods within the  $A$  phase, for which we use subindex  $(A, i)$ , for  $i = 0, \dots, \infty$ . Second, the number of periods the economy spends in the  $A$  region before transiting to  $D$  matters for the initial conditions of the  $D$  phase. Thus we use the subindex  $(D, j, i)$ , where  $i$  stands for the length of time in the  $A$  phase before the transition into the  $D$  phase.

One can show that if initial resources are below a critical level, which we assume to be the case here, entrepreneurs' resources dwindle over time as the appreciation continues. In these circumstances,  $\phi_{A,i}$  and  $\phi_{D,0,i}$  rise, with  $\lim_{i \rightarrow \infty} \phi_{A,i} = \infty$  and  $\lim_{i \rightarrow \infty} \phi_{D,0,i} < \infty$ . It follows from these statements and first order condition (23) that along the path

$$\phi_{A,i} > \phi_{D,0,i} \tag{24}$$

and their ratio (at least eventually) rises over time. Thus,

$$p_{A,i} = 1 + \beta\delta \frac{\phi_{D,0,i}}{\phi_{A,i}} f < 1 + \beta\delta f \tag{25}$$

and asymptotes from above to the zero profit (loss) level:

$$\lim_{i \rightarrow \infty} p_{A,i} = 1.$$

Given the resources inherited from the  $A$  phase, dynamics within the  $D$  phase are identical to those described in the complete markets case. Figure 8 illustrates the path of  $n$  for three scenarios that differ in the duration of the appreciation phase (2, 8 and 16 periods). As expected, the number of export units declines as the appreciation continues. Moreover, a longer appreciation phase difficults the recovery phase. In the example portrayed in the figure, an appreciation lasting two periods is followed by a recovery that takes approximately five periods. In contrast, an appreciation lasting 16 periods is followed by a recovery that takes about ten periods.

The main reason for the difference in recovery time is that the longer the appreciation lasts, the harder it becomes for the export sector to jump-start the recovery during the depreciation phase. For example, note that in the 16-periods appreciation case, it takes about five periods to start a significant recovery of the export sector, while in the 2-periods appreciation case the recovery is immediately brisk.

