Sovereign Default and Debt Renegotiation

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Abstract

We develop a small open economy model to study sovereign default and debt renegotiation. The model features both endogenous default risk and endogenous debt recovery rates. Sovereign bonds are priced to compensate creditors for the risk of default and the risk of debt restructuring. We find that both equilibrium debt recovery rates and sovereign bond prices decrease with the level of debt. In a quantitative analysis, the model successfully accounts for the volatile and countercyclical bond spreads, countercyclical current account and other empirical regularities of the Argentine economy. The model also replicates the dynamics of bond spreads during the recent debt crisis in Argentina.

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Markets for sovereign debt of emerging economies have developed rapidly over the past few decades. Associated with the enormous growth of sovereign debt markets have been the recurrent large-scale sovereign debt crises.[1] To resolve debt crises in the absence of an international bankruptcy law, the defaulting countries and lenders usually renegotiate over the reduction of defaulted debt.[2] Despite the importance of post-default debt renegotiation to sovereign borrowing and default, the existing literature does not contain a model that adequately captures the strategic considerations at play in the international capital markets. It remains a challenge to incorporate both sovereign default and debt renegotiation into a dynamic equilibrium model and to account for the dynamics of sovereign bond spreads in emerging economies.

In this paper, we develop a small open economy model to investigate the connection between sovereign default, debt renegotiation, and interest rates in a dynamic borrowing framework. The model features both endogenous default risk and endogenous debt recovery rates. With this model, we theoretically and quantitatively study the determination of debt recovery rates and how debt renegotiation interacts with a country's default decision. Moreover, in a quantitative exercise we analyze the valuation of sovereign bonds and map the model to Argentine data.

In the model, a risk-averse country and risk-neutral competitive financial intermediaries trade one-period discount bonds. The country faces stochastic endowments and has an option to default. Default may result in the loss of future access to capital markets, or lead to direct sanctions imposed by the lenders. However, through renegotiation over debt reduction, the inefficient sanctions can be lifted and the defaulting country can restore its reputation, regaining access to capital markets once the renegotiated debt is repaid in full.

[1] There are 84 events of sovereign default from 1975 to 2002 according to Standard and Poor's (2002). One recent example is the Argentina's default on the international bonds of over $82 billion in 2001.

[2] The most recent renegotiation is the Argentine sovereign debt restructuring closed in 2005. See Chuhan and Sturzenegger (2003) for a description of sovereign debt renegotiations between 1980 and 2000.

In the meantime, the lenders can recover at least a part of the defaulted debt. Debt recovery rates, which are endogenously determined in a Nash bargaining game, affect a country's ex ante incentive to default. In equilibrium, sovereign bonds are priced to compensate the lenders for both the risk of default and the risk of debt restructuring.

We first establish the existence of a recursive equilibrium in the model economy. We analytically characterize the equilibrium bond prices and equilibrium debt recovery schedule. The debt recovery rates decrease with indebtedness, and there is no debt reduction for small-scale debt default. We also show that default may arise in equilibrium, and a country is more likely to default if it has a higher level of debt. Finally, interest rates increase with the level of debt due to the higher default probability and lower debt recovery rate.

We use the model to analyze quantitatively the sovereign debt of Argentina from 1994 to 2001. The model successfully accounts for the high volatility of the Argentine bond spreads. Moreover, it generates the countercyclicality of bond spreads, which is found to be an important feature of emerging economies.[3] In the model, when a country gets a bad shock, default risk is higher, and the expected debt recovery rate is smaller. Therefore, the sovereign bond spreads are higher in recessions. Furthermore, the model generates volatile consumption and a countercyclical current account, which are in line with the data. We also show that the model can replicate the time series of Argentine bond spreads from 1994 to 2001. In addition, we quantitatively examine the role of debt renegotiation. We demonstrate that the changes in bargaining power have a great impact on debt recovery rates and bond spreads as well as on the sovereign borrowing.

In the literature, most studies on sovereign default focus on the question of why countries repay their debt. In their pioneering work on sovereign debt, Eaton and Gersovitz (1981) argue that a country's incentive to make repayments is to preserve its reputation

[3]Neumeyer and Perri (2004) and Uribe and Yue (2005) document the countercyclical country interest rates for emerging markets. They show that countercyclicality of sovereign bond spreads exacerbates the business cycle fluctuations in these countries.

as a good borrower. Grossman and Van Huyck (1987), Atkeson (1991), Cole and Kehoe (1998), Kletzer and Wright (2000), and Wright (2002) analyze other aspects of reputation mechanism. An alternative explanation is that the country's debt repayment motive comes from the creditors' threat of direct sanctions, as Bulow and Rogoff (1989a) first point out. Cole and Kehoe (2000) also assume direct default cost in their sovereign default study. However, all these papers assume that a country either fully repays its debt or defaults completely, incurring the default penalties. The manner in which a debt crisis is resolved plays no role in the country's default decision in these papers.

Regarding the impact of debt renegotiation on international lending and borrowing, Bulow and Rogoff (1989b) present a model of sovereign debt renegotiation in which direct sanctions are lifted through a continuous bargaining. Fernandez and Rosenthal (1990) analyze debt renegotiation by assuming that the borrowing country gains improved future access to capital markets when the renegotiated debt is repaid in full. Recent studies focus on the implication of the sovereign debt renegotiation reforms, including Collective Action Clauses (CACs) and Sovereign Debt Restructuring Mechanism[4] (see Eichengreen, Kletzer and Mody (2003), Weinschelbaum and Wynne (2005) and Bolton and Jeanne (2004)). However, the dynamic bargaining games analyzed in this literature are embedded in a static borrowing model. Therefore, a country's consideration for its future borrowing plays no role in the renegotiation. The distinguishing feature of our paper relative to the two strands of literature above is that we incorporate both sovereign default and debt renegotiation into a dynamic borrowing model.

Several recent studies quantitatively analyze sovereign debt crises in emerging economies (see Gibson and Sundaresan (2001), Arellano (2004), Aguiar and Gopinath (2004), Bai and Zhang (2005), Tomz and Wright (2005)). Our paper is closest to Arellano (2004) and Aguiar

[4]Collective Action Clauses (CACs) in sovereign bond contracts are majority renegotiation clauses, under which the changes endorsed by a specified majority of bond holders are binding on all bondholders. Sovereign Debt Restructuring Mechanism is a statutory code proposed by IMF.

and Gopinath (2004).[5] Both papers explore the connection between sovereign default, interest rates and output fluctuations in Argentina. Yet they rule out debt renegotiation and assume a zero debt recovery rate. Our equilibrium model not only endogenizes debt recovery rates, but also quantitatively accounts for the volatile and countercyclical sovereign bond spreads as well as the high default frequency.

