

Optimal Fiscal Policy and Amplification in a Small Open Economy*

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October 18, 2005

Abstract

We study optimal fiscal policy in a small open economy characterized by two main frictions – incomplete financial markets and an inability of the government to commit to policy. Our main contribution is to show that in this environment, the best sustainable policy can amplify and prolong shocks to output. In particular, the government’s credibility not to expropriate foreign capital endogenously varies with the state of the economy and may be “scarcest” during recessions. This increased threat of expropriation depresses investment and prolongs downturns.

*We thank comments and suggestions from Andy Atkeson, Doireann Fitzgerald, Pierre-Olivier Gourinchas, Hugo Hoppenhayn, Pat Kehoe, Narayana Kocherlakota, Enrique Mendoza, Marcelo Oviedo, Chris Phelan, Ken Rogoff, Paul Romer, Andy Rose, Aleh Tsyvinsky, Carlos Vegh and Ivan Werning. We also thank seminar participants at Berkeley, Board of Governors, Cornell, Harvard, NBER summer institute, SED, Minneapolis Fed, New York University, Boston University, New York Federal Reserve Bank, Northwestern University and University of California at Santa Cruz for comments.

Introduction

We study optimal fiscal policy in a small open economy (SOE) characterized by two main frictions – incomplete financial markets and an inability of the government to commit to policy. Incomplete markets provides an incentive to use fiscal policy to proxy for missing insurance markets and the lack of commitment tempts the government to confiscate foreign capital. Our main contribution is to show that in this environment, the best sustainable policy can amplify and prolong shocks to output. In particular, the government’s credibility not to expropriate capital endogenously varies with the state of the economy and is “scarcest” during recessions. This increased threat of expropriation depresses investment during downturns.

The empirical motivation for our model is the pattern observed in many emerging markets: Governments allow foreign capital to earn large returns in booms but confiscate capital income during crises. The most recent crisis in Argentina in January 2002 is a dramatic illustration of this phenomenon. During the crisis, Argentina repudiated contracts, froze prices on privately owned utilities, and imposed taxes on exports. Measures of expropriation risk as calculated by the Heritage Foundation and Fraser institute deteriorated sharply. A similar deterioration of property rights is observed in other emerging market crises.

The oscillation between pro-growth policies and populism observed in many developing economies seems to contribute to (rather than stabilize) the volatility of output. Our paper rationalizes such behavior by focusing on two key characteristics that distinguish emerging market economies: inadequate insurance markets and the inability of governments to commit to future policy promises.

Our baseline model has a government implementing fiscal policy on behalf of risk averse domestic agents who lack access to financial markets and do not own capital. Risk neutral foreigners invest capital in the economy that is immobile for one period and has an opportunity cost given by the world interest rate. The government uses linear taxes/subsidies to transfer income between foreign capitalists and domestic agents. The government is assumed to run a balanced budget. These assumptions generate a sparse structure that isolates our mechanism. We show that the mechanism remains relevant in richer settings.

Uncertainty in the benchmark model is driven by a stochastic endowment process. The endowment shock generates a risk that the domestic agents cannot insure. The government can provide insurance by using a combination of transfers and taxes on capital income. A useful expositional feature of the additive endowment shock is that the marginal product of capital is independent of the shock’s realization. That is, the first-best capital stock is

acyclical. This feature of the model allows us to starkly isolate the role of fiscal policy in generating investment fluctuations.

If the government could commit, the optimal fiscal policy (the Ramsey solution) does not distort capital in this economy (similar to Judd 1985 and Chamley 1986) but does provide insurance to the domestic agents. The Ramsey insurance scheme exploits the fact that capital is perfectly elastic *ex ante* but inelastic *ex post*. The Ramsey program imposes taxes on capital that vary across states of nature but have an expected payment of zero (as in Zhu (1992) and Chari et al (1994)). Investment is therefore constant at the first best level. Note that (as in Judd 1985), the optimality of zero capital taxation is preserved even in an environment where the government does not internalize the welfare of foreign capitalists.

In the Ramsey solution, the government completely insures domestic agents period by period. It does so by taxing foreign capital when the endowment is low and subsidizing capital when it is high. That is, the government drives down the *ex post* return to capital if the realized endowment shock is low. We observe this implemented in practice in many ways – higher taxes, failure to pay out nominal promises on contracts, confiscation, etc. Nevertheless, investors are willing to bear this risk *ex ante* as long as the returns to capital are sufficient when times are good.

The government uses taxes to transfer resources across states within a period. However, given the absence of financial assets, it cannot transfer resources across periods to smooth consumption inter-temporally. Therefore, consumption of domestic agents under the Ramsey plan still varies over time.

What if the government cannot commit to the Ramsey plan? Given that the capital stock is fixed for one period, the government is tempted to tax capital at the highest possible rate and to redistribute the proceeds to the workers. We follow Chari and Kehoe (1990) and use sustainable equilibria as our solution concept.

The best sustainable equilibrium for the government is supported by the threat that any deviation will be punished by reversion to the worst sustainable equilibrium. The worst equilibrium is easily characterized in our SOE environment. Specifically, the sustainable equilibrium that delivers the lowest payoff to the government is one where taxes on capital are at their highest possible level for all histories.

Incentive compatible allocations are those which offer the government no benefit from deviation when facing as a punishment reversion to the worst equilibrium. The best sustainable equilibrium can then be characterized by the incentive compatible allocation that maximizes the government's objective function.

A main result of the paper characterizes the sustainability of the Ramsey plan. Under the Ramsey plan, consumption is increasing in the expected endowment. That is, a higher expected endowment allows the government to transfer a greater amount of income while delivering to the capitalists their outside option. When endowment shocks are persistent, a low endowment today implies low consumption tomorrow in the Ramsey plan. Therefore, the lower the endowment shock today, the greater the government's incentive to deviate tomorrow. In other words, the government's credibility regarding taxation of next period's capital income is lowest during a bad endowment realization.

The reason why this is the case is as follows. If the government deviates tomorrow, the amount of income it expropriates depends only on next period's realized shock and the stock of capital. Under the Ramsey plan, capital is independent of the history of shocks. Therefore, consumption following a deviation from the Ramsey plan is independent of previous states. Given Markovian shocks and the fact that capital adjusts after one period, the continuation values tomorrow (under the Ramsey plan or after deviation) are only a function of next period's shock. However, as noted above, tomorrow's consumption under the Ramsey plan is greater following a higher endowment realization today. Taken together, the government has the greatest incentive to deviate after a low realization. This is the sense in which credibility is "scarce" during downturns, depressing investment.

Note that this reasoning, as well as the formal proof, does not require strong assumptions regarding the specific shape of the government's objective function (other than strict concavity). Moreover, the result requires minimal characterization of the continuation values.

A common feature of models with incentive compatibility constraints is that these constraints tend to bind in high states. This stems from the nature of insurance – payments are made in high states and benefits are received in low states. However, we show that despite this, investment may be distorted in low states even when capital is undistorted in high states.

The main result concerns the sustainability of efficient investment. We use both numerical and analytical techniques to explore conditions under which investment remains pro-cyclical when far from the efficient level. We also show that there are regions of the parameter space in which investment is larger during low realizations of the shock.

We also extend the model to consider static insurance markets and productivity shocks. Lastly, we discuss the case when the government has access to a bond.

In an important paper in the international business cycle literature, Kehoe and Perri (2001) consider a model of risk sharing across two countries with limited commitment. Dif-

ferently from Kehoe and Perri (2001), we study a small open economy and emphasize the role of the government in generating amplification. Also related is Lane and Tornell (1999) who study a model without commitment where parties have access to a common savings technology. They restrict attention to Markov equilibria, and do not study the issue of credibility. Our paper is also related to Phelan and Stacchetti (2001), where a full blown policy game is analyzed for the case without uncertainty.

The paper is organized as follows. In Section 1 we present the model with endowment shocks and describe the full commitment solution. Section 2 presents the limited commitment results. Section 3 extends the model to static insurance markets and productivity shocks. Section 4 discusses the case where the government has access to a bond and Section 5 concludes.

1 Model

Time is discrete and runs to infinity. The economy is composed of a government and two types of agents: domestic agents and foreign capitalists. Domestic agents (or “workers”) are risk averse and supply inelastically l units of labor every period for a wage w . In addition they receive an endowment shock, z every period that has the following properties:

Assumption 1 $z \in Z$ follows a Markov process and the finite set Z has a highest element $\bar{z} < \infty$ and a lowest element $\underline{z} > 0$.

Let $z^t = \{z_0, z_1, \dots, z_t\}$ be a history of endowment shocks up to time t . Denote by $q(z^t)$ the probability that z^t occurs.

The expected lifetime utility of workers is given by

$$\sum_{t=0}^{\infty} \sum_{z^t} \beta^t q(z^t) u(c(z^t))$$

where $c(z^t)$ is their consumption in history z^t and u is a standard utility function with $u' > 0, u'' < 0$.

Assumption 2 (Segmented Capital Markets) *Workers do not have access to financial markets. Their consumption is given by*

$$c(z^t) = z_t + w(z^t)l + T(z^t)$$

where $T(z^t)$ are transfers received from the government at history z^t and $w(z^t)$ is the competitive wage at history z^t .

There is a mass of risk-neutral foreign capitalists that supply capital, but no labor. The foreign capitalists own competitive domestic firms that produce by hiring domestic labor and using foreign capital. The production function F is of the standard neoclassical form:

$$y = F(k, l)$$

where F is constant returns to scale with $F_k > 0$ and $F_{kk} < 0$.