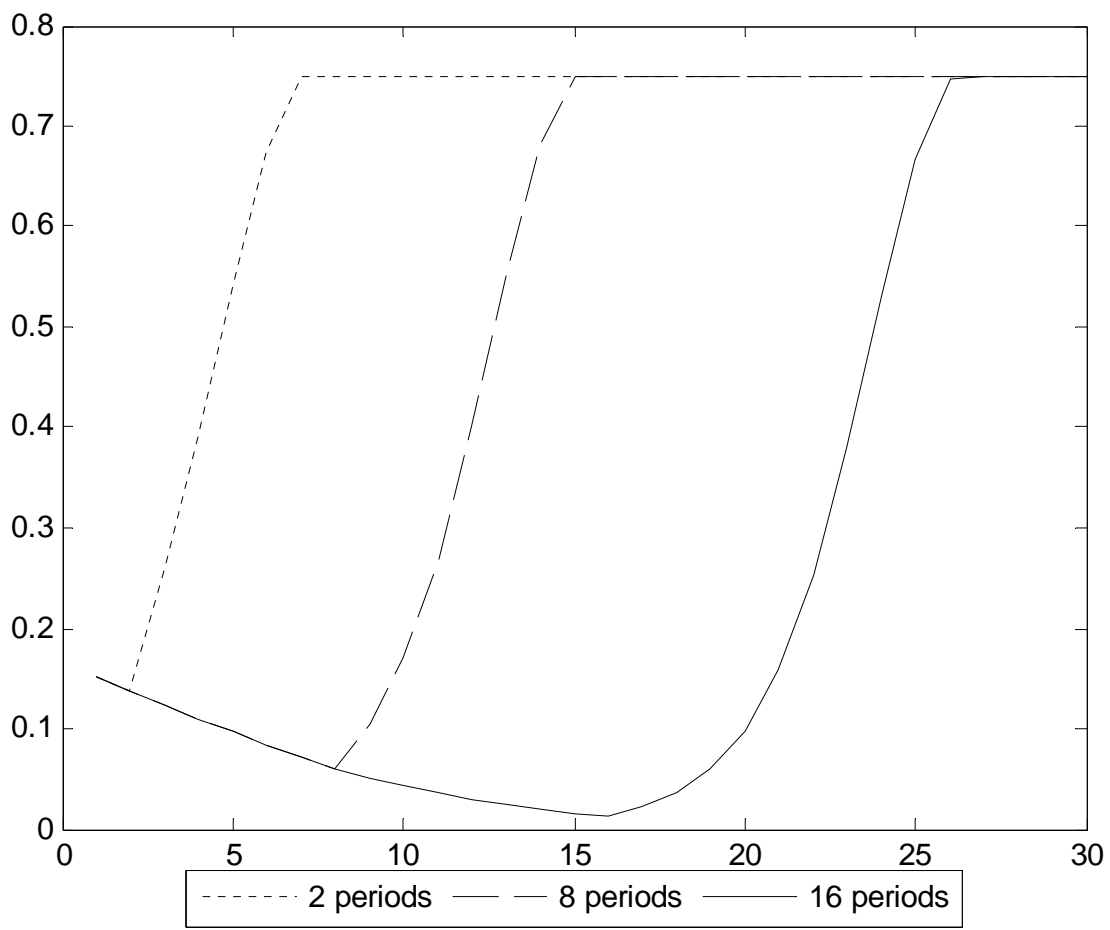


Figure 8: Dynamics of  $n$  with Incomplete Markets

### 4.2.2 Optimal Exchange Rate Intervention

What are the implications of dwindling financial resources for the optimal policy package? The analytics of this case are complex but the intuition is not. Let us develop the answer in two steps.

**A quasi-incomplete markets approximation** Consider as an intermediate step to gauge the form of the optimal policy, the effect of lowering  $a_0$  in the complete markets case. It is important to notice, however, that for this to be a reasonable approximation of the incomplete markets case we must keep  $\kappa$  constant across the scenarios. We do this by offsetting the impact of a decline in  $a_0$  on  $\kappa$  with an increase in  $e$ .

Figure 9 shows the ratio of second best exchange rate to the competitive equilibrium as a function of  $a_0$ . Panel (a) shows the ratio in the  $A$  region, while panel (b) shows the ratio at  $t_{D,0}$  (for ease of exposition the axis for  $a_0$  is inverted). In both panels there is a reference dashed line at one. Since in the incomplete markets case resources dwindle as the appreciation phase continues, one can think of the horizontal axis as a proxy for the duration of the appreciation phase. With this interpretation in mind we see that early on, when resources abound, there is limited space for intervention during the appreciation and intervention takes place primarily when the economy transits to the  $D$  phase. In this region, the optimal policy exhibits an over-overshooting (the ratio of second best to competitive equilibrium real exchange rate is below one in panel (b)). As time goes by, intervention in the appreciation phase rises, and hence the need for intervention in the  $D$  phase subsides.

**The incomplete markets case** The true incomplete markets case is more complex because intervention at any stage of the appreciation phase affects all the remaining periods of the appreciation phase as well as the first period of the  $D$  phase. However we can still characterize the second best numerically.

[TO BE COMPLETED]

## 5 Overoptimism and Frictions in Nontradables

In this section we consider two extensions that have similar policy implications. The first one replaces the taste shock device for a consumption distortion, while the second one considers frictions in the nontradable sector or labor market. In each case the core of our analysis

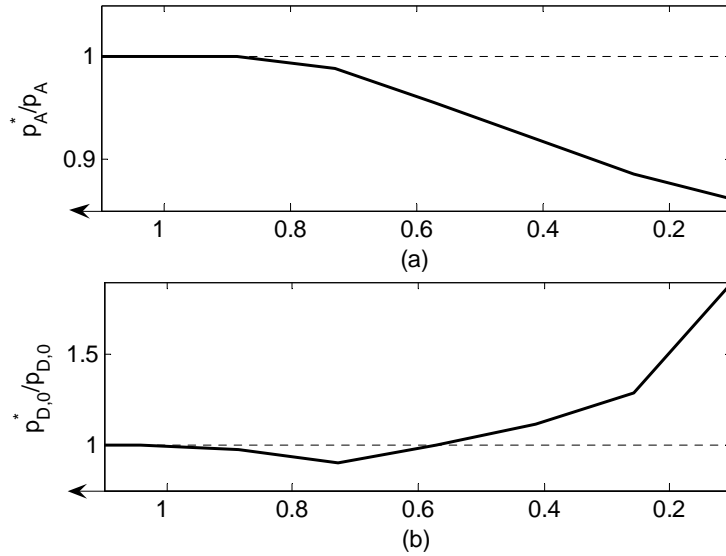


Figure 9: Initial Wealth and Intervention

goes through, but now the optimal policy reallocates a larger share of the intervention to the appreciation phase. In a well defined sense, the policy moves away from ex-post intervention and toward prevention.