The remainder of the paper is organized as follows. In Section 1, we describe the model environment. Section 2 presents the sovereign borrower and lenders' problems and defines a recursive equilibrium. We then demonstrate the existence of a recursive equilibrium and characterize the equilibrium bond prices and debt recovery rates. Section 3 provides the model calibration and the results of the quantitative analysis. We conduct sensitivity analysis and policy experiments in Section 4. Finally, Section 5 offers concluding remarks. The proofs are in the Appendix.

1 The Model Environment

We study sovereign default and debt renegotiation in a dynamic model of a small open economy. We consider a risk-averse sovereign country that cannot affect the world risk free interest rate. The country's preference is given by the following utility function:

$$E_0 \sum_{t=0}^{\infty} \beta^t u\left(c_t\right) \tag{1}$$

where $0 < \beta < 1$ is the discount factor of the sovereign, c_t denotes the consumption in period t and $u : \mathbb{R}_+ \rightarrow \mathbb{R}$ is the period utility function, which is continuous, strictly increasing, strictly concave, and satisfies the Inada conditions. The discount factor reflects both pure time preference and the probability that the current sovereignty will survive into

[5]The framework in these papers is similar to the consumer default model developed in Chatterjee, Corbae, Nakajima and Rios-Rull (2002)

the next period.[6] In each period the country receives an exogenous endowment of the single non-storable consumption good y_t. The endowment y_t is stochastic, drawn from a compact set Y. $\mu_y(y_t|y_{t-1})$ is the probability distribution function of a shock y_t conditional on the previous realization y_{t-1}.

International financial intermediaries are risk-neutral and have perfect information on the country's endowment and asset position. We also assume that they behave competitively on the international capital markets and can borrow or lend as much as needed at a constant world risk-free interest rate r. Each period, one financial intermediary is randomly selected to trade with the sovereign government.[7]

Capital markets are incomplete. The sovereign government and financial intermediaries can borrow or lend only via one-period zero-coupon bonds.[8] The face value of a discount bond is denoted as b', specifying the amount to be repaid next period. When the sovereign government purchases bonds, $b' > 0$, and when it issues new bonds, $b' < 0$. The set of bond face values is $B = [b_{\min}, b_{\max}] \subset R$, where $b_{\min} \leq 0 \leq b_{\max}$. We set the lower bound $b_{\min} < -\frac{\overline{y}}{r}$, which is the largest debt level that the country could repay. The upper bound $b_{\max}$ is the highest level of assets that the country may accumulate, and it exists when the interest rates on a country's saving are sufficiently small compared to the time discount factor. Let $q(b', y)$ be the price of a bond with face value b' issued by the sovereign with an endowment shock y. The bond price function will be determined in equilibrium.

[6] Grossman and Van Huyck (1988) construct a model of sovereign borrowing where the time discount factor is a product of time preference coefficient and the government's survival probability of staying in power next period.

[7] An alternative assumption is that all intermediaries lend to the country and that there exists a creditor coordinating mechanism in the debt renegotiation. That is, we rule out the strategic "holdout" behavior of creditors in the post-default debt renegotiation.

[8] Our model has incomplete markets. In contrast, in the literature on optimal contract with limited enforcement, for example Alvarez and Jermann (2000), bonds are state-contingent. In their model, there is endogenous risk of default due to the enforcement problem. Yet default does not arise in equilibrium.

1.1 Default Option and Renegotiation on Debt Reduction

We assume that the financial intermediaries always commit to repay their debt. But the sovereign government is free to decide whether to repay its debt or to default. We denote the country's default history by a discrete variable $h \in \{0, 1\}$. Let $h = 0$ stand for no default on a country's record, that is, a good credit score; whereas $h = 1$ indicates an unresolved sovereign default in the country's credit history, or a bad credit score.

If a country with a good credit score ($h = 0$) defaults on its debt $b < 0$, then it does not pay anything this period. The present value of its debt is reduced to a fraction $\alpha(b, y)$, which is the debt recovery rate determined in debt renegotiation. However, the country cannot borrow or lend in the current period. Moreover, the country's credit score deteriorates in the next period ($h' = 1$).

If a country has a bad credit score ($h = 1$) and unpaid debt arrear $b < 0$, we assume that it cannot save or borrow. Thus default incurs the reputation cost of financial exclusion, as discussed in Eaton and Gersovitz (1981).[9] We also assume that there is a loss equal to a fraction λ_d of endowment for a country with a default record.[10] However, the defaulting country can restore its reputation by repaying the reduced debt. We assume that once the debt arrear is cleared, the country's credit score is upgraded and it regains its access to capital markets.[11] Thus, the resumption of the international credit relationship is endogenous, depending on the amount of the debt arrear and the country's economic condition. This is distinct from the permanent financial exclusion assumption in Eaton and Gersovitz (1981) and an exogenous probability for the defaulting country to re-access capital market

[9] This assumption can be rationalized if the creditors can seize the country's assets accumulated in the default periods, or the creditors can collude (see Wright (2002)). In either case, defaulting countries do not have the saving opportunity analyzed in Bulow and Rogoff (1989a).

[10] The defaulting country may not obtain advanced technology, direct investment, or foreign aid from other countries, which reduces its output. Reputation spillover analyzed in Cole and Kehoe (1998) also lead to output loss.

[11] Fernandez and Rosenthal (1990) and Cole, Dow and English (1995) analyze models in which the defaulting country regain access to capital markets by making partial debt repayment.

assumed in Arellano (2004) and Aguiar and Gopinath (2004).

The debt recovery schedule $\alpha(b,y)$ is determined in the post-default renegotiation and depends on the defaulted debt value b and endowment shock y. We model the renegotiation using a Nash bargaining game. Upon the bargaining agreement, the present value of defaulted debt is reduced to a fraction $\alpha(b,y)$ of the unpaid debt b. Should the renegotiation fail, international investors lose their investment. In this case, the financial intermediaries impose direct sanctions $\lambda_s y$ on the country, which is in addition to output loss $\lambda_d y$ due to the initial default.[12] The country is excluded from the financial markets forever from that period on.