The capitalists have access to financial markets. We assume a small open economy where the capitalists face the exogenous world interest rate of r^* . We assume that capital is installed before the endowment shock and tax rate are realized and cannot be moved until the end of the period. For simplicity we assume the depreciation rate is 0. Capital profits are denoted $\pi(z^t)$, where

$$\pi(z^t) = F(k(z^{t-1}), l) - w(z^t)l$$

We make the following assumption about the government's objective function:

Assumption 3 (Redistributive Government) *The government's objective function is to maximize the lifetime utility of the workers.*

We model the government as benevolent towards domestic workers. Alternatively, we could assume that the government maximizes the utility of a subset of agents, such as political insiders or public employees. The analysis will make clear that our results extend to these alternative objective functions as long as the favored agents are risk averse and lack access to capital markets.

The government taxes capital profits at a linear rate $\tau(z^t)$ and transfers the proceeds to the workers $T(z^t)$. We assume the government does not have access to international financial markets:

Assumption 4 (Balanced Budget) *The government runs a balanced budget at every state.*

$$\tau(z^t)\pi(z^t) = T(z^t)$$

This is an important assumption. We discuss relaxing this constraint in Section 4.

Taking as given a tax rate plan $\tau(z^t)$, firms maximize after-tax profits,

$$E_0 \sum_t \left(\frac{1}{1+r^*} \right)^t (1 - \tau(z^t)) \pi(z^t).$$

Profit maximization implies the following two first order conditions:

$$F_l(k(z^{t-1}), l) = w(z^t) \quad (1)$$

$$r^* = E[(1 - \tau(z^t)) | z^{t-1}] F_k(k(z^{t-1}), l), \quad (2)$$

where $E[. | z^{t-1}]$ indicates expectation conditional on history z^{t-1} and F_i denotes the partial derivative of F with respect to $i = k, l$.

According to equation (2), the expected return to capitalists from investing in the domestic economy should equal the world interest rate, r^* . Given the additive nature of the endowment shock, optimal capital is a constant in a world without taxes. We now proceed to characterize the optimal fiscal policy under commitment.

1.1 Optimal Taxation under Commitment

Suppose that the government can commit at time 0 to a tax policy $\tau(z^t)$ for every possible history of shocks z^t . This (Ramsey) plan is announced before the initial capital stock is invested. The government chooses $c(z^t)$, $k(z^t)$, and $\tau(z^t)$ to maximize

$$\sum_{t=0}^{\infty} \sum_{z^t} \beta^t q(z^t) u(c(z^t))$$

subject to

$$c(z^t) = z_t + w(z^t)l + T(z^t) \quad (3)$$

$$\tau(z^t) (F(k(z^{t-1}), l) - w(z^t)l) = T(z^t) \quad (4)$$

$$F_l(k(z^{t-1}), l) = w(z^t) \quad (5)$$

$$r^* = E[(1 - \tau(z^t)) | z^{t-1}] F_k(k(z^{t-1}), l) \quad (6)$$

By combining the worker and government budget constraints with the optimal labor

condition of the firms (equation 5), we obtain

$$c(z^t) = z_t + F(k(z^{t-1}), l) - (1 - \tau(z^t)) F_k(k(z^{t-1}), l) k(z^{t-1}) \quad (7)$$

We have used the constant returns to scale assumption and Euler's theorem, $F(k, l) = F_k k + F_l l$. Taking expectations of equation (7) and substituting in equation (2), we obtain a single aggregate constraint in expectation,

$$E[z_t | z^{t-1}] + F(k(z^{t-1}), l) - E[c(z^t) | z^{t-1}] - r^* k(z^{t-1}) = 0 \quad (8)$$

This constraint states that the sum of the expected endowment and produced output should equal the sum of expected consumption and payments to capitalists. The following lemma uses equation (8) to simplify the constraint set:

Lemma 1 *For any $c(z^t)$ and $k(z^{t-1})$ that satisfy (8), there exists a function $\tau(z^t)$ such that (7) and (6) are satisfied.*

Proof. For a given $c(z^t)$ and $k(z^{t-1})$, define $\tau(z^t)$ as the solution to (7). The fact that (6) holds follows directly from (8). ■

The previous result exploits the fact that capitalists only care about the expected return to capital. Given an expected ex post profit, the government can use taxes to transfer resources to workers across states.

The problem of the government under commitment is then to maximize

$$\sum_{t=0}^{\infty} \sum_{z^t} \beta^t q(z^t) u(c(z^t))$$

subject to (8).

Proposition 1 *Under commitment, the optimal fiscal policy*

(i) *Provides full intra-period insurance to the workers:*

$$c(\{z_t, z^{t-1}\}) = c(\{z'_t, z^{t-1}\}) \text{ for all } (z_t, z'_t) \in Z_t \times Z_t \text{ and } z^{t-1} \in Z^{t-1}, \text{ and}$$

(ii) At the beginning of every period, the expected capital tax payments are zero:

$$E [\tau (z^t) | z^{t-1}] = 0$$

Proof. The Lagrangian of the problem is

$$\sum_{t=0}^{\infty} \beta^t \left[\sum_{z^t} q(z^t) u(c(z^t)) + \sum_{z^{t-1}} q(z^{t-1}) \gamma(z^{t-1}) \left\{ \begin{array}{l} E[z_t | z^{t-1}] + F(k(z^{t-1}), l) \\ -E[c(z^t) | z^{t-1}] - (r^*) k(z^{t-1}) \end{array} \right\} \right]$$

where z^{t-1} evaluated at $t = 0$ refers to the initial information set. Notice that if $\gamma(z^{t-1})$ is non-negative the Lagrangian is concave on c, k . The first order conditions for the maximization of the Lagrangian are

$$\begin{aligned} u'(c(z^t)) &= \gamma(z^{t-1}) \\ F_k(k(z^{t-1}), l) &= r^* \end{aligned}$$

where the first condition implies that $c(\{z_t, z^{t-1}\}) = c(\{z'_t, z^{t-1}\})$ for all $(z_t, z'_t) \in Z_t \times Z_t$ and the second condition implies that $E[\tau(z^t) | z^{t-1}] = 0$ ■

Proposition 1 shows that the government can insure all the intra-period risk the workers are facing without distorting investment: $F_k(k(z^{t-1}), l) = r^*$. In this model, it is efficient to set expected tax payments on capital equal to zero, a result well known in the Ramsey taxation literature (Judd (1985), Chamley (1986) and the stochastic version in Zhu (1992)). Chari, Christiano and Kehoe (1994) obtain a similar result in a business cycle model.

A quick corollary follows,

Corollary 1 *Under commitment, (i) realized capital taxes are countercyclical:*

$$\tau(z_t, z^{t-1}) > \tau(z'_t, z^{t-1}) \quad \text{for } z_t < z'_t$$

(ii) *If $E[z_t | z^{t-1}]$ is increasing in z^{t-1} , then $\tau(z_t, z^{t-1})$ is increasing in z^{t-1} .*

Proof. From (7) it is possible to solve for the tax rate

$$\tau(z_t, z^{t-1}) = \frac{E[z_t | z^{t-1}] - z_t}{r^* k}$$

Since k is independent of z_t and z_{t-1} , the results follow. ■

The government taxes capitalists and transfers to workers in low endowment states while transferring from workers to capitalists in high endowment states. It does so in such a way that the expected tax burden on capital is zero and the workers are fully insured across states within a period. The government exploits the fact that capital is ex post inelastic to transfer capital income across states so that worker consumption is equalized. The ex ante elasticity of capital provides the necessary incentive to keep average tax payments at zero. The results in this section tell us that a government with commitment would not amplify shocks through its tax policy. Investment will be a constant at the optimal level.

We now turn to the important question of the best policy in the absence of commitment.

2 Optimal Taxation with Limited Commitment

Once the investment decision by the capitalists has been made, the government would like to tax capital as much as possible and redistribute the proceeds to the workers. Thus, the optimal tax policy under commitment may not be dynamically consistent. As is standard in the literature, we model the economy as a game between the capitalists and the government and use sustainability (Chari and Kehoe 1990) as our solution concept. In this section, we characterize the best sustainable equilibria of the game.

We assume the following,

Assumption 5 (A Maximum Tax Rate) *At any state z , the tax rate on capital cannot be higher than $\bar{\tau} > 0$.*

Throughout the analysis, we assume that $\bar{\tau}$ is greater than the maximal tax rate under the Ramsey plan (the Ramsey plan is feasible).

Let h_{t-1} be the history of tax policies and endowment shocks up to the beginning of period t : $h_{t-1} = \{(\tau_s, z_s) | s = 0, \dots, t-1\}$. As shown by Chari and Kehoe (1990), we do not need to include the capitalists' previous investment decisions in the definition of the history. A government's policy rule at time t is a function $\tau_t(h_{t-1}, z_t)$ that maps previous histories and the current shock into a tax rate less than or equal to $\bar{\tau}$. A capitalist's investment rule at time t is a function $k(h_{t-1})$ that maps previous histories into a capital level. A government policy plan is a sequence of policy rules $\sigma = \{\tau_0, \tau_1, \dots\}$. A capitalist's investment plan $\kappa = \{k_0, k_1, \dots\}$ is a sequence of investments rules.

Definition 1 *A **sustainable equilibrium** is a pair (σ, κ) such that:*

(i) Given a policy plan σ and any history h_{t-1} , the associated investment rule under κ , $k_t(h_{t-1})$, is the value of k that solves

$$r^* = E [(1 - \tau(h_{t-1}, z_t)) F_k(k, l) | z^{t-1}] \quad (9)$$

(ii) Given κ , for any history (h_{t-1}, z_t) , the continuation of the policy plan σ maximizes the expected lifetime utility of the workers from t onwards subject to the budget constraint (7).