### 5.1 Consumers' overoptimism

In reality, consumption binges rarely occur by themselves. In the international context, they often come as a response to a rise in national income due to a positive terms of trade shock in commodity producing countries, or due to a large increase in the supply of capital flows to the country. Adding external income shocks directly onto our complete markets, rational representative agent setup, would have no impact on consumption behavior. We need to add some “friction” on the consumption side as well.

The simplest extension along these lines is to replace the taste shocks for income (terms of trade) shocks  $e(s_t)$  but assume that consumers are overoptimistic with respect to the

expected duration of the high income phase  $A$ :<sup>11</sup>

$$\delta^{cons} < \delta.$$

In such case the first order conditions of the competitive equilibrium are identical to those of the taste shock context, with a reinterpretation of  $\theta_A > 1$  as a function of the gap  $\delta - \delta^{cons}$ . If the social planner does not share in the consumers' optimism, then it would be justified to implement some sort of saving policy, with the goal of reducing not only  $c_A^N$  but also  $c_A^T$ . Still, the intertemporal reorganization of  $c_t^N$  dimension of this policy has the same implications and considerations as in our analysis earlier. The main difference with that discussion is that since the social planner sees  $\theta_A$  as a distortion, it has more margin for intervention in the appreciation phase than before. This conclusion follows directly from its new first order condition:

$$u'(1 - n_A) = \lambda p_A^{fb} < \theta_A u'(1 - n_A).$$

## 5.2 Rigidities in the nontradables sector

More generally, a larger share of the intervention will be shifted toward the  $A$  phase whenever there are frictions in the nontradable sector. This is typically the case, for example, in the sudden stops literature, particularly when liabilities are dollarized. The latter limits the possibility and desirability of implementing a large overshooting in  $D$ , even if short lived.

Another example is the presence of a real wage rigidity, either as the result of a distortion or of a reservation wage.<sup>12</sup> Yet another is simply that some of the inputs of production in the nontradables sector are tradables.

Let us develop the simplest of these examples and assume that workers have a reservation wage of  $w$  units of tradable goods, which is not binding except, possibly, during the overshooting phase. Suppose that this is indeed the case for the optimal policy (but not for

---

<sup>11</sup>Alternatively, we could introduce procyclical consumption (or short horizons) through non-representative agents. The extreme version of this formulation is one where consumers live for only one period and must consume their income in that period. The social planner Pareto-weights a generation  $t$  periods from the current one by  $\beta^t$ . If no intergenerational transfer mechanism other than through the real exchange rate is available, then we are again in the situation just described. The constrained goal of the social planner is to reallocate consumption away from non-tradables during the appreciation phase. Relative to the pure taste shock scenario, a larger share of the adjustment is done in the  $A$  phase, in order to reduce the burden of the adjustment on the first generation in the  $D$  phase.

<sup>12</sup>See Blanchard (2006b) for a more thorough discussion of wage rigidities and appreciations; and Blanchard (2006a) for an application to the case of Portugal.

the competitive equilibrium): In the over-overshooting scenario, the social planner would like to bring  $p_{D,0}^{sb}$  below  $w$ , but it cannot. What is the impact of this binding constraint on the optimal policy? In particular, how much of the intervention is reallocated to the appreciation phase? Let us return to the complete markets environment to answer the latter question. We know that in this context the social planner's first order condition in the  $A$  phase is:

$$\theta_A u'(1 - n_A) = \lambda p_A^{fb} = \lambda(1 + \beta \delta f). \quad (26)$$

It follows immediately that

$$p_A^{sb,w} < p_A^{sb}$$

since the binding constraint must necessarily lower  $\kappa$  relative to the unconstrained case, and this implies that  $\lambda = u'(\theta_A \kappa)$  rises. In turn, the latter implies that  $n_A$  increases, which given the firms financial constraint can only be achieved with a larger intervention that drops the real exchange rate below that of the unconstrained case.