2 Recursive Equilibrium

In this section, we define and characterize a dynamic recursive equilibrium.

2.1 Sovereign Government's Problem

The sovereign government's objective is to maximize the expected lifetime utility of a domestic representative agent. The government makes its default decision and determines its assets for next period, given the current asset position b and the endowment shock y. Figure 1 displays the timing of the government's decisions within one period in the model economy. Let $v(b,h,y) : \mathcal{L} \to \mathcal{R}$ be the life-time value function for the country that starts the current period with the credit score h, asset position b, and endowment shock y, where $\mathcal{L} = B \times \{0\} \times Y \cup B_{-} \times \{1\} \times Y$.[13] We restrict the space of bond price functions to be $Q = \left\{q|q(b,y) : B \times Y \to \left[0, \frac{1}{1+r}\right]\right\}$, and the space of debt recovery schedules to be $A = \{\alpha|\alpha(b,y) : B_{-} \times Y \to [0,1]\}$. Given any bond price function $q \in Q$ and debt recovery

[12] Creditors may litigate in foreign courts or apply trade sanctions, forcing the country to conduct its trade in roundabout ways to avoid seizure, as discussed in Bulow and Rogoff (1989b) and Rose (2002).

[13] Note that only the country with a good credit score can have savings,

schedule $\alpha \in A$, the sovereign government solves its optimization problem.

For $b \geq 0$ and $h = 0$, the country has a good credit score and savings from last period. The government receives payments from the financial intermediaries and determines its next-period asset position b' to maximize utility. Thus, the value function is

$$v(b, 0, y) = \max_{c, b' \in B: c + q(b', y) b' = y + b} u(c) + \beta \int v(b', 0, y') \mu(y'|y) \tag{2}$$

For $b < 0$ and $h = 0$, the country has a good credit score and outstanding debt. If the government honors its debt obligation, it chooses its next-period asset position b' and consumes. If the government defaults, it cannot borrow or save in the current period. Moreover, the country's credit score deteriorates to $h' = 1$. But the government gets its debt reduced to $\alpha(b, y) b$, and its next period debt position is $\alpha(b, y) b (1 + r)$. The government determines to default or not optimally. Its optimal value function is:

$$v(b, 0, y) = \max \left\{ v^r(b, 0, y), v^d(b, 0, y) \right\} \tag{3}$$

where the $v^r(b, 0, y)$ is the value function if the government does not default:

$$v^r(b, 0, y) = \max_{c, b' \in B: c + q(b', y) b' = y + b} u(c) + \beta \int v(b', 0, y') \mu(y'|y)$$

and the $v^d(b, 0, y)$ is the value of default:

$$v^d(b, 0, y) = u(y) + \beta \int v(\alpha(b, y) b (1 + r), 1, y') \mu(y'|y)$$

For $h = 1$, the country has a bad credit score and unpaid debt arrear $b < 0$. The country is financially excluded, and its endowment suffers a proportional loss of $\lambda_d y$. The government chooses to optimally pay back the debt arrear. We assume that the creditors can enforce the payment of interests that accrued with the unpaid debt.[14] If the government repays partially, its next-period credit score remains bad. The debt arrear rolls over at the interest rate r. The value function is thus

$$v(b, 1, y) = \max_{c, b' \in [b,0]: c + \frac{b'}{1+r} = (1-\lambda_d) y + b} u(c) + \beta \int_Y v(b', 1, y') \, d\mu(y'|y) \tag{4}$$

Lastly, when all the debt arrear is paid, the country regains its full access to the markets. That is for $b = 0$ and $h = 1$, the value function is:

$$v(0, 1, y) = v(0, 0, y) \tag{5}$$

The sovereign government's default policy can be characterized by default sets $D(b) \subset Y$. Default set is the set of endowment shock y's for which default is optimal given the debt position b.

$$D(b) = \left\{ y \in Y : v^r(b, 0, y) \leq v^d(b, 0, y) \right\}$$

In the model economy, the country may have an incentive to default because the default option and debt renegotiation introduce contingencies into non-contingent sovereign debt contracts and facilitate interstate consumption smoothing. But intertemporal consumption smoothing is hurt due to higher interest rates and more limited access to the financial markets after default.

[14] This assumption is reasonable because it is common to pledge a renegotiated debt contract with risk-free bonds. For instance, according to the Brady plan, bonds which are issued to exchange for defaulted bank loans are backed with U.S. Treasury zero coupon bonds. See Chuhan and Strurzenegger (2003) for details.

2.2 The Debt Renegotiation Problem

We model the debt renegotiation as a generalized Nash bargaining game. The threat point of the bargaining game is that the country stays in autarky and the creditors get nothing. The expected value of autarky to the country, $v^{aut}(y)$, is given in a recursive form:

$$v^{aut}(y) = u\left(\left(1 - (\lambda_d + \lambda_s)\right) y\right) + \beta \int_Y v^{aut}(y')\, d\mu(y'|y) \tag{6}$$

Note that permanent autarky implies that a country has no access to capital markets and faces output loss plus direct sanctions, governed by $\lambda_d + \lambda_s$.

We denote the country's surplus in the Nash bargaining by $\Delta^B(a; b, y)$, which is the difference between the value of accepting the debt recovery rate a and the value of rejecting it, given the country's debt level b and endowment y.

$$\Delta^B(a; b, y) = \left[u(y) + \beta \int_Y v\left(a(1+r)b, 1, y'\right) d\mu(y'|y)\right] - v^{aut}(y) \tag{7}$$

The term in the bracket is the expected life-time utility of defaulting when the debt recovery rate is a. The surplus to the country comes from two sources. First, the direct output loss is smaller under the renegotiation agreement because no sanctions are imposed. Second, although the country's credit score becomes bad in the next period, the expected length of financial exclusion is finite. Thus, the defaulting country gains from losing the access to capital markets for a temporary periods rather than permanently.