To characterize the best sustainable equilibrium, we first study the worst equilibrium. In this model, the worst equilibrium is easy to characterize.

2.1 The Worst Equilibrium

Define σ_W as a tax policy that sets τ equal to $\bar{\tau}$ at every history. Define κ_W as the investment plan that sets capital to k_W at each history, where k_W solves $r = (1 - \bar{\tau})F_k(k_W, l)$. Then the following holds:

Proposition 2 (Worst Equilibrium) *The pair (σ_W, κ_W) is a sustainable equilibrium. In particular, of all sustainable equilibria, after any history h_{t-1} , (σ_W, κ_W) generates the lowest payoff to the government.*

Proof. To show that (σ_W, κ_W) is an equilibrium, note that if the capitalists believe that the government will tax at the maximum rate in the next period, then investing k_W is a best response. Note that if after any history (h_{t-1}, z_t) , if the government believes that the capitalists will follow the investment plan κ_W in the future, then it is optimal for the government to tax at the maximum rate today.

To show that this equilibrium is a lower bound for the the government's payoff, note first that in any equilibrium at any possible history, we know that $k(z^t) \geq k_W$. This is because taxes are restricted to be at most $\bar{\tau}$ and the expected after-tax marginal product of capital would be greater than r^* for any $k < k_W$. Given that $c(k) = z + F(k, l) - (1 - \bar{\tau})F_k(k, l)$ is increasing in k , taxing at $\bar{\tau}$ generates a consumption at least as high as $c(k_W)$. Starting from any k , the government, by taxing at $\bar{\tau}$, can thus guarantee a payoff at least as high as that of (σ_W, κ_W) . ■

Let $W(h_{t-1})$ be the payoff to the government at the beginning of period t after a history h_{t-1} under the equilibrium (σ_W, κ_W) . Given the Markovian nature of the endowment shocks and that tax rates are always $\bar{\tau}$ in any history in this equilibrium, we can redefine W as a function of the current realization $W(z_{t-1})$. We can use this function W to recursively generate the sustainable equilibria. We turn to the characterization of these equilibria in the next subsection.

2.2 The Best Sustainable Equilibrium

We can characterize the set of sustainable equilibrium payoffs by using reversion to the worst equilibrium as the punishment to government deviations (see Chari and Kehoe 1990). We are interested in the equilibrium that provides the government with its highest payoff. We refer to this as the best sustainable equilibrium. We denote strategies in this equilibrium as (σ^*, κ^*) .

To set notation, we define the value function of the government under the best equilibrium as follows. Let $V(h_{t-1})$ denote the government's expected payoff after history h_{t-1} at the beginning of period t before shocks and investment are realized. This value function holds under any history in which the government has not deviated from strategy σ^* . It is possible to show that best equilibrium payoff can be attained through stationary strategies. To see this, note that the punishment W is stationary, as are the budget constraint (8) and the government's objective function. Therefore, for any sustainable equilibrium with time dependent strategies there exists a sustainable equilibrium with stationary strategies that achieve at least as high a payoff. Given this and the Markov nature of z , we can redefine V as a function of the current shock, $V(z_{t-1})$. The best equilibrium payoff V can be shown to solve the following Bellman equation:

$$V(z_{t-1}) = \max_{k, c(\cdot)} E[u(c(z_t)) + \beta V(z_t) | z_{t-1}] \quad (10)$$

subject to

$$E[z_t | z_{t-1}] + F(k, l) - E[c(z_t) | z_{t-1}] - r^*k = 0 \quad (11)$$

$$u(c(z_t)) + \beta V(z_t) \geq u(\bar{c}(z_t, k)) + \beta W(z_t), \forall z_t \in Z \quad (12)$$

for

$$\bar{c}(z_t, k) = z_t + F(k, l) - (1 - \bar{\tau})F_k k \quad (13)$$

and where W is the continuation value of the government in the worst equilibrium.

Equation (11) is the aggregate resource constraint of the government and the inequalities (12) are the participation constraints. Note that the presence of concave functions of choice variables on both sides of constraint (12) implies that the constraint set is not convex. However, since the Bellman operator in (10) is monotone, a numerical solution can be found by iterating down using the full commitment payoff as the initial guess for V . Subsection 2.5 describes the results of a numerical analysis.

As a first step in characterizing the best equilibrium, we prove a Folk theorem.

Proposition 3 (A Folk theorem) *There exists a $\beta^* \in (0, 1)$ such that for all $\beta \geq \beta^*$ the Ramsey solution is sustainable and it is not sustainable for $\beta \in [0, \beta^*)$.*

Proof. Note that the Ramsey and the worst equilibrium's allocations are independent of the value of β . Let $c^R(z_{t-1})$ denote the consumption at time t under the Ramsey plan, conditional on z_{t-1} . Recall that the consumption under the Ramsey plan at time t is independent of z_t . Let $c^W(z_t)$ denote the consumption allocations under the worst equilibrium given a current endowment z_t . For a given β , define $\Omega(z_{t-1}, \beta)$ as β times the difference in the government's payoff between the Ramsey allocation and the worst equilibrium:

$$\Omega(z_{t_0}, \beta) \equiv \sum_{t=t_0}^{\infty} \beta^{t+1-t_0} \sum_{z^t} q(z^t|z_{t_0}) \left[u(c^R(z_t)) - \sum_{z_{t+1}} q(z_{t+1}|z^t) u(c^W(z_{t+1})) \right] \quad (14)$$

The terms in square brackets on the right hand side of (14) are strictly positive. This follows from the optimality of the Ramsey plan plus the fact that $k^* > k^W$ given that $\bar{\tau} > 0$. This implies that Ω is strictly increasing in β , is equal to zero when $\beta = 0$ and approaches infinity as β approaches one.

We can write the participation constraints at the Ramsey allocation as

$$u(c^R(z_{t-1})) - u(\bar{c}(z_t, k^*)) \geq -\Omega(z_t, \beta). \quad (15)$$

As the right hand side of (15) is increasing in β , if this constraint is satisfied at β^0 , then it is satisfied at any $\beta > \beta^0$. When $\beta = 0$, the right hand side of (15) is zero and the constraint will not hold for some z . When $\beta \rightarrow 1$, the right hand side of (15) approaches minus infinity, implying there is a $\beta^* < 1$ for which all the participation constraints are satisfied at the Ramsey allocation for $\beta \geq \beta^*$, and at least one constraint is violated at the Ramsey allocation for $\beta < \beta^*$. ■

When the government is sufficiently patient, the Ramsey solution is sustainable. As before, this will imply a fiscal policy that does not distort capital. The interesting question is however, what happens when the government is not sufficiently patient to sustain the Ramsey solution, nor impatient enough that the worst equilibrium is the unique sustainable equilibrium.

The following lemma is the first step towards an answer. Let $k(z)$ and $c(z'|z)$ be the respective policy rules that solve the Bellman equation (10) at state z .

Lemma 2 *In a best equilibrium,*

- (i) *For all states z , $F_k(k(z), l) \geq r^*$;*
- (ii) *For any state z_{t-1} , if the participation constraints (12) are slack for a subset $Z_o \subset Z$, then $c(z|z_{t-1}) = c(z'|z_{t-1})$ for all $(z, z') \in Z_o \times Z_o$;*
- (iii) *If for some $(z, z') \in Z \times Z$ we have that $c(z|z_{t-1}) \neq c(z'|z_{t-1})$, then $F_k(k(z_{t-1}), l) > r^*$.*

Proof. A necessary condition for an optimum is that there exist multipliers $\lambda(z) \geq 0$ and γ such that

$$\gamma\{F_k(k, l) - r^*\} - \sum_{z_t} \lambda(z_t) u'(\bar{c}(z_t, k)) \bar{c}_k(z_t, k) = 0 \quad (16)$$

Another necessary condition for an optimum is that

$$(q(z_t|z_{t-1}) + \lambda(z_t)) u'(c(z_t)) - \gamma q(z_t|z_{t-1}) = 0 \quad (17)$$

$$\Leftrightarrow (1 + \lambda(z_t)/q(z_t|z_{t-1})) u'(c(z_t)) = \gamma \quad (18)$$

This implies that $\gamma \geq 0$. Using the definition of \bar{c}_k (equation 13), we have that $\bar{c}_k > 0$. Equation (16) then implies (i).

For part (ii), note for all $z_0 \in Z_o$, $\lambda(z_0) = 0$ and from (17), $u'(c(z_0)) = \gamma$. Strict concavity of u implies the result.

For part (iii), note that if $c(z_t)$ is not constant for all $z_t \in Z$ at an optimum (by the hypothesis of part (iii)) then strict concavity implies that $\lambda(z_t) > 0$ for some z_t . Given that $\lambda(z) \geq 0$, with strict inequality for at least one $z \in Z$, together with the fact that u' and \bar{c}_k are strictly positive, equation (16) implies (iii). ■

The first part of the lemma states that capital never exceeds the first best level. Benhabib and Rusticini (1997) show that in a deterministic closed economy model of capital taxation

without commitment, there are situations where capital is subsidized in the long run, pushing capital above the first best level. In our case, with an open economy, such a situation never arises.

Part two states that the planner will always implement insurance across states to the extent possible. If two states have unequal consumptions and slack constraints, it is a strict improvement (due to risk aversion) to narrow the gap in consumption.

The final part of the lemma states that if the government fails to achieve perfect insurance, it will also distort capital. To see the intuition for this result, suppose that capital were at its first best level but consumption was not equalized across states. The government could distort capital down slightly to relax the participation constraints. This has a second order effect on total resources in the neighborhood of the first best capital stock. However, the relaxation of the participation constraints allows the government to improve insurance. Starting from an allocation without perfect insurance, this generates a first order improvement in welfare.