## 6 Final Remarks

The purpose of this paper is to show how financial frictions lend support to the view that persistent appreciations may justify intervention, even if agents are fully rational and forward looking. The reason for the intervention is not to improve the health of the export sector per se, as our social planner is primarily concerned about consumers (workers), but a pecuniary externality within consumers. By putting excessive cost pressure on financially constrained export firms during the appreciation phase, consumers reduce these firms' ability to recover once the factors behind the appreciation subside. The result is a severe overshooting and real wage collapse at that stage, which hurts consumers more than they gain from the extra consumption during the appreciation.

We show that the optimal policy goes beyond exchange rate stabilization during the appreciation phase. On one hand, the intensity of the intervention during the appreciation varies over time. On the other, the intervention also carries over to the depreciation phase, where in some cases it may be optimal to exacerbate the initial overshooting in exchange for a faster recovery. In abstract, this policy path can be implemented through an appropriate sequence of taxes and subsidies to nontradable consumption. In reality, the flexibility of such policies is limited, leaving to expenditure policy and central bank's reserves management

most of the burden. While these are not perfect substitutes for taxes and subsidies, much of our insights still carry over to them.

When thinking about policy, it is worth noting the distinction between an “appreciation” and an “overvaluation.” The latter is an elusive concept in practice but it has a well defined meaning in ours: an overvaluation refers to a situation where the exchange rate is higher than it is socially optimal. However, this gap is not limited to an appreciation episode as it can also take place during a depreciation phase. The over-overshooting result is an example of an overvaluation during the depreciation phase. A wage rigidity is an example of when such overvaluation cannot be cured fully with intervention within the depreciation phase. The latter example also illustrates the linkages between current interventions and *future* overvaluations.

We note also that our model uses a single reason –a financial constraint– for constrained production in the appreciation and depreciation phases. However, some of our conclusions extend to other scenarios as well. In particular, we could replace the financial constraint in the depreciation phase for a technological time to build assumption. In such case, the overshooting is also directly linked to excessive export destruction in the appreciation phase and there is a reason for intervention. The main difference in this instance is that the optimal policy does not prolong the intervention into the depreciation phase.

Finally, while our analysis focuses on the real exchange rate, it seems suitable for other important relative prices within an economy. For example, a real estate boom can have important cost consequences for sectors that compete with the construction sector for inputs and factors of production. More broadly, ours is a model of the optimal management of sectoral reallocation in the presence of temporary but persistent shocks, when some sectors have limited financial resources and technological flexibility.

## 7 Appendix: Technical Assumptions, Lemmata and Propositions

### 7.1 Entrepreneur's problem in recursive form

Let  $V(a, n^-; s^t)$  denote the expected utility of an entrepreneur in state  $s^t$  who is holding  $a$  units of cash and  $n^-$  production units. For all the equilibrium price sequences we will consider in the paper, it is possible to show that this expected utility is finite for any pair  $a, n^-$  and satisfies the following Bellman equation:

$$\begin{aligned}
 V(a, n^-; s^t) &= \max_{\substack{c^{T,e}, n, \\ \{a(s_{t+1})\}_{s_{t+1} \in \mathcal{S}}}} c^{T,e} + \beta \sum_{s_{t+1} \in \mathcal{S}} \pi(s_{t+1}|s_t) V(a(s_{t+1}), n; \langle s^t, s_{t+1} \rangle) \quad (27) \\
 \text{s.t.} \quad &c^{T,e} + q(s^t)n + \beta \sum_{s_{t+1} \in \mathcal{S}} \pi(s_{t+1}|s_t) a(s_{t+1}) \leq (1 - p(s^t))n + a + q(s^t)n^- \\
 &a(s_{t+1}) \geq 0, \quad n \geq 0, \quad c^{T,e} \geq 0.
 \end{aligned}$$

**Lemma 1** *The value function  $V(a, n^-; s^t)$  takes the linear form:*

$$V(a, n^-; s^t) = \phi(s^t) \cdot (a + q(s^t)n^-).$$

**Proof.** If the  $V$  on the right of the max operator in (27) is linear, then the problem is linear, so linearity of the value function can be shown by an induction argument. Notice that  $a$  and  $n^-$  enter only in the first constraint in the form  $[a + q(s^t)n^-]$ . To show that the constant term is zero in the linear expression for  $V$ , notice that if an entrepreneur starts with  $a = 0$  and  $n^- = 0$ , the only feasible path is zero investment and consumption, so  $V(0, 0; s^t) = 0$ . ■

The first order conditions (5)-(7) can be derived from problem (27) using the linearity of  $V$  and the envelope theorem, which implies that the Lagrange multiplier on the budget constraint is equal to  $\phi(s^t)$ .