The surplus to the risk-neutral financial intermediaries is the present value of recovered debt.

$$\Delta^L(a; b, y) = -ab \tag{8}$$

If the lenders have all the bargaining power, then they could extract debt repayments up to the full amount of a country's cost of default. If, on the other hand, the borrowing country can make take-it-or-leave-it offers, then it gets a complete debt reduction in the bargaining. To analyze the general case, we assume that the borrower has a bargaining power θ and the lenders have a bargaining power $(1-\theta)$. The bargaining power parameter θ summarizes the institutional arrangement of debt renegotiation. To ensure that the bargaining problem is well defined, we define the bargaining power set $\Theta \subset [0,1]$ such that for $\theta \in \Theta$ the renegotiation surplus has a unique optimum for any debt position b and endowment shock y.

Given debt level b and endowment y, the debt recovery rate $\alpha(b,y) \in A$ solves the following bargaining problem:

$$\alpha(b,y) = \arg\max_{a\in[0,1]} \left[\Delta^B(a;b,y)^{\theta}\left(\Delta^L(a;b,y)\right)^{1-\theta}\right] \tag{9}$$

$$\text{s.t. } \Delta^B(a;b,y) \geq 0$$

$$\Delta^L(a;b,y) \geq 0$$

One remark is that we assume the renegotiation takes place only once for one default event. This assumption is reasonable because repeated renegotiations are seldom observed in reality. Our assumption keeps the model tractable in analyzing the interaction between default and renegotiation.[15]

[15] An alternative way of modelling renegotiation is to allow for continuous costly renegotiations. In that case, the number of rounds of bargaining is a state variable and the pricing problem becomes more complicated. Our model with one-round bargaining is a special case and contains most results on the interaction between default and renegotiation. Allowing for continuous costless renegotiation generates either risk-free debt or no international lending.

2.3 International Financial Intermediaries' Problem

Taking the bond price function as given, the financial intermediaries choose the amount of debt b' to maximize their expected profit π. Their expected profit is given by

$$\pi\left(b',y\right)=\begin{cases} q\left(b',y\right)b'-\frac{1}{1+r}b' & \text{if } b'\geq 0 \\ \frac{\left[1-p\left(b',y\right)+p\left(b',y\right)\cdot\gamma\left(b',y\right)\right]}{1+r}\left(-b'\right)-q\left(b',y\right)\left(-b'\right) & \text{if } b'<0 \end{cases} \tag{10}$$

where $p\left(b',y\right)$ is the expected probability of default for a country with an endowment y and indebtedness b', and $\gamma\left(b',y\right)$ is the expected recovery rate, given by the expected proportion of defaulted debt that the creditors can recover, conditional on default.

Because we assume that the sovereign debt market is completely competitive, the financial intermediaries' expected profit is zero in equilibrium. Using the zero expected profit condition, we get

$$q\left(b',y\right)=\begin{cases} \frac{1}{1+r} & \text{if } b'\geq 0 \\ \frac{\left[1-p\left(b',y\right)\right]}{1+r}+\frac{p\left(b',y\right)\cdot\gamma\left(b',y\right)}{\left(1+r\right)} & \text{if } b'<0 \end{cases} \tag{11}$$

When the country lends to the intermediaries, $b'\geq 0$, the sovereign bond price is equal to the price of a risk-free bond $\frac{1}{1+r}$. When the country borrows from the intermediaries, $b'<0$, there exist the risks of default and debt restructuring. The sovereign bond is priced to compensate the financial intermediaries for bearing both risks.[16]

Since $0\leq p\left(b',y\right)\leq 1$ and $0\leq\gamma\left(b',y\right)\leq 1$, the bond price $q\left(b',y\right)$ lies in $\left[0,\frac{1}{1+r}\right]$. The interest rate on sovereign bonds, $r^{s}\left(b',y\right)=\frac{1}{q\left(b',y\right)}-1$, is bounded below by the risk-free rate. The difference between the country interest rate and the risk free rate is the country's credit spread , $s\left(b',y\right)=r^{s}\left(b',y\right)-r$.

[16] The price functions for consumer debt in Chatterjee et al. (2002) and sovereign debt in Arellano (2004) and Aguiar and Gopinath (2004) are a special case of our model. In that case, debt recovery rate is zero, and the default risk only pins down the bond price.

2.4 Recursive Equilibrium

We now define a stationary recursive equilibrium in the model economy.

Definition 1 *A recursive equilibrium is a set of functions for (i) the sovereign government's value function $v^*(b,h,y)$, asset holdings $b'^*(b,h,y)$, default set $D^*(b)$, consumption $c^*(b,h,y)$ (ii) recovery rate $\alpha^*(b,y)$ and (iii) pricing function $q^*(b,y)$ such that*

1. Given the bond price function $q^*(b,y)$ and debt recovery rate $\alpha^*(b,y)$, the country's asset holding $b'^*(b,h,y)$, consumption $c^*(b,h,y)$ and default set $D^*(b)$ satisfy the sovereign government's optimization problem (2), (3), and (4).

2. Given the bond price function $q^*(b,y)$, the recovery rate $\alpha^*(b,y)$ solves the debt renegotiation problem (9).

3. Given the recovery rate $\alpha^*(b,y')$, the bond price function $q^*(b,y)$ satisfies the zero expected profit condition for intermediaries (11), where the default probability $p^*(b',y)$ and expected recovery rate $\gamma^*(b,y)$ are consistent with the sovereign's default policy and renegotiation agreement.

In equilibrium, the default probability $p^*(b',y)$ is related to the sovereign government's default policy in the following way:

$$p^*(b',y) = \int_{D^*(b')} d\mu(y'|y) \tag{12}$$

The expected recovery rate $\gamma^*(b,y)$ in equilibrium is determined by

$$\begin{aligned} \gamma^*(b',y) &= \frac{\int_{D^*(b')} \alpha^*(b',y')\, d\mu(y'|y)}{\int_{D^*(b')} d\mu(y'|y)} \\ &= \frac{\int_{D^*(b')} \alpha(b',y')\, d\mu(y'|y)}{p^*(b',y)} \end{aligned} \tag{13}$$

The numerator is the expected proportion of debt that the investors can recover, and the denominator is the default probability.

We can establish the existence of a recursive equilibrium in the model economy.[17]

Theorem 1 *Given any bargaining power $\theta \in \Theta$, a recursive equilibrium exists.*

Proof. See Appendix. ∎

2.5 Characterization of a Recursive Equilibrium

We now proceed to establish some properties of a recursive equilibrium.