The lemma indicates that outside the Ramsey allocation, capital will be distorted down and full insurance will be unattainable. We now turn to the question in which states will the Ramsey allocation fail to be sustainable.

2.3 Persistent Shocks and Amplification

A careful look at the government's program (10) reveals that what links one period to the next is the conditional distribution over next period's endowment. In particular, as seen from the budget constraint (8), the current shock determines the expected resources to be divided next period between domestic and foreign agents.

In a world where the current endowment shocks are signals about the distribution of endowment shocks tomorrow, the promises of future taxation will be functions of the current state. How do these promises change with the state of the economy? Is it harder for a government to make promises of not taxing capital following good times or bad?

We begin by defining “persistence” and “full support” in our framework:

Definition 2 (Persistent Shocks and Full Support) *The endowment shocks are **persistent** if $E(z'|z)$ is strictly increasing in z . The process z has **full support** if for every pair $(z', z) \in Z \times Z$, $q(z'|z) > 0$.*

Our main proposition is as follows:

Proposition 4 (Distortion in Bad States) *Suppose that the endowment process is persistent and has full support. In a best sustainable equilibrium, if $k(z) = k^*$ for some $z \in Z$, then $k(z') = k^*$ for all $z' > z$.*

Proof. Consumption under full commitment can be written as:

$$c^R(z) = E(\hat{z}|z) + F(k^*, l) - rk^*$$

where k^* is such that $F_k(k^*, l) = r$. As stated before, consumption under commitment is independent of the current realization of the endowment shock, \hat{z} (perfect intra-period insurance). The fact that $k(z) = k^*$ implies that the first best capital level is attained following a z shock. We know then from lemma (2) and the full support assumption, that for any \hat{z} following z , $c(\hat{z}|z) = c^R(z)$. This implies that all the participation constraints following z are satisfied:

$$u(c^R(z)) \geq u(\bar{c}(\hat{z}, k^*)) + \beta(V(\hat{z}) - W(\hat{z})), \forall \hat{z} \in Z. \quad (19)$$

The full support assumption guarantees that following z , a participation constraint exists and is satisfied for every element of Z . Note that all terms on the right hand side of (19)–the continuation values as well as the deviation consumption – depend only on \hat{z} and not on z . Note as well that $c^R(z)$ depends on z only through $E(\hat{z}|z)$. Therefore, the persistence assumption implies that $c^R(z)$ is increasing in z . Therefore, for $z' > z$, $c^R(z') > c^R(z)$. It follows from (19) that the participation constraints are satisfied at the Ramsey allocations for all $z' > z$. ■

We postpone discussion of the intuition of this result until we prove one more proposition. The above proposition concerns the case when the Ramsey allocation is sustainable following certain states but is not sustainable following others. The theorem then characterizes the nature of these two sets. This leaves open the question regarding whether such a situation ever arises. That is, we have not ruled out the possibility that either the Ramsey allocation is sustainable for all states or for no state. Proposition 5 proves that there always exist discount rates for which the main proposition is relevant.

Define $V(z|\beta)$, and $W(z|\beta)$ as the best equilibrium and worst equilibrium value functions (as before), but in this notation we make explicit that the payoffs are functions of β . Then as a first step toward Proposition 5, we note that at the β^* of the Folk theorem,

Lemma 3 (Continuity) *For all $z \in Z$, the value function, $V(z|\beta)$, is continuous at β^* .*

Proof. The Folk theorem states that the Ramsey allocation is sustainable for $\beta \geq \beta^*$. Recall that the Ramsey allocation is invariant to β and the payoffs depend on β only through the direct discounting. This implies that $V(z|\beta)$ is right continuous at β^* .

To prove left continuity at β^* , we first define the following quantities:

$$\begin{aligned}\hat{c}(z, k) &\equiv E(z'|z) + F(k) - rk \\ \hat{V}(z, k|\beta) &\equiv \sum_{t=0}^{\infty} \sum_{z^t} \beta^t q(z^t|z) u(\hat{c}(z^t, k)) \\ \hat{H}(z', z, k|\beta) &\equiv u(\hat{c}(z', k)) + \beta \hat{V}(z', k, \beta) - u(\bar{c}(z', k)) - \beta W(z'|\beta)\end{aligned}$$

That is, $\hat{c}(z, k)$ is the consumption that satisfies the budget constraint (8) when consumption is constant across states next period and capital is k . $\hat{V}(z, k|\beta)$ is the expected payoff from consuming \hat{c} every period, holding constant k . Finally, $\hat{H}(z', z, k|\beta)$ is non-negative when the consumption plan $\hat{c}(z, k)$ together with the constant capital stock k satisfy the participation constraint at z' following z . Several remarks follow directly from these definitions:

- (a) For all $z \in Z$, $\hat{V}(z, k|\beta)$ is continuous in β and k ;
- (b) $\hat{V}(z, k^*, \beta^*) = V(z|\beta^*)$, $\forall z \in Z$, as k^* is optimal and sustainable at β^* ;
- (c) $\hat{H}(z', z, k^*|\beta^*) \geq 0$, $\forall (z', z) \in Z \times Z$, or the Ramsey allocation is sustainable at β^* .

We will use the fact that

$$\hat{H}_k(z', z, k^*|\beta) < 0, \forall (z', z) \in Z \times Z, \quad (20)$$

where $\hat{H}_k = \frac{\partial \hat{H}}{\partial k}$. This follows from (a) k^* maximizes $\hat{c}(z, k)$ and $\hat{V}(z', k, \beta)$, and (b) $\bar{c}_k > 0$. Equation (20) implies that small reductions in k from k^* strictly relax the participation constraints (which may already be slack). This plus the fact that \hat{V} is continuous in k and both \hat{V} and W are continuous in β implies that there exist $k_0 < k^*$ and $\beta_0 < \beta^*$ such that the participation constraints hold in the set $N_0 \equiv (k_0, k^*] \times (\beta_0, \beta^*]$,

$$\hat{H}(z', z, k, \beta) \geq 0, \forall (k, \beta) \in N_0$$

By definition, continuity of $\hat{V}(z, k|\beta)$ in k and β imply for each $\varepsilon > 0$, there exists a neighborhood N_1^ε of (k^*, β^*) , such that:

$$|\hat{V}(z, k|\beta) - \hat{V}(z, k^*|\beta^*)| < \varepsilon, \forall z \in Z \text{ and } \forall (k, \beta) \in N_1^\varepsilon.$$

Define $N_\varepsilon = N_0 \cap N_1^\varepsilon$. This set is non-empty. Moreover, by construction of N_0 , the incentive constraints are satisfied at allocations associated with elements of N_ε . That is, $\hat{V}(z', z, k|\beta) \leq V(z|\beta) \leq V(z|\beta^*)$, $\forall (k, \beta) \in N_\varepsilon$, where the second inequality follows from the fact that $V(z|\beta)$ is increasing in β . Moreover, by definition of N_1^ε , $|\hat{V}(z, k|\beta) - V(z|\beta^*)| < \varepsilon$. Taken together:

$$|V(z|\beta) - V(z|\beta^*)| < \varepsilon, \forall z \in Z \text{ and } \forall (k, \beta) \in N_\varepsilon$$

This implies that $V(z|\beta)$ is also left continuous at β^* . ■

Now, we show that there exists a range of β s for which our main proposition is relevant.

Proposition 5 *Suppose that the endowment process is persistent and has full support. Then, there exists β_0 such that for all $\beta \in (\beta_0, \beta^*)$, we can define an associated $z_\beta < \bar{z}$ with $k(z) < k^*$, $\forall z \leq z_\beta$ and $k(z) = k^*$, $\forall z > z_\beta$.*

Proof. Define $IC^R(z', z|\beta)$:

$$IC^R(z', z|\beta) \equiv u(c^R(z)) + \beta V(z'|\beta) - u(\bar{c}(z', k^*)) - \beta W(z'|\beta).$$

By definition (and full support) note that at β^* , $IC^R(z', \underline{z}|\beta^*) = 0$ for some $z' \in Z$. Note as well that persistence implies $c^R(\bar{z}) > c^R(\underline{z})$. Therefore, $IC^R(z', \bar{z}|\beta^*) > IC^R(z', \underline{z}|\beta^*) \geq 0$, $\forall z' \in Z$. By continuity in β of $V(z|\beta)$ at β^* (from previous lemma), we can find a β_0 such that for all $\beta \in (\beta_0, \beta^*)$, $IC^R(z', \bar{z}|\beta) \geq 0$, $\forall z' \in Z$. Therefore, for each $\beta \in (\beta_0, \beta^*)$, there exists at least one element of Z such that $k(z) = k^*$. By the Folk theorem and part (iii) of Lemma 2, there is at least one $z \in Z$ such that $k(z) < k^*$. The additional properties of z_β follow from Proposition 4. ■

The intuition behind Proposition 4 is as follows. If shocks have persistence, consumption in the Ramsey plan will be higher following a higher endowment shock. The deviation consumption and the continuation values depend only on the realized shock next period. Thus, the gains to deviation will be greater in any state following a recession. Consequently, the government may not be able to commit to the Ramsey taxes (which are zero on average) following a low shock. Therefore, capitalists expect average taxes to be positive leading to sub-optimal investment. The proposition implies that the inability to commit may result in an economy in which capital fluctuates despite a constant optimal capital level.

Proposition 5 tells us more: Given any strictly concave utility function, and any shock process that satisfies persistence and full support, there always exists a non-empty region of

government's impatience (as measured by β) such that in the best sustainable equilibrium, investment is distorted for low realizations of the endowment and undistorted for high.