### 7.2 Proof of Proposition 1

The cutoff is given by

$$\hat{a}^{fb} = \frac{\left(p_A^{fb} - 1\right) n_A^{fb} + \beta \delta f \left(n_D^{fb} - n_A^{fb}\right) - \beta \delta \left(1 - p_D^{fb}\right) n_D^{fb}}{1 - \beta(1 - \delta)},$$

where  $\{p_s^{fb}\}_{s \in \mathcal{S}}$  and  $\{n_s^{fb}\}_{s \in \mathcal{S}}$  are defined in the text.

Let us conjecture, and verify, that the prices in the text are equilibrium prices. Under these prices one can show, by guessing and verifying, that  $V(a, n^-; s^t) = a + q(s^t) n^-$ , i.e., that  $\phi(s^t) = 1$ , and, at each node  $s^t$ , the entrepreneur is indifferent between any feasible portfolio  $n, \{a(s_{t+1})\}$ . If the entrepreneur begins with  $a_0$ , he can consume the difference  $a_0 - \hat{a}^{fb}$  and then adopt the following rule: choose  $n(s^t) = n_A, a(D|s^t) = f \left( n_D^{fb} - n_A^{fb} \right) - \left( 1 - p_D^{fb} \right) n_D^{fb}$  and  $a(A|s^t) = \hat{a}^{fb}$ , for each history  $s^t = \{A, A, \dots, A\}$ ; choose  $n(s^t) = n_D, a(D|s^t) = 0$  for each history  $s^t = \{A, \dots, A, D, \dots, D\}$ . These decisions are feasible and, given these decisions, it is straightforward to check that all markets clear.

### 7.3 Characterization of the constrained equilibrium

First let us establish the following preliminary lemma.

**Lemma 2** *Define the function*

$$H(n) \equiv fn - \left( 1 - \frac{\kappa}{1-n} \right) n - x,$$

*the equation  $H(n) = 0$  has a unique solution  $n^* \in (0, 1)$ , for each  $\kappa > 0$  and  $x > 0$ . Moreover,  $H(n) > 0$  for each  $n > n^*$ . The solution  $n^*$  is increasing in  $x$ . If  $x = 0$  the equation can have one or two solutions, one of which is  $n = 0$ . The properties above apply to the largest solution.*

**Proof.** A solution exists because  $H$  is continuous in  $[0, 1)$ ,  $H(0) = -x$  and  $\lim_{n \rightarrow 1} H(n) = \infty$ . Consider the case  $x > 0$ . Let  $n^*$  be a solution, then  $f - \left( 1 - \frac{\kappa}{1-n^*} \right) > 0$  must hold. If  $n > n^*$ ,  $H'(n) = f - \left( 1 - \frac{\kappa}{1-n} \right) + \kappa \frac{n}{(1-n)^2} > 0$  because  $f - \left( 1 - \frac{\kappa}{1-n} \right) > f - \left( 1 - \frac{\kappa}{1-n^*} \right) > 0$  and  $\kappa \frac{n}{(1-n)^2} > 0$ . This implies that  $H(n) > 0$  for each  $n > n^*$ , and the solution is unique. The comparative statics follow from the implicit function theorem. When  $x = 0$  the solution  $n^* = 0$  is trivial. If there is another solution  $n^* > 0$ , the properties stated can be proved following the steps of the case  $x > 0$ . ■

**Proposition 6** *Suppose Assumptions 1 and 2 hold and  $a_0 < \hat{a}^{fb}$ . Then, there exists an equilibrium with the following properties.*

- (i) *The equilibrium price of non-tradables and the number of firms are constant during the A phase.*

(ii) The equilibrium price of non-tradables and the number of firms are (weakly) increasing in the  $D$  phase.

(iii) In the  $D$  phase the price of non-tradables converges to  $p_D^{fb}$  in finite time, at which point the number of firms stabilizes.

**Proof.** We will proceed in three steps. First, we define a map  $T$  for the coefficient  $\kappa$ . Second, we derive some properties of this map. Finally, we show that this map has a unique fixed point and from this fixed point we can construct an equilibrium with the desired properties.

*Step 1.* Define

$$\underline{\kappa} = \frac{e}{2} \frac{1 - \beta(1 - \delta)}{(1 - \beta)\theta_A + \beta\delta},$$

and let  $\kappa^{fb}$  be defined as in the text.

Fix a value for  $\kappa \in [\underline{\kappa}, \kappa^{fb}]$  and construct an equilibrium as follows.