Theorem 2 *For a bargaining power $\theta \in \Theta$, there exists a threshold $\bar{b}(y) \leq 0$ such that the equilibrium debt recovery function α satisfies*

$$\alpha^*(b, y) = \begin{cases} \frac{\bar{b}(y)}{b} & \text{if } b \leq \bar{b}(y) \\ 1 & \text{if } b \geq \bar{b}(y) \end{cases}$$

Proof. See Appendix. ∎

The intuition for Theorem 2 is the following: After default, the defaulting country cares about the total amount of reduced debt, which affects the expected duration of financial exclusion. At the same time, the financial intermediaries are solely concerned with the total recovery on defaulted debt. Therefore, the bargaining on debt recovery rate is equivalent to the renegotiation over the reduced debt. There is an optimal value of reduced debt that maximizes total renegotiation surplus. Hence, debt recovery rates decrease inversely with the amount of defaulted debt, and there is no debt reduction for debt levels smaller than the threshold.

[17] Krueger and Uhlig (2004) show the existence of optimal contracts with endogenous outside options when a defaulting agent can start a new credit relationship with a competing financial intermediary. The equilibrium definition and the proof of existence in our paper is related to their work.

Despite the limited data availability on sovereign debt renegotiation, some observations from the recent sovereign bond exchanges are largely consistent with Theorem 2's prediction. Table 1 shows the scale of debt crises and debt recovery rates for Ukraine, Pakistan, Ecuador, Russia, and Argentina. The debt recovery rate is lower for a more severe debt crisis, both in terms of the dollar amount and relative to the country's output and foreign reserves. Thus the model prediction is in line with the empirical observations. Cole, Dow and English (1995) find similar results in their analysis about the role of debt renegotiation settlement in signalling a government's type.

The government's incentive to default depends on the *ex post* renegotiation agreement on debt reduction. Given the equilibrium debt recovery schedule $\alpha(b, y)$, characterized by Theorem 2, and the endowment shock y, the value function of a defaulting country is independent of the level of debt if it is larger than $\bar{b}(y)$. Therefore, we can show that the default set increases with the country's indebtedness and the equilibrium default probability increases with the level of debt.

Theorem 3 *Given an equilibrium debt recovery schedule $\alpha^*(b, y)$ and an endowment $y \in Y$, for $b^0 < b^1 \leq \bar{b}(y)$, if default is optimal for b^1, then default is also optimal for b^0. That is $\overline{D}^*(b^1) \subseteq \overline{D}^*(b^0)$.*

Proof. See Appendix. ■

Theorem 4 *Given an equilibrium debt recovery schedule $\alpha^*(b, y)$ and an endowment $y \in Y$, the sovereign government's probability of default in equilibrium satisfies $p^*(b^0, y) \geq p^*(b^1, y)$, for $b^0 < b^1 \leq \bar{b}(y) \leq 0$.*

Proof. See Appendix. ■

Given the endogenous debt recovery rates, our model predicts that default probability increases with the level of debt. Eaton and Gersovitz (1981), Chatterjee et al. (2002) and

Arellano (2004) all obtain similar results, although they assume a zero debt recovery rate and rule out the possibility of debt renegotiation.

We also characterize the equilibrium bond price schedule.

Theorem 5 *Given an endowment $y \in Y$, for $b^0 < b^1 \leq \bar{b}(y) \leq 0$, an equilibrium bond price $q^*(b^0, y) \leq q^*(b^1, y)$.*

Proof. See Appendix. ■

In equilibrium, bond prices depend on both the risk of default and the expected debt recovery rates. For a high level of debt, the default probability is higher, but the expected debt recovery rate is lower. Therefore, equilibrium bond prices decrease with indebtedness. This result is consistent with the empirical evidence, for example, Edwards (1984).

The next theorem characterizes the debt arrear repayment policy of a defaulting country.

Theorem 6 *Given an endowment $y \in Y$, if there exists a level of debt $\widetilde{b} < 0$ that satisfies*

$$\begin{aligned} & \sup_{b'<0} \left((1-\lambda_d)\, y + \widetilde{b} - \frac{b'}{1+r} \right) + \beta y^{1-\sigma} \int_G v(b', 1, y')\, d\mu(y'|y) \qquad (14) \\ = \; & u\left((1-\lambda_d)\, y + \widetilde{b} \right) + \beta y^{1-\sigma} \int_G v(0, 0, y')\, d\mu(y'|y) \end{aligned}$$

then for all $b \in B_-$ and $b > \widetilde{b}$, it is strictly optimal for the defaulting country to repay its debt arrear in full, and for all $b \in B_-$ and $b < \widetilde{b}$, a partial repayment is strictly optimal.

Proof. See Appendix. ■

This theorem implies that if the sovereign government fully repays the debt arrear b and regains access to financial markets next period, then it also chooses to do so with a lower level of debt arrear. If the government decides not to repay the debt in full, it will do the same for higher debt arrear. With the above theorem, the expected duration of financial exclusion increases with the amount of debt arrear.

A last remark about the theoretical model concerns the number of equilibria. There may exist multiple equilibria: one equilibrium features high default risk, low debt recovery rates, and corresponding low bond price schedule; another equilibrium may support low default risk, high debt recovery rates, and high bond price schedule. In the subsequent quantitative analysis, we compute an equilibrium with high debt recovery rates and high bond prices.[18]

3 Quantitative Analysis

In this section, we calibrate the model to analyze quantitatively the sovereign debt of Argentina.

3.1 Calibration

We define one period as a quarter. The utility function for the sovereign government has constant relative risk-aversion (CRRA). So

$$u(c) = \frac{c^{1-\sigma} - 1}{1 - \sigma} \tag{15}$$

where σ is the coefficient of risk aversion. We set the risk aversion coefficient to 2, which is standard in the macroeconomics literature. We set the risk-free interest rate r to 1%, the average quarterly interest rates on 3 month US treasury bills. The output loss parameter λ_d is set to 2% , which is in line with the output contraction estimated by Sturzenegger (2002).