The proposition presents a general result. It does not require any particular shape of the utility function other than concavity. In particular, we do not need a full characterization of the continuation values V and W . However, this generality relies on fairly strong assumptions regarding asset markets, in particular the absence of government debt. We discuss this further in Section 4.

Remark 1 *A common feature of models of insurance with limited commitment is that the participation constraints tend to bind when the endowment is high. This results from the fact that insurance calls for payments during booms and inflows during downturns. However, in precisely an environment that emphasizes insurance, we show that distortions of investment can occur during recessions despite undistorted investment during booms. In fact, even if the participation constraints only bind in high endowment states, and these states are least likely following a recession (i.e. persistence), it is during recessions that the capital margin will be first distorted.*

Remark 2 *The propositions' results rely on the fact that the utility function is strictly concave. If utility is linear, the government is "free" to allocate consumption across states to minimize the temptation to tax capital ex-post and maximize expected output. We can show that in this case, investment is a (perhaps sub-optimal) constant independent of the history of shocks. There is no propagation of the shocks through fiscal policy. It is then, the desire to provide insurance to risk averse agents (in the absence of commitment) that motivates the government to distort investment downward during recessions.*

Remark 3 *If the endowment follows an i.i.d. process, distortions to investment are independent of the current shock. Under an i.i.d. process, the conditional distribution of tomorrow's endowment is independent of today's realization. This implies that V and W are constants, as is consumption under the Ramsey plan. Therefore, gains from deviation are independent of previous shocks. Investment may be distorted, but will be constant.*

2.4 Going Beyond Our Main Result: Two Shocks Case

Proposition ?? characterizes what would happen around the first best, once incentive compatibility constraints start binding. Our result was quite strong and general, independent of

the shape of the utility function: as long as the shocks are persistence, capital is distorted first in states where the endowment is small. Recessions are thus amplified by policy.

However, it is also of interest to know what would happen when we move far away from the first best allocation. Is it possible to characterize more what the efficient allocation would look like? A quick answer would be to simulate the economy and provide some numerical characterizations of the efficient allocation. Eventually we will turn to this in the next section. However, first, we will provide some more analytical results concerning the case with only two endowment shocks: $z_H > z_L$.

We will first state the following assumption about the utility function.

Assumption 6 (IARA) *The utility function u has Non-decreasing Absolute Risk Aversion (IARA).*

Note that exponential utility satisfy our assumption 6.

When all incentive constraints bind.

We will analyze the case where all incentive constraints are binding. This implies that

$$u(c(z_i|z_j)) = u(d(k_j) + z_i) - \beta(W_i - V_i)$$

Plugging this back in the value equation, we have that we can rewrite the value equation as

$$W_i = \sum_z q(z|z_i) [u(d(k_i) + z) + \beta V(z)]$$

Subtracting the autarky value equation we get an expression for $W - V$,

$$W_i - V_i = \sum_z q(z|z_i) [u(d(k_i) + z) - u(d(k_m) + z)]$$

Using this, we can then rewrite the budget constraint in the coming from a high shock as follows

$$H_H(k_H, k_L) = F(k_H) - rk_H + \sum_z q(z|z_H) \left[z - C \left(u(d(k_H) + z) - \beta \sum_{z'} q(z'|z) [u(d(k(z)) + z') - u(d(k_m) + z')] \right) \right] \geq 0$$

where $C(u(x)) = x$. Define H_L as,

$$H_L(k_H, k_L) = F(k_L) - rk_L + \sum_z q(z|z_L) \left[z - C \left(u(d(k_L) + z) - \beta \sum_{z'} q(z'|z) [u(d(k(z)) + z') - u(d(k_m) + z')] \right) \right] \geq 0$$

Given that consumption is pinned down by (and increasing in) the capital levels, the objective is to find the highest pair k_H, k_L that satisfy $H_L(k_H, k_L) \geq 0$ and $H_H(k_H, k_L) \geq 0$. Suppose that $k_H < k_L$. Given that C is an increasing function, it follows that increasing k_H strictly relaxes the budget constraint in the low state:

$$0 \leq H_L(k_H, k_L) < H_L(k_L, k_L)$$

What about in the high state? Note that given that $u(d(k) + z') - u(d(k_m) + z')$ is decreasing in z' , and that C is increasing, we have that,

$$\begin{aligned} & z_H - C \left[u(d(k) + z_H) - \beta \sum_{z'} q(z'|z) [u(d(k) + z') - u(d(k_m) + z')] \right] \\ & \geq z_H - C [(1 - \beta) u(d(k) + z_H) + \beta u(d(k_m) + z_H)] \end{aligned}$$

Similarly we can show that

$$\begin{aligned} & z_L - C \left[u(d(k) + z_L) - \beta \sum_{z'} q(z'|z) [u(d(k) + z') - u(d(k_m) + z')] \right] \\ & \leq z_L - C [(1 - \beta) u(d(k) + z_L) + \beta u(d(k_m) + z_L)] \end{aligned}$$

where the inequalities are strict if $k > k_m$.

Now, we have our first lemma,

Lemma 4 *Suppose that u satisfy assumption (6), then*

$$z - C [(1 - \beta) u(d(k) + z) + \beta u(d(k_m) + z)]$$

is increasing in z .

Proof. Define G to be

$$G(a) = (1 - \beta) d(k) + \beta d(k_m) + a - CE(d(k) + a, d(k_m) + a)$$

where CE is the certainty equivalent of a lottery that pays $d(k) + a$ with probability $(1 - \beta)$ and pays $d(k_m) + a$ otherwise; given utility function u . If u satisfy IARA, then G is increasing in a . Given that by definition

$$CE = C[(1 - \beta) u(d(k) + z_H) + \beta u(d(k_m) + z_H)]$$

this implies the result. ■

The following lemma tell us more about the incentive constraint in the high state

Lemma 5 *Suppose that u satisfy assumption (6), then*

$$H_H(k, k) \geq H_L(k, k)$$

Proof. From the previous lemma we have that

$$\begin{aligned} z_H - C \left[u(d(k) + z_H) - \beta \sum_{z'} q(z'|z) [u(d(k) + z') - u(d(k_m) + z')] \right] \geq \\ z_L - C \left[u(d(k) + z_L) - \beta \sum_{z'} q(z'|z) [u(d(k) + z') - u(d(k_m) + z')] \right] \end{aligned}$$

Persistence ($q(z_H|z_H) > q(z_H|z_L)$), now implies the following,

$$\begin{aligned} \sum_z q(z|z_H) \left[z - C \left(u(d(k) + z) - \beta \sum_{z'} q(z'|z) [u(d(k) + z') - u(d(k_m) + z')] \right) \right] \geq \\ \geq \sum_z q(z|z_H) \left[z - C \left(u(d(k) + z) - \beta \sum_{z'} q(z'|z) [u(d(k) + z') - u(d(k_m) + z')] \right) \right] \end{aligned}$$

By the definition of H_H and H_L we thus have

$$H_H(k, k) \geq H_L(k, k)$$

which is what we wanted to proof ■

Now we are ready to proof that $k_H \geq k_L$.

Proposition 6 *Suppose that u satisfy assumption (6), then if all incentive constraints bind, in an efficient allocation $k(z_H) \geq k(z_L)$. Even more is some cooperation is achieved, $k(z_L) > k_M$, then $k(z_H) > k(z_L)$*

Proof. For the first part, we proceed by contradiction. Suppose that in an efficient allocation $k(z_H) = k_H < k(z_L) = k_L$. This implies that

$$0 \leq H_L(k_H, k_L) < H_L(k_L, k_L)$$

So there exists a $\hat{k} < k_L$ such that

$$0 < H_L(\hat{k}, \hat{k})$$

by applying the previous lemmas. Using the previous lemma we have that (\hat{k}, \hat{k}) also satisfy the budget constraint in the high state

$$H_H(\hat{k}, \hat{k}) \geq H_L(\hat{k}, \hat{k}) \geq 0$$

So, we have found an allocation of capital levels (\hat{k}, \hat{k}) that is strictly higher than the original allocation, and that satisfy the budget constraints. Given that consumption levels are monotonic functions of the capital levels, this new allocation provides a strictly higher utility level, a contradiction of efficiency.

For the second part, note that if $k(z_H) = k(z_L) = k > k_M$, then $H_H(k, k) > H_L(k, k) \geq 0$ where the strict inequality comes from the full support of the shocks. The budget constraint in the high state is thus slacked, which implies that we can increase k_H a bit, relaxing the constraint in the low state, and hence being able to increase k_L as well, which is an strict improvement. ■

When all incentives constraints bind, a sufficient condition for an increasing capital level is that the utility function has non decreasing absolute risk aversion. The exponential will satisfy then this condition. Note however that IARA is a sufficient condition, not necessary. In particular, we do not have a result for a utility function that satisfies DARA instead: it is possible that the efficient capital level when all incentive constraints bind is still increasing.¹

¹IARA(DARA) is also a sufficient condition for the capital level to be weakly increasing(decreasing) in the

However, we are not able to analytically characterize more of this problem. The next section presents a numerical analysis.

2.5 Numerical Analysis

We now turn our attention to a numerical analysis that illustrates these results and consider combinations of p and β , for which a general combination of incentive compatibility constraints can bind. To be clear, this is not meant to be a calibration exercise, but only an illustration of the solution, since it is not possible to completely analytically characterize the solution of the game.