*Phase A.* If

$$(1 - \beta(1 - \delta)) a_0 > \left(p_A^{fb} - 1\right) \left(1 - \frac{\kappa\theta_A}{p_A^{fb}}\right), \quad (28)$$

then set  $p_A$  equal to  $p_A^{fb}$ , set  $n_A = \hat{n}_A = \left(1 - \frac{\kappa\theta_A}{p_A^{fb}}\right)$  and

$$a_{D,0} = \frac{1}{\beta\delta} \left[ (1 - \beta(1 - \delta)) a_0 - \left(p_A^{fb} - 1\right) n_A \right] > 0. \quad (29)$$

Notice that  $\hat{n}_A > 0$ . Since  $\kappa \leq \kappa^{fb}$  we have  $1 - \frac{\kappa\theta_A}{p_A^{fb}} \geq 1 - \frac{\kappa^{fb}\theta_A}{p_A^{fb}} > 0$ , where the last inequality follows from assumption (A1).

If (28) does not hold, then set  $p_A$  equal to the solution of

$$(1 - \beta(1 - \delta)) a_0 = (p_A - 1) \left(1 - \frac{\kappa\theta_A}{p_A}\right), \quad (30)$$

(which has a unique solution in  $[1, p_A^{fb}]$ ), set  $n_A = \left(1 - \frac{\kappa\theta_A}{p_A}\right)$ , and set

$$a_{D,0} = 0.$$

Notice that when  $p_A = \kappa\theta_A$ , the RHS of (30) is zero, therefore  $p_A \in [\kappa\theta_A, p_A^{fb}]$  and  $n_A \geq 0$ .

*Phase D.* Define

$$\hat{n}_D = 1 - \frac{\kappa}{p_D^{fb}}.$$

Construct the sequence  $\{n_{D,j}\}$  that satisfies:

$$f(n_{D,0} - n_A) = \left(1 - \frac{\kappa}{1 - n_{D,0}}\right) n_{D,0} + a_{D,0} \quad (31)$$

$$f(n_{D,j} - n_{D,j-1}) = \left(1 - \frac{\kappa}{1 - n_{D,j}}\right) n_{D,j} \text{ for } j = 1, 2, \dots, J \quad (32)$$

until  $n_{D,J+1}$  is larger than  $\hat{n}_D$ . From then on set:

$$n_{D,j} = \hat{n}_D \text{ for all } j > J.$$

Letting  $x = a_{D,0} + fn_A$ , Lemma 2 ensures that (31) has a solution for  $n_{D,0}$  (if  $a_{D,0} + fn_A = 0$ , pick the solution with the largest  $n_{D,0}$ ). To show that  $n_{D,0} \geq n_A$  consider the following: Either  $H(\hat{n}_D) \leq 0$ , and the solution will be larger than  $\hat{n}_D$ . In this case the economy converges to  $\hat{n}_D$  immediately and  $\hat{n}_D \geq \hat{n}_A \geq n_A$  from assumption (A2). If, instead  $H(\hat{n}_D) > 0$  then  $H(n_{D,0}) = 0$ . Notice that

$$H(n_A) = \left(\frac{\kappa}{1 - n_A} - 1\right) n_A - a_{D,0} \leq \left(p_D^{fb} - 1\right) n_A - a_{D,0} < 0$$

where the first inequality follows from the following chain of inequalities

$$\frac{\kappa}{1 - n_A} = \frac{\kappa\theta_A}{1 - n_A\theta_A} \frac{1}{\theta_A} < p_A^{fb} \frac{1}{\theta_A} < p_D^{fb} < 1,$$

(the second inequality in the chain follows from assumption (A2)). Therefore, Lemma 2 implies that  $n_{D,0} > n_A$ . In a similar way, it is possible to prove that (32) implies  $n_{D,j} \geq n_{D,j-1}$  for each  $j$ .

From these two steps we obtain a sequence  $p_A, \{p_{D,j}\}$ , which can then be substituted in expression (2), to obtain a  $\kappa'$ . This defines a map  $T : [\underline{\kappa}, \kappa^{fb}] \rightarrow [\underline{\kappa}, \kappa^{fb}]$ .