The endowment process is calibrated to the Argentine quarterly output from 1980Q1

[18]Our equilibrium selection is reasonable because if financial intermediaries offer different bond price schedules to the sovereign government, the government always prefers to borrow at the lowest interest rate. Because of competition among lenders, only lenders with the lowest interest rates can trade on the markets.

to 2003Q4, which are seasonally adjusted real GDP data from the Ministry of Finance (MECON). To capture the stochastic trend in GDP, we model the output growth rate as an AR(1) process:

$$\log g_t = \left(1 - \rho_g\right) \log(1 + \mu_g) + \rho_g \log g_{t-1} + \varepsilon_t^g \tag{16}$$

where growth rate is $g_t = \frac{y_t}{y_{t-1}}$, growth shock is $\varepsilon_t^g \overset{iid}{\sim} N\left(0, \sigma_g^2\right)$, and $\log(1 + \mu_g)$ is the expected log gross growth rate of the country's endowment.[19] We estimate the endowment process to match the average growth rate, as well as the standard deviation and autocorrelation of HP detrended output.

Because a realization of the growth shock permanently affects endowment, the model economy is nonstationary. In the quantitative analysis, we detrend the model by the lagged endowment level y_{t-1}. The detrended counterpart of any variable x_t is thus $\widehat{x}_t = \frac{x_t}{y_{t-1}}$.

In the last part of the calibration, we pick the time discount factor β, sovereign government's bargaining power θ, and direct sanctions parameter λ_s to match the average default frequency, average debt recovery rate, and debt service-to-output ratio of Argentina, using a simulated method of moments (SMM). Reinhart, Rogoff and Savastano (2003) report four episodes of sovereign defaults in Argentina's external debt from 1824 to 1999. In 2001, Argentina defaulted a fifth time on its external debts, making its average default frequency 2.78% annually or 0.69% quarterly. The average debt recovery rate is estimated as the first available bid price for defaulted bonds 30 days after Argentine default in 2001. According to the Moody's (2003) report, Argentina's average recovery rate is 28%.[20] The debt service-to-output ratio includes both short term debt and debt service on long term debt,

[19] Aguiar and Gopinath (2004) introduce shocks to the endowment growth rate in their study of sovereign default and interest rates.

[20] In the recent bond exchanges for the Argentine defaulted debt, the recovery rate is 30%. According to Moody's (2003), the average recovery rate is 41% for sovereign borrowers.

which is calculated using data from the World Bank.[21] For the period 1980-2003, the ratio of Argentina's debt service to its gross national income is 9.54%.

The time discount factor β is found to be 0.740. This high degree of impatience helps to generate frequent defaults.[22] It also reflects the high political instability in Argentina with 14 presidents from 1981 to 2004. The government's quarterly survival probability is 84.7%, which makes the value of pure time discount factor 0.873. The sanctions parameter is 1.22%, which shows that the creditors have some power to impose direct sanctions on the country. Finally, the bargaining power is 0.83, which shows that Argentina has a more favorable position in debt renegotiation than the international investors. Table 2 presents the statistics for Argentina that we use as the calibration target. Table 3 summarizes the calibration results.

3.2 Comparison of Model to Data

We feed the endowment process to the model and conduct 2000 simulations. In each round, we simulate the model for 1000 periods and extract the last 50 observations to explore the behavior of the model economy in the stationary distribution. Overall, the model matches the Argentine interest rate volatility, consumption volatility, as well as the correlations between interest rates, output, consumption and current account. Table 4 compares the model statistics with the data statistics.

The bond spreads data are quarterly spreads on Argentine foreign currency denominated 3-year bonds from 1994Q2 to 2001Q4, taken from Broner, Lorenzoni and Schmukler (2004).[23] Broner, Lorenzoni and Schmukler find that the sovereign bond spreads have a

[21] We calibrate the model to debt service-to-GDP ratio in stead of debt stock-to-GDP ratio because debt stock (including long-term debt) is the total discounted value of debt service over future years. Our model studies quarterly debt. Thus, debt service-to-GDP ratio is the approporiate debt index for calibration.

[22] Arellano (2004) and Aguiar and Gopinath (2004) also use a low discount factor to generate high default frequency for Argentina.

[23] We are grateful to Broner, Lorenzoni and Schmukler for kindly providing their dataset.

significant term structure variation. The average bond spreads and volatility for Argentine foreign bonds decrease with the bond maturity for the period of 1994Q2 to 2001Q4.[24]

The model simulation closely matches the volatility of the Argentine interest rates in the data, which has been found hard to explain in the literature. The model can account for about 78% of volatility in the 3-year bond spreads in the data. This improves the result in the previous studies on sovereign bond spreads(see Arellano (2004) and Aguiar and Gopinath (2004)). In our model, the bond spreads are jointly determined by the default probabilities and debt recovery rates. Therefore, allowing for debt renegotiation breaks the one-to-one matching from default probabilities to bond spreads even though lenders are risk neutral. The debt recovery rates are correlated with default probability. In particular, default probability is higher when a larger fraction of debt reduction is expected in the post-default renegotiation. Hence, the endogenous debt renegotiation amplifies the default risk and thus the volatility of bond spreads.

The average annual bond spread is 1.84% in the simulation, which is about 45% of the average spread in the data. Although the spread average is lower than the data statistic, it is conceivably higher than the results in recent studies. This shows that endogenous debt renegotiation increases average bond spreads predicted in an equilibrium model. Note that the term premium between 3-month bonds analyzed in our model and 3-year bonds in the data should be taken into account in the comparison. [25] Moreover, our model assumes risk neutral creditors, so the predicted bond spreads do not include risk premium, which may increase bond spreads in the data.

[24] According to the estimated bond spread curve in Broner et al.(2004), the average bond spreads increase from 4.08% for 3-year bonds to 6.24% for 12-year bonds. The corresponding volatilities increase from 1.68% to 3.12%. The commonly used J.P. Morgan's Emerging Markets Bond Indices uses bonds with maturities of 7-10 years. Because the calibrated model generates interest rates for 3-month bonds, we compare the model prediction to short-term bond spreads.

[25] In our model, the mean spread is equal to the product of average default probability and average debt reduction rate in the model. Since the default frequency in the data is 2.78% and the average debt reduction rate is 72%, the average bond spreads in the stationary distribution is about 2%.