We consider two discrete values for z : z_H and z_L . To solve the problem numerically, we iterate on (10), where the initial guess $W^0(z)$, is the value function for the case with full commitment. Since the value in the case with full commitment will necessarily be at least as great as the value with limited commitment, and since the bellman operator is monotone, starting with $W^0(z)$ we should converge monotonically down to the maximized value with limited commitment. Given the continuation value $W^0(z)$, the government chooses tax rates as functions of today's shock and the previous one: $t(z_t|z_{t-1})$, such that it maximizes (10) subject to (11) and (12). This generates a new value $W^1(z)$. We repeat this procedure until $|W^{i+1} - W^i| < \varepsilon$, where ε is a small number.

Table 1 in Appendix B lists the values for the numerical example. We assume that the transition matrix is symmetric, with p the probability of continuing in the current state. For a single value of p (0.89), we solve for capital stocks, k_H and k_L and plot the ratio $\frac{k_H}{k_L}$ for various values of the discount factor, β . As β converges to 1, the first best level of capital stock is attained regardless of the state and the ratio is 1. This follows from the folk theorem. At the other extreme, when β converges to 0, the only sustainable equilibrium is the Markov (punishment) equilibrium and the capital stock in each state equals the same constant. As β decreases below 1, we initially have the case of our main theorem, that first the IC 's start to bind in any state following the low state. So for a range of β just below the Ramsey range, the first best capital stock is attained in the high state, and it is distorted in the low state. Consequently, the ratio is greater than 1. At the other extreme, when β is greater than the Markov range, when all constraints are still binding, we have shown that a sufficient

endowment level in the case where there is full persistence of the shocks. To see this note that the incentive constraints in the case of full persistence are given by $u(F(k) - rk + z) \geq \beta u(F(k) - (1 - \tau)F'(k)k + z) + (1 - \beta)u(F(k_M) - rk_M + z)$ where R.H.S. can be written as a certainty equivalent relation and the result will follow. We thank Ivan Werning for this point.

condition for $k_H > k_L$ is non-decreasing absolute risk aversion in the utility function². There is an intermediate range of β 's, where the solution is much harder to describe analytically. In this range, some of the IC's still bind and the trade off between providing more insurance vs. raising the capital stock can result in k_H going below k_L . In the figure below we have chosen parameter values for which k_H goes below k_L , but this is not always the case in this range.

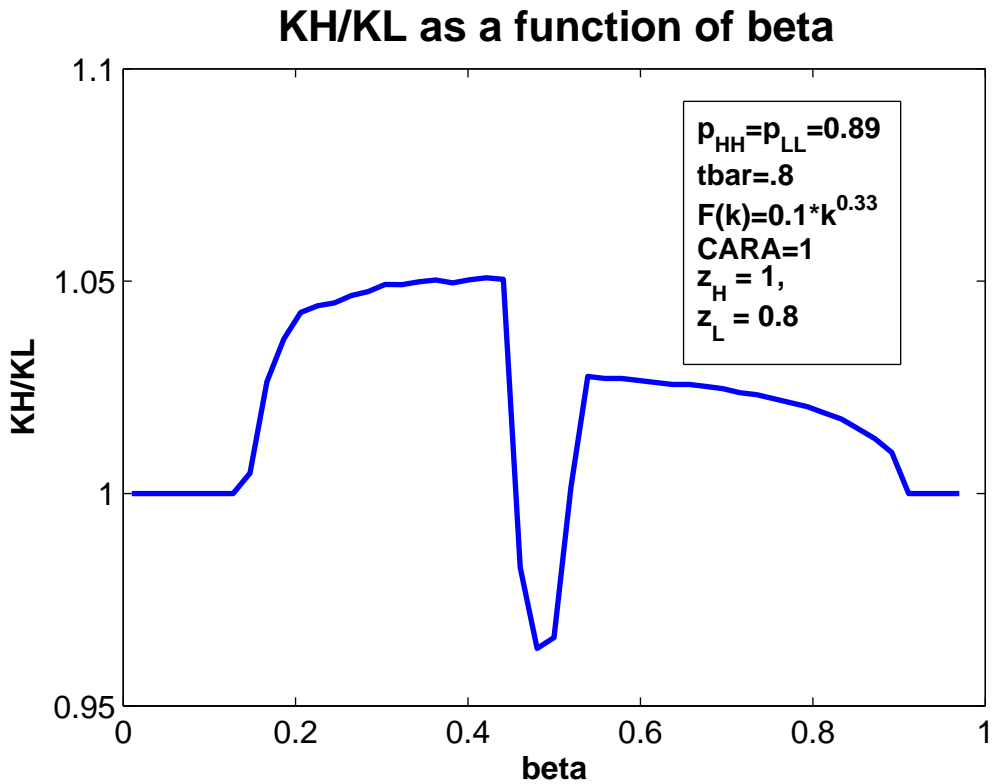


Figure 1

In Figure 2, we plot the graph where we allow p to also vary. For a given β , when $p = 0.5$, this is the i.i.d case and the ratio is one. At the other extreme, when there is full persistence, $p = 1$, we have shown that the ratio is greater than or equal to one in the case of increasing absolute risk aversion (it equals one when the utility function is exponential), and is strictly less than one in the case of decreasing absolute risk aversion (power utility).

²We have simulated cases for power utility (a common example of decreasing absolute risk aversion), and we have found that when all incentive binds, the capital ratio is also above one.

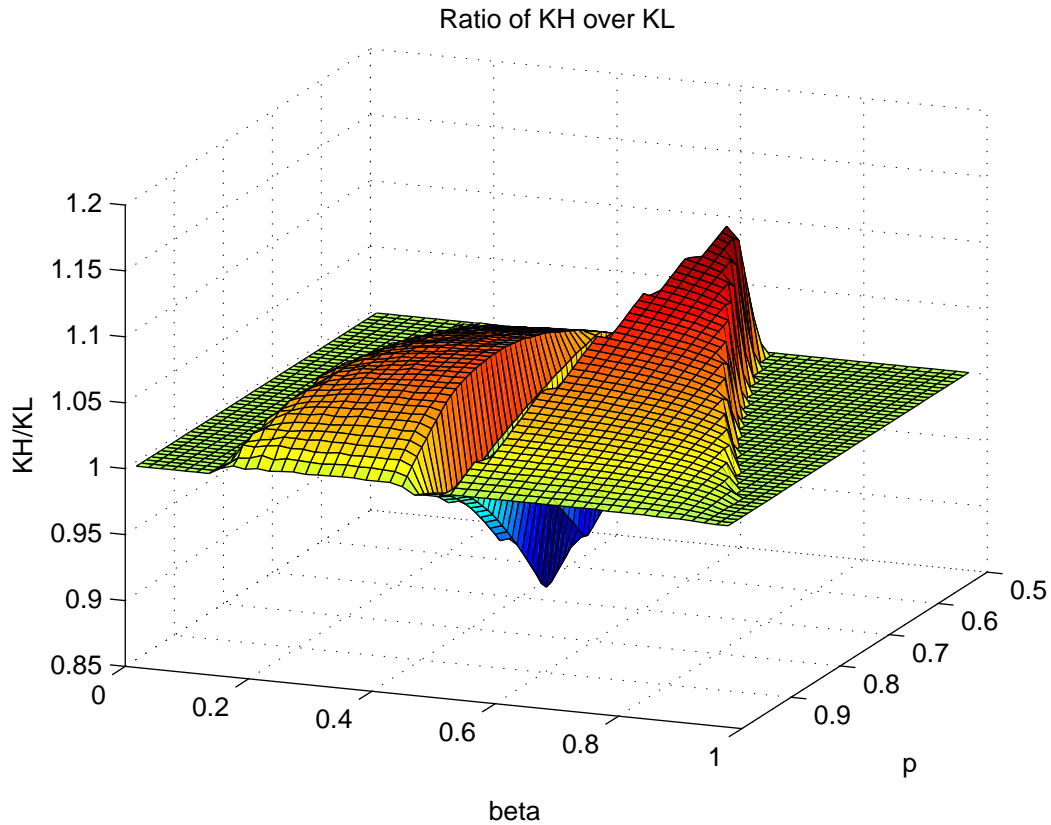


Figure 2

3 Extensions

3.1 Static Insurance Markets

An extension we consider is to determine if the government could have improved on the limited commitment outcome if it had access to static insurance markets. That is, suppose the government can buy and sell insurance claims $a(z^t)$ with $E(a(z^t)|z^{t-1}) = 0$. This insurance can be used to smooth the consumption of workers across states within a period, but not across periods. We show, that as long as the government has the same limited commitment issues related to the insurance as it does with the tax contracts, the availability of static insurance markets will not improve on the equilibrium outcome previously described. That is, any welfare level that can be attained through the use of static insurance contracts can

be replicated through the tax and transfer policy.

Consumption is now given by

$$c(z^t) = z_t + F(k(z^{t-1}), l) - (1 - \tau(z^t))F_k \quad (21)$$

If the government deviates, it also loses its insurance claims

$$\bar{c}(z_t, k) = z_t + F(k, l) - (1 - \bar{\tau})F_k(k, l)k \quad (22)$$

We can now state the following proposition.

Proposition 7 *For any equilibrium with $\{\tau(z^t), a(z^t)\}$ there exists an equilibrium $\tilde{\tau}(z^t)$ that uses no insurance ($\tilde{a}(z^t) = 0$) and delivers the same utility at any history.*

Proof. Define $\tilde{\tau}$ as

$$c = z + F(k, l) - (1 - \tau)F_k(k, l)k + a \equiv z + F - (1 - \tilde{\tau})F_k(k, l)k = \tilde{c}$$

This implies

$$\tilde{\tau} = \tau - \frac{a}{F_k k}$$

By construction, $c = \tilde{c}$. Since $E(\tilde{\tau}) = E(\tau)$ capital stock is the same under both allocations. The deviation consumption $\bar{c}(z, k)$ is unchanged. So the new $\tilde{\tau}$ is an equilibrium delivering the same allocation. ■

So, having access to static insurance markets does not change the incentive compatible allocations available to the government, and the results in the previous section still hold.