*Step 2.* It can be shown that the map  $T$  is continuous. Furthermore, let us prove that

$$T(\kappa(1 + \Delta)) < (1 + \Delta)T(\kappa).$$

In the construction in Step 1, an increase in  $\kappa$  leads to a (weak) reduction in  $n_A$  and  $n_{D,j}$  for all  $j$  (for the initial conditions of phase  $D$  notice that if (28) is satisfied, then, using the definition of  $p_A^{fb}$ , it is possible to show that  $a_{D,0} + fn_A$  is independent of  $\kappa$ ; if (28) is not satisfied, then an increase in  $\kappa$  leads to a decrease in  $n_A$ ). But since  $n_A = 1 - \theta_A\kappa/p_A$ ,  $n_{D,j} = 1 - \kappa/p_{D,j}$ , the prices  $p_A$  and  $p_{D,j}$  increase less than proportionally than  $\kappa$ . Therefore,  $\kappa'$  increases less than proportionally.

*Step 3.* Define the following map for  $z \equiv \log(\kappa)$  :

$$z' = \tilde{T}(z) \equiv \log(T(\exp^z)).$$

Step 2 shows that this map is continuous and has slope smaller than 1. Therefore this map has a unique fixed point (uniqueness is not needed for the statement of this proposition, but will be useful in the next propositions). Let  $\kappa$  be the fixed point and consider the prices and quantities constructed in Step 1. To ensure that they are an equilibrium, it remains to check that the sequence of prices and quantities are optimal for the entrepreneur. Derive the marginal utility of money at  $t_{D,0}$  from the recursion:

$$\phi_{D,j} = \beta \frac{f}{f - (1 - p_{D,j})} \phi_{D,j+1}. \quad (33)$$

By construction we have  $p_{D,j} \leq p_D^{fb}$ , which implies that  $\phi_{D,j} \geq 1$ . Moreover, entrepreneurs' consumption and cash savings are zero until the point where  $\phi_{D,j} = 1$ .

To check optimality in phase  $A$ , notice that

$$\phi_A = \frac{\beta \delta f}{p_A - 1} \phi_{D,0}$$

and  $\phi_A > \phi_{D,0}$  iff  $p_A < p_A^{fb}$ . ■

### 7.3.1 Proof of Propositions 2 and 3

The following lemma provides a useful preliminary result.

**Lemma 3** *The equilibrium value of  $\kappa$  is (weakly) increasing in  $a_0$ .*

**Proof.** Let  $T(\kappa; a_0)$  be the mapping  $[\underline{\kappa}, \kappa^{fb}] \rightarrow [\underline{\kappa}, \kappa^{fb}]$  defined in the proof of Proposition 6, indexed by the initial wealth  $a_0$ . Choose two values  $a'_0 < a''_0$ . Let  $\kappa'$  and  $\kappa''$  be the corresponding equilibrium values of  $\kappa$ . Now, fixing  $\kappa'$  we want to show that  $T$  is monotone in  $a_0$ , i.e.,  $T(\kappa'; a''_0) \geq T(\kappa'; a'_0)$ .

If (28) holds at  $a'_0$ , then an increase in  $a_0$  leaves  $p_A$  unchanged, and it increases  $a_{D,0}$  (from 29) and leaves  $n_A$  unchanged. If (28) does not hold, an increase in  $a_{D,0}$  leads to an increase in  $p_A$ , and an increase in  $a_{D,0} + fn_A$ , since  $a_{D,0}$  either remains zero or becomes positive and  $n_A$  increases. In both cases,  $a_{D,0} + fn_A$  increases. This means that, in phase  $D$ , there will be a (weak) increase in  $n_{D,j}$  for all  $j$ , and, thus, a (weak) increase in  $p_{D,j}$  for all  $j$ . Therefore,  $T(\kappa'; a''_0) \geq T(\kappa'; a'_0) = \kappa'$ .

This implies that  $T(\kappa; a_0'')$  has a fixed point in  $[\kappa', \kappa^{fb}]$ . Since  $T$  has a unique fixed point and  $T(\kappa''; a_0'') = \kappa''$ , by construction, this implies  $\kappa'' \geq \kappa'$ . ■

Now we can prove the two propositions. Consider first Proposition 2. Suppose that at  $a_0'$  we have  $p_A = p_A^{fb}$  in equilibrium. This means that (28) holds at  $a_0'$ . Since  $\kappa'' \geq \kappa'$ , (28) holds *a fortiori* for  $a_{D,0}'', \kappa''$ , it follows that at the new equilibrium  $p_A = p_A^{fb}$  and  $a_{D,0} > 0$ .