The third result is that the model accounts for the relation between bond spreads, outputs and current account in the data. Current account data are from MECON from 1980Q1 to 2003Q4 and are calculated as a ratio of output. Bond spreads are negatively correlated with output and positively correlated with current account. Moreover, the current account is countercyclical in the model, although the magnitude of correlation in the model is lower than in the data. Because the growth shock is persistent, when the country gets a good shock, its permanent income increases even more. So the country has an incentive to borrow more.[26] The bond spreads increase with the level of debt. However, a good economic shock also shifts up the bond price schedule and thus decreases bond spreads. The results show that the shift in bond price schedule dominates. Hence, bond spreads are countercyclical and positively correlated with current account.[27]

The model also generates volatile consumption at the business cycle frequency. The consumption data are seasonally adjusted from 1980Q1 to 2003Q4, taken from MECON. The consumption volatility is higher than output volatility in the data, which is a common feature of emerging economies (see Neumeyer and Perri (2004)). In our model, a good endowment shock increases permanent income more than proportionally, so the country borrows to consume more, and vice versa. Therefore, consumption is more volatile than endowment in our model. However, the model does not generate a current account as volatile as in the data.

We plot the bond price, default probability, and debt recovery rate schedule in equilibrium. Figure 2 presents the bond price functions for a country with the highest and the lowest endowment shock in the current period. It shows that the bond price increases with the assets-to-output ratio (or decreases in the debt service-to-output ratio) and increases with the endowment shock. Figure 3 plots the default probabilities. For a country with a

[26] Atkeson (1991) develop a model with limited enforcement and moral harzard to explain this pattern of capital outflow.

[27] Aguiar and Gopinath (2004) explore this effect in their model with shocks to endowment growth.

very low level of debt, there is virtually no default, regardless of the endowment shock. The default probability increases with indebtedness. We also find that the default probability is higher for a country that experiences a bad economic shock. Figure 4 plots the debt recovery schedule. For a defaulting country with a good economic shock, the debt recovery rate is higher, and vice versa. Such recovery rates contribute to the countercyclicality of interest rates because the ex ante incentive to default depends on *ex post* debt reduction. The higher debt reduction increases the country's ex ante default incentive when it receives a bad shock. A higher default probability and a lower debt recovery rate generate a higher sovereign bond spread, and thus a negative correlation between spreads and endowment. Figure 4 also displays the relation between debt recovery rates and the level of debt. If a country defaults with a small amount of debt, there is no debt reduction. As the amount of defaulted debt increases, the debt recovery rate decreases.

We also show that the model can replicate the recent Argentine debt crises and the time series of Argentine bond spreads over the past 10 years. We feed the Argentine GDP growth rate into the model and compare the time series of bond spreads. Figure 5 plots the H-P detrended output, the 3 year Argentine bond spreads, and the simulated bond spreads from 1994Q2 to 2001Q4. The figure demonstrates that the model can explain the recent Argentinian default episode. Before a default occurs, the country faces volatile and countercyclical interest rates. When the country gets a really bad shock, the model generates a default on the country's debt, as what we observe in Argentina in the last quarter of 2001.

Lastly, regarding the length of financial exclusion, the results show that the country is punished one period after default on average.[28] Gelos et al (2003) find that it takes less than one year for defaulted countries to regain access to international financial markets in the 1990s. Since the renegotiation agreement in the model is reached immediately after

[28] We get longer periods of financial exclusion when the country has lower bargaining power.

default, the financial exclusion periods in the model does not include delay in renegotiation and thus is shorter than what we observe in the data.

4 Additional Model Implications

In this section, we examine how the equilibrium changes with the bargaining power. Results of sensitivity analysis are also presented.

4.1 Bargaining Power and Collective Action Clauses

We compute the model economy for different bargaining powers. The results are summarized in Table 5. The bargaining power parameter has a direct impact on debt recovery rate. It is intuitive that higher bargaining power for the country results in a lower debt recovery for lenders. Keeping other things fixed, the lower recovery rate increases the average bond interest rates. On the other hand, the lower debt recovery rate shifts down the bond price schedule. As a result, borrowing is discouraged and thus the debt-to-output ratio is smaller. With less borrowing, both the default probability and the bond interest rates decreases, ceteris paribus. Therefore, the increasing bargaining power for the country has two opposite effects on the bond interest rates. How the equilibrium interest rates change depends on which effect dominates. Table 5 shows that the average interest rates do not change monotonically with the bargaining powers.

For a country with full bargaining power, the debt renegotiation always results in a zero debt recovery rates. In this case, borrowing cost increases dramatically with debt. The debt service-to-output ratio is 2% and annual default probability is 0.08%. When the country has small bargaining power, for example $\theta = 0.35$, the debt recovery rate is very high. In this case, the bond price, as a function of indebtedness, becomes much flatter. Therefore, our model can generate a wide range of debt-to-output ratios, as shown for $\theta = 0.35$.

The results in Table 5 with different bargaining powers can be viewed as outcomes of policy experiments. To evaluate the impact of different policies on the country's welfare, we calculate the country's ex ante utilitarian welfare in the stationary distribution. We also compute the consumption increment ϕ that makes the country indifferent between the economy with a certain bargaining power and the benchmark economy. Let Λ^0 denote benchmark welfare, and Λ^p denote welfare in the model economy with a given bargaining power. Consumption increment ϕ satisfies the following:

$$\phi = \left(\frac{\Lambda^p + 1/\left(1-\sigma\right)\left(1-\beta\right)}{\Lambda^0 + 1/\left(1-\sigma\right)\left(1-\beta\right)} \right)^{1/(1-\sigma)} - 1$$

If $\phi > 0$, the country is better off with the new bargaining power than in the benchmark case. The converse also holds. The results show that having a higher bargaining power slightly improves the country's welfare. When a country has a higher bargaining power, the lower default frequency leads to less deadweight loss and smaller consumption volatility, therefore consumption increases.

Our results shed light on the impact of reform in sovereign bond restructuring on the international financial market. In the recent debate, voluntary and market-friendly debt restructuring clauses in bond contracts are viewed as an improvement of the debt restructuring process. One example is the use of Collective Action Clauses (CACs). Because sovereign bond holders are diverse, and one investor may only hold a small fraction of debt, in the event of default one investor would always find it incentive compatible to hold the debt rather than to cooperate in the renegotiation. This "hold out" problem results in a costly renegotiation and reduces the country's bargaining power. However, CACs can align bondholders' incentives by specifying a majority rule that binds all bondholders to eliminate the "hold out" problem. Through the experiments on our model, we find that when the sovereign borrower has higher bargaining power, the country's borrowing cost does not necessarily increase. The amount of sovereign debt issued on the market is also affected and the extent of risk sharing differs with the bargaining power. These results are consistent with the recent empirical findings on bond issuance and spreads (see Eichengreen, Kletzer and Mody (2003)).