3.2 Productivity Shocks

This far we have modeled the shocks z as an endowment shock. We now consider the case where z is a productivity shock. The production function is

$$y = zF(k, l) \quad (23)$$

The consumers budget constraint is simply

$$c(z^t) = w(z^t)l + T(z^t)$$

Profit maximization by firms and capitalists investment decision imply the following two conditions:

$$z_t F_l (k (z^{t-1}), l) = w (z^t) \quad (24)$$

$$r^* = E [(1 - \tau (z^t)) z_t | z^{t-1}] F_k (k (z^{t-1}), l) \quad (25)$$

The main deviation from the previous set up is that now the optimal level of capital will vary with the state z^{t-1} , as long as there is some persistence in the state. However, all the previous Lemmas and Propositions, with the exception of Proposition (??), follow through with small alterations to the proof. For instance, as in Proposition (1), when the government has full commitment, workers are completely insured intra-period. Further ex ante taxes, $E(z_t \tau (z^t))$, on capital equal 0 and capital is at its first best level. $k^*(z)$ is increasing in z .

Proposition (??) changes because $\Delta_{z_{t-1}} (z_{t-1}, z_t)$ needs to take into account the fact that $k^*(z_{t-1})$ is increasing in z_{t-1} .

Consumption under full commitment can be written as:

$$c^*(z_t | z_{t-1}) = E(z_t | z_{t-1}) F(k^*(z_{t-1}), l) - r^* k^*(z_{t-1})$$

where $k^*(z)$ satisfies equation (25). As stated before, consumption at time t under commitment is independent of the realization of the productivity shock at time t , z_t .

Autarkic consumption similarly can be written as

$$\bar{c}(z_t, k^*(z_{t-1})) = z_t F(k^*(z_{t-1}), l) - \frac{z_t (1 - \bar{\tau})}{E(z_t | z_{t-1})} r^* k^*(z_{t-1})$$

Since there is more capital following a boom, in the first best case, there can be greater temptation to deviate following a boom, since there is more to tax. $\bar{c}(z_t, k^*(z_{t-1}))$ is increasing in k^* ($\bar{c}_k(z_t, k^*(z_{t-1})) = \bar{\tau} z_t k^*$). Proposition (??) can now be restated as follows.

Proposition 8 *Suppose $E(z_t | z_{t-1})$ is increasing in z_{t-1} , $\Delta(z_{t-1}, z_t)$ is increasing in z_{t-1} if any of the following statements hold: (i) The utility function is of the form $u(c) = \frac{c^\theta}{\theta}$ with $\theta \leq 0$ and the expected capital share $\frac{r^* k^*(z_t)}{E(z_t | z_{t-1}) F(k^*(z_{t-1}), l)}$ is weakly decreasing in z_{t-1} . (ii) The production function is Cobb Douglas, $F(k, l) = k^\alpha l^{1-\alpha}$ and the shocks are such that $\frac{z_t}{E(z_t | z_{t-1})} \leq \frac{(1-\alpha)}{\alpha}$.*

Proof. See Appendix A. ■

Proposition (8) provides sufficient conditions for our main result that distortions begin to

appear first in recessions. Since the expected capital share is simply the constant α when the production function is Cobb Douglas, condition (i) states that as long as the CRRA utility function has a risk aversion parameter greater than or equal to 1, if the production function is Cobb Douglas it is necessarily the case that capital is first distorted in a recession. In the case when we do not require the utility function to have the CRRA form, then we must place restrictions on the persistence of the productivity process relative to the curvature of the Cobb Douglas production function. The condition $\frac{z_t}{E(z_t|z_{t-1})} \leq \frac{(1-\alpha)}{\alpha}$, suggests that the lower the curvature of the production function (smaller α), the easier it is for the condition to be satisfied.

4 The Role of Debt

An important ingredient for the amplification effect is that in the Ramsey plan, higher shocks today generate higher levels of consumption next period. This cyclical behavior in the level of consumption next period makes the Ramsey plan harder to sustain after a lower shock, and distortions in the capital margin appear first after low endowment states. As easily observed, the balanced budget restriction imposed on the government is fundamental in delivering the cyclical behavior of next period consumption under the Ramsey plan. If the government had access to inter-temporal financial instruments, it would smooth out that variation. However, by restricting ourselves to the balanced budget case, we were able to maintain sufficient tractability so as to highlight the mechanism behind amplification.

Consider now the case where the government has access to a risk free bond which it can use to smooth consumption across time. With this instrument, in addition to the taxes, the government can completely smooth worker's consumption across time and across states if it had commitment. Taxes will be counter-cyclical, as before. However, now, with promised consumption no longer a function of the previous state, the government's incentives to deviate from the Ramsey prescription will be independent of that state. Consequently, distortions will appear everywhere simultaneously.

Note that this result does not directly over-turn our previous results. It is expected then that a situation where the government's access to financial markets is not perfect, will also be characterized by a fiscal policy that distorts capital first in the low states and amplifies the business cycle. Since the level of financial access of emerging economies is arguably far from perfect, one can conjecture that the realistic case is somewhere in the middle between budget balance and full access to financial markets, where promised consumption will still be cyclical and distortions will still appear first after lower realizations of the endowment.

To clarify the arguments, we present in this section the case where the government has access to a risk free bond and full commitment to both taxes and debt.

The value function of the government under full commitment now solves the following Bellman equation:

$$W(z_{t-1}, b(z^{t-1})) = \max_{k, c(\cdot), b(\cdot)} E [u(c(z_t, b(z^t))) + \beta W(z_t, b(z^t)) | z_{t-1}]$$

subject to

$$E [c(z^t)] = E [z_t | z^{t-1}] + F(k(z^{t-1}), l) - r^* k(z^{t-1}) + (1 + r^*) b(z^{t-1}) - E [b(z^t)] \quad (26)$$

where $b(z^{t-1})$ is the level of assets accumulated at the end of period $t - 1$. Define $\lambda(z^{t-1})$ to be the lagrange multiplier on (26). The first order conditions are

$$c(z^t) : \quad u'(c(z_t, b(z^t))) = \lambda(z^{t-1}) \quad (27)$$

$$k(z^{t-1}) : \quad F_k(k(z^{t-1}), l) = r^* \quad (28)$$

$$b(z^t) : \quad \lambda(z^{t-1}) = \beta W_b(z_t, b(z_t)) \quad (29)$$

The envelope condition is,

$$W_b(z_{t-1}, b(z^{t-1})) = (1 + r^*) \sum_{z_t} q(z_t | z^{t-1}) \lambda(z^{t-1}) \quad (30)$$

Combining (29), (27) and (30) we obtain

$$u'(c(z_t, b(z^t))) = \beta(1 + r^*) E_t [u'(c(z_{t+1}, b(z^{t+1})))] \quad (31)$$

Also, from (27), we have that consumption is equalized across states.

$$c(\{z_t, b(z_t, z^{t-1}), z^{t-1}\}) = c(\{z'_t, b(z'_t, z^{t-1}), z^{t-1}\}) \quad (32)$$

If we assume $\beta(1 + r^*) = 1$, from (31) and (32) we have that consumption is equalized across time and across states. From (28) we have that capital is at the first best level and constant.

From the constraint (26), recursive substitutions of $b(z^t)$ and the law of iterated expec-

tations, we can solve for the constant level of consumption in the Ramsey plan:

$$c^* = r^*b(z^{t-1}) + F(k^*, l) - r^*k^* + \left(\frac{r^*}{1+r^*}\right) E \left[\sum_{n=0}^{\infty} \left(\frac{1}{1+r^*}\right)^n z_{t+n} \middle| z_{t-1} \right] \quad (33)$$

Since $F(k^*, l) - r^*k^*$ is a constant and $E \left[\sum_{n=0}^{\infty} \left(\frac{1}{1+r^*}\right)^n z_{t+n} \middle| z_{t-1} \right]$ is increasing in z_{t-1} , this implies that $b(z^{t-1})$ is decreasing in z^{t-1} .

We can solve for the level of savings/borrowing using equation (33),

$$\begin{aligned} b(z_t, z^{t-1}) - b(z_{t-1}, z^{t-2}) &= \left(\frac{1}{1+r^*}\right) \times \\ &\times \left(E \left[\sum_{n=0}^{\infty} \left(\frac{1}{1+r^*}\right)^n z_{t+n} \middle| z_{t-1} \right] - E \left[\sum_{n=0}^{\infty} \left(\frac{1}{1+r^*}\right)^n z_{t+n+1} \middle| z_t \right] \right) \end{aligned}$$

which implies that $b(z_t, z^{t-1}) - b(z_{t-1}, z^{t-2}) = 0$ if $z_t = z_{t-1}$.

Note that in the case with persistence, the government in the Ramsey plan will dissave in a high state following a low state and will save in a low state following a high state.

Using the budget constraint,

$$c(z^t) = z_t + F(k(z^{t-1}), l) - (1 - \tau(z^t))F_k(k(z^{t-1}), l)k(z^{t-1}) + (1 + r^*)b(z^{t-1}) - b(z^t)$$

and the constant consumption equation (33) we can solve for the tax rate,

$$\tau(z^t) = \frac{1}{r^*k} \left[\frac{r^*}{1+r^*} E \left[\sum_{n=0}^{\infty} \left(\frac{1}{1+r^*}\right)^n z_{t+n} \middle| z_{t-1} \right] - z_t + b(z_t, z^{t-1}) - b(z_{t-1}, z^{t-2}) \right]$$

It follows that in the Ramsey plan, the government taxes capitalists in low states and subsidizes them in high states as before. Accordingly, taxes are counter-cyclical.