Consider next Proposition 3. Suppose that at  $a_0'$  we have  $p_{D,0} = p_D^{fb}$  in equilibrium. This means that the following inequality holds

$$fn_D' \leq \left(1 - \frac{\kappa'}{1 - \hat{n}_D'}\right) \hat{n}_D' + fn_A' + a_{D,0}' \quad (34)$$

where  $\hat{n}_D' = 1 - \kappa'/p_D^{fb}$ .

Now we want to prove the following inequality

$$fn_A'' + a_{D,0}'' \geq fn_A' + a_{D,0}' \quad (35)$$

If (28) holds at  $a_0'$  then some algebra (using the definition of  $p_A^{fb}$ ) shows that

$$fn_A'' + a_{D,0}'' = fn_A' + a_{D,0}' = \frac{1}{\beta\delta} [(1 - \beta(1 - \delta)) a_0].$$

If (28) does not hold at  $a_0'$ , then we have  $a_{D,0}'' \geq 0 = a_{D,0}'$ . Furthermore, we can show that  $n_A'' \geq n_A'$ . Notice that  $\kappa''$  greater than  $\kappa'$  only because, on average, equilibrium prices are larger. However,  $p_{D,j}' = p_D^{fb}$  at  $a_0'$ , and  $p_{D,j}'' \leq p_D^{fb}$ . This implies that

$$\frac{p_A''}{p_A'} \geq \frac{\kappa''}{\kappa'},$$

and since  $n_A = 1 - \kappa/p_A$  this implies  $n_A'' \geq n_A'$ . Therefore, (35) holds in all cases. This implies that (34) also holds at  $a_0''$ , and therefore  $p_{D,0} = p_D^{fb}$ .

Notice that if (28) holds at  $a_0'$  then, we can proceed as in the proof of Proposition 2 and show that  $a_{D,0}'' > a_{D,0}'$  and  $n_A'' = n_A' = \hat{n}_A$ .

[TO BE COMPLETED]

## 8 Appendix: Algorithms and Parametric Assumptions

[TO BE COMPLETED]

## References

- [1] Aghion, P., P. Bacchetta, and A. Banerjee, “Financial Development and the Instability of Open Economies,” MIT mimeo 2003.
- [2] Blanchard, O.J. “Adjustment within the Euro: The difficult case of Portugal,” MIT mimeo, February 2006a.
- [3] Blanchard, O.J. “Current Account Deficits in Rich Countries,” MIT mimeo, October 2006b.
- [4] Burnstein, A., M. Eichenbaum, and S. Rebelo, “Large Devaluations and the Real Exchange Rate,” *Journal of Political Economy* August 2005.
- [5] Caballero, R.J. and A. Krishnamurthy, “International and Domestic Collateral Constraints in a Model of Emerging Market Crises,” *Journal of Monetary Economics* 8 (3), pp. 513-548, 2006.
- [6] Caballero, R.J. and A. Krishnamurthy, “Smoothing Sudden Stops,” *Journal of Economic Theory* 119(1), November 2004, pp. 104-127
- [7] Corden, W.M., “Booming Sector and Dutch Disease Economics: Survey and Consolidation,” *Oxford Economic Papers* 36 (1984), 359-380.
- [8] Farhi, E., M. Golosov, and A. Tsyvinski, “A Theory of Liquidity and Regulation of Financial Intermediation,” MIT mimeo 2006.
- [9] Geanakoplos, J. and H.M. Polemarchakis. “Existence, Regularity, and Constrained Suboptimality of Competitive Allocations When the Asset Market Is Incomplete.” in *General Equilibrium Theory*. Vol. 2. ed. by Gerard Debreu, 67-97, 1996. (First published 1986.)
- [10] Golfajn, I. and R. Valdes, “The Aftermath of Appreciations,” *Quarterly Journal of Economics* 114(1), February 1999, pp. 229-262.
- [11] Gourinchas, P.O., , R. Valdes, and O. Landerretche, “Lending Booms: Latin America and the World,” *Economia*, 1(2), Spring 2001.
- [12] Krugman, P., “The Narrow Moving Band, the Dutch Disease, and the Competitive Consequences of Mrs. Thatcher,” *Journal of Development Economics* 27 (1987), 41-55.
- [13] Lorenzoni, G. “Inefficient Credit Booms,” MIT mimeo 2006.

- [14] Wijnbergen, S. van, “The Dutch Disease: A Disease After All?” *The Economic Journal*, March 1984.