In the case when a high state follows a low one, the government borrows and subsidizes capitalists. When a low state follows a high state, the government taxes capitalists and also saves for the future. The feature that the government borrows in a high state and saves in a low state is consistent with the evidence of counter-cyclical budget balances that is observed in the data for developing country governments. With this policy, the government achieves a perfectly smoothed consumption profile for the workers and does not distort the capital margin.

What about the incentives to deviate from the Ramsey plan? Suppose that following a deviation, the government loses its assets and does not repay its debts. The value after a deviation is then as before (given by $V(z_t)$). So, the incentive constraint is now

$$u(c(z^t)) + \beta W(z^t) \geq u(\bar{c}(k, z_t)) + \beta V(z^t)$$

Note that under the Ramsey plan,

$$u(c(z^t)) + \beta W(z^t) = \frac{u(c^*)}{1 - \beta}$$

which is constant independent of z^t and t . The gains to deviating from the Ramsey plan at any state z^t is independent of the previous shocks z^{t-1} , and the arguments in previous sections do not directly apply.

Remark *Note that this analysis of the incentive constraints only applies when $\beta(1 + r^*) = 1$. In this case, there are parameter values where the Ramsey plan is sustainable, and we can ask the question (as in previous sections) of in which states the incentive constraints would bind “first”. However, when $\beta(1 + r^*) < 1$, the Ramsey plan would require a falling consumption profile, and it would never be incentive compatible. So when $\beta(1 + r^*) < 1$, some incentive constraints will always bind in any continuation game for any parameter values. We conjecture that in this case, as the government would eventually always hit borrowing limits, fiscal policy would amplify the cycle.*

To summarize, we find that at one extreme, when the government cannot borrow or save, in the case with limited commitment, distortions first appear following a low state. At the other extreme, when the government can perfectly insure consumption across time and states, distortions would appear everywhere simultaneously. Consequently, we conjecture that in the intermediate and more realistic case when financial access is less than perfect and consumption is higher following a high shock relative to a low shock, we should obtain distortions and amplification similar to the budget balance case. We do not however prove this in this paper and leave it for future research.

5 Conclusion

In this paper we have explored the question of optimal fiscal policy in an open economy when the government lacks commitment and markets are incomplete.

To provide a clear exposition of our mechanism underlying procyclicality and amplification, we considered a parsimonious specification in our benchmark model. The workers in this model are subject to endowment shocks that they cannot insure. The government, who cares only about the workers provides insurance through the use of linear taxes on labor and capital. The government taxes capital and transfers to the workers in recessions. To prevent capital distortions, the government then taxes labor and subsidizes capital in booms. The insurance motive then generates counter-cyclical taxes or pro-cyclical fiscal policy. When the government has full commitment to its tax policy, it is able to provide intra-period insurance to the workers without distorting capital, by setting the expected tax rate on capital to zero. In this environment, the capital stock is a constant.

It is when the government lacks commitment that we find that fiscal policy can be distortionary and investment varies with the realization of the endowment shock. We show that the incentive to deviate in any state today depends not only on the realized state but also on the path the economy experienced before arriving at this state. This result arises because when the government is restricted to running a balanced budget, consumption in any period is greater following a boom than a recession, as long as there is some persistence in the endowment shock. Consequently, the gains to deviating and expropriating capital at the maximum possible rate is greater following a recession. In this environment, the government is less able to commit to not expropriating capital following recessions and distortions in capital first appear here.

Since an important part of the amplification effect arises because a higher shock today leads to higher consumption tomorrow, we discuss how the results would change when the government is not restricted to running a balanced budget. We conjecture that, as long as financial access is less than perfect, and the government cannot perfectly smooth consumption over time, our amplification mechanism should hold. We present a brief analysis of this in Section 4 and leave the proof of the conjecture to future research.

Our model is based on two key features that we think are important in characterizing emerging markets: imperfect access to financial markets and high impatience rates that limits the governments commitment to its tax policy. As such, the paper provides a rationale for the behavior of these governments: even a benevolent government would amplify the cycle.

Appendix A: Proof for Proposition 14.

Suppose $E(z_t|z_{t-1})$ is increasing in z_{t-1} , $\Delta(z_{t-1}, z_t)$ is increasing in z_{t-1} if any of the following statements hold: (i) The utility function is of the form $u(c) = \frac{c^\theta}{\theta}$ with $\theta \leq 0$ and the expected capital share $\frac{r^*k^*(z_{t-1})}{E(z_t|z_{t-1})F(k^*(z_{t-1}), l)}$ is weakly decreasing in z_{t-1}

Proof for part (i): We need to show that the difference between first best consumption and deviation consumption at time t is increasing in z_{t-1} . First best consumption in any state z_t is

$$c^*(z_t) = E(z_t|z_{t-1})F(k^*(z_{t-1}), l) - r^*k^*(z_{t-1})$$

Deviation consumption is,

$$\bar{c}(z_t|z_{t-1}) = z_t F(k^*(z_{t-1}), l) - (1 - \bar{\tau}) z_t F_k(k^*(z_{t-1}), l) k^*(z_{t-1})$$

Alternatively, we need

$$\Delta = u[z_t(F - F_k k + \bar{\tau} F_k k)] - u[E(z_t|z_{t-1})(F - F_k k)]$$

to be decreasing in z_{t-1} .

$$\Delta = (F - F_k k)^\theta \left[u(z_t) \left[1 + \bar{\tau} \frac{1}{\frac{F}{F_k k} - 1} \right]^\theta - u(E(z_t|z_{t-1})) \right]$$

$$\log \Delta = \theta \log(F - F_k k) + \log \left[u(z_t) \left[1 + \bar{\tau} \frac{1}{\frac{1}{(F_k k/F)} - 1} \right]^\theta - u(E(z_t|z_{t-1})) \right]$$

In the preceding equations we have used the utility function form, $u(c) = c^\theta/\theta$.

Now, $F - F_k k = F_l l$, and as long as θ is negative, $(F - F_k k)^\theta$ would be decreasing in k . Given that k is increasing in z_{t-1} , then $(F - F_k k)^\theta$ would be decreasing in z_{t-1} as well.

Now, $u(E(z_t|z_{t-1}))$ is increasing in z_{t-1} . So, we need that

$$u(z_t) \left[1 + \bar{\tau} \frac{1}{\frac{1}{(F_k k/F)} - 1} \right]^\theta$$

be decreasing in z_{t-1} . Given $\theta < 0$, and $u(z)$ is negative, this implies

$$1 + \bar{\tau} \frac{1}{\frac{F_k k}{F} - 1}$$

should be decreasing in z_{t-1} . Note that $1 + \bar{\tau} \frac{1}{\frac{F_k k}{F} - 1}$ is decreasing in $\frac{F_k k}{F}$, the capital share.

So, if the capital share is weakly decreasing in k then, $u(z_t) \left[1 + \bar{\tau} \frac{1}{\frac{F_k k}{F} - 1} \right]^\theta$ is also weakly decreasing in k as long as θ is negative and the proposition goes through. A special case is when the production function is Cobb-Douglas and the share of capital is a constant.

Proof for part (ii) Suppose the production function is of the Cobb Douglas form.

$$y = zk^\alpha l^{1-\alpha}$$

Since,

$$E(z_t|z_{t-1})F_k(k^*(z_{t-1})) = E(z_t|z_{t-1})\alpha(k^*)^{\alpha-1} = r^*$$

we can rewrite the first best consumption as

$$c^*(z_t) = \left(\frac{1-\alpha}{\alpha} \right) r^* k^*$$

$$\Delta(z_{t-1}, z_t) = u\left(\left(\frac{1-\alpha}{\alpha}\right)r^*k^*\right) - u(z_t k^*(z_{t-1})^\alpha [(1-\alpha) + \bar{\tau}\alpha]) \equiv H(k^*)$$

The preceding equation is a function of k^* . Since k^* is increasing in z_{t-1} , we only need to check for the conditions under which $H'(k^*) \geq 0$.

$$\begin{aligned} H'(k^*) &= u'(c^*) \left(\frac{1-\alpha}{\alpha} \right) r^* - u'(\bar{c}) [(1-\alpha) + \bar{\tau}\alpha] z_t \alpha (k^*)^{\alpha-1} \geq 0 \\ &= k^* (u'(c^*) c^* - u'(\bar{c}) \bar{c} \alpha) \geq 0 \end{aligned}$$

This requires

$$\frac{u'(c^*) c^*}{u'(\bar{c}) \bar{c}} \geq \alpha \tag{34}$$

Since $\bar{c} > c^*$, $\frac{u'(c^*)}{u'(\bar{c})} \geq 1$, a sufficient condition is $\frac{c^*}{\bar{c}} \geq \alpha$.

$$\frac{c^*}{\bar{c}} = \frac{(1-\alpha) E(z_t|z_{t-1})}{z_t [(1-\alpha) + \bar{\tau}\alpha]} \geq \frac{(1-\alpha) E(z_t|z_{t-1})}{z_t}$$

A further sufficient condition is $\frac{(1-\alpha)E(z_t|z_{t-1})}{z_t} \geq \alpha$, which implies

$$\frac{z_t}{E(z_t|z_{t-1})} \leq \frac{(1-\alpha)}{\alpha}$$

Appendix B: Numerical Example

Table 1: Numerical Example: Parameters

World interest rate	0.1
Depreciation rate	0
CARA coefficient	1
Punishment Tax	0.7
Capital Share	0.33
z_H	1
z_L	0.8

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