4.2 Sensitivity Analysis

We study the sensitivity of our results to some key structural parameter values. The first panel in Table 6 reports the effect of risk free interest rates. A higher risk free rate implies higher borrowing costs for the country. Thus the country borrows less and defaults less frequently. At the same time, the average recovery rate changes slightly with the risk free rate. The total effect of lower default risk and a small change in recovery rate is that the bond spreads decrease with the risk free rate. This is consistent with what Eichengreen and Mody (1998) find in their empirical study of sovereign bond spreads.

The results in the benchmark model are sensitive to the choice of time discount factor, as shown in the second panel in Table 6. Because a more patient sovereign government cares more about its reentry to capital markets in the future, the value of renegotiation agreement is relatively higher than the cost of repaying more reduced debt. Therefore, the bargaining results in a higher recovery rate. The default probability also decreases when the country is more patient because the intertemporal consumption smoothing is highly valued. Accordingly, the average bond spreads decrease with the discount factor.

Finally, we examine the sensitivity of the model to changes in sanctions and output loss in default. First, when the creditors do not have any sanction technology (i.e. $\lambda_s = 0$), the sovereign government implicitly has a higher value at its threat point in bargaining. Therefore, debt renegotiation results in a smaller recovery rate and the bond price schedule shifts down. In this case, the country's debt-to-output ratio and default probability are lower. Overall, average bond spreads decrease. Second, an increase in the output loss in default (λ_d) lowers default probability because of the higher default penalty. But the debt recovery rate decreases because the creditors' sanction threat becomes relatively less important. As a result, the debt-to-output ratio increases. Average bond spreads decrease because the drop in default probability dominates.

5 Conclusion

It is well observed that sovereign debt crises have a great impact on the borrowing countries and international capital markets. Therefore, it is crucial to understand the sovereign default risk and the role of debt crises resolution in the sovereign debt markets. This paper studies sovereign default and debt renegotiation in a small open economy model. This model allows us to investigate the interaction between default and debt renegotiation within a dynamic borrowing framework. We find that debt recovery rates decrease with indebtedness and, in turn, affect the country's *ex ante* incentive to default. In equilibrium, sovereign bonds are priced to compensate creditors for the risks of default and restructuring. Consistent with the empirical evidence, the model predicts that interest rates increase with the level of debt.

We use the model to analyze quantitatively the sovereign debt of Argentina. The model successfully accounts for the high bond spreads, countercyclical country interest rates, and other key features of the Argentine economy. The model also replicates the dynamics of bond interest rates during the recent Argentine debt crisis. Furthermore, we demonstrate that the changes in bargaining power have a great impact on debt recovery rates as well as on the sovereign bond spreads, shedding light on the policy implications of sovereign debt restructuring procedure. Overall, our study points out the importance of analyzing the connection between default and renegotiation in understanding sovereign debt market.

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Table 1:Recent Debt Crises and Bond Exchanges

Country	Pakistan	Ukraine	Russia	Ecuador	Argentina
Time of default	Dec 98	Sep 98	Nov 98	Aug 99	Nov 01
Defaulted debt (billion $)	0.75	2.7	73	6.6	82.3
Defaulted debt/output	0.10	0.09	0.12	0.46	0.32
Defaulted debt/reserves	0.37	1.82	2.65	3.54	5.64
Debt recovery rate	100%	100%	64%	60%	30%

Table 2: Target Statistics for Argentina (1980.1-2003.4)

Statistics (quarterly)	Source	Data	Model
Panel A			
World risk-free interest rate	US Treasury-bill interest rates	1%	1%
Output loss in debt crises	Sturzenegger (2002)	2%	2%
Panel B			
Average output growth rate	MECON	0.420	0.42%
Output standard deviation	MECON	4.35%	4.35%
Output autocorrelation	MECON	0.82	0.82
Panel C			
Average debt service/output	the World Bank	9.54%	9.69%
Default frequency	Reinhart et al. (2003)	0.69%	0.54%
Average recovery rate	Moody's (2003)	28%	28%

Note: Data are from World Bank, Chuhan and Sturzenegger (2003), and Moody's (2003).

Table 3: Model Parameter Values in the Model

Parameter	Symbol	Value
Panel A		
Coefficient of Risk Aversion	σ	2
Risk Free Interest Rate	r	1%
Output Loss in Default	λ_d	2%
Panel B		
Average Endowment Growth	μ_g	0.42%
Std Dev. to Endowment Growth Shock	σ_g	2.53%
Endowment Growth AR(1) coefficient	ρ_g	0.41
Panel C		
Time Discount Factor	β	0.740
Sanction Threat	λ_s	1.22%
Bargaining Power	θ	0.83

Table 4: Model Statistics for Argentina

Non-target Statistics	Data	Model
Average Bond Spreads (annual)	4.08%	1.84%
Bond Spreads Std. Dev.(annual)	1.68%	1.32%
Correlation between Bond Spreads and Output	-0.12	-0.18
Correlation between Bond Spreads and Current Account	0.49	0.54
Correlation between Current Account and Output	-0.88	-0.14
Consumption Std. Dev./Output Std. Dev.	1.15	1.03
Current Account Std. Dev. (annual)	5.40	2.32

Table 5: Statistics for Different Bargaining Powers

bargaining power	recovery rate	$\frac{\text{debt}}{\text{output}}$	default prob.	mean s (annualized)	consumption increment relative to benchmark
θ=0.35	89.01%	93.14%	5.36%	1.28%	-0.356%
θ=0.7	33.68%	13.46%	1.32%	0.96%	-0.034%
$\boldsymbol{\theta}$=0.83	**27.93%**	**9.69%**	**2.16%**	**1.84%**	–
θ=0.9	19.64%	7.83%	1.32%	1.24%	0.025%
θ=1	0	2.25%	0.08%	0.12%	0.062%

Table 6: Sensitivity Analysis for Benchmark Model

	Default prob.	Recovery rate	debt/output	Mean s	Std.(s)
Risk free rate					
$\mathbf{r = 0.01}$	**2.16%**	**27.93%**	**9.69%**	**1.84%**	**1.32%**
$r = 0.02$	0.77%	27.37%	8.98%	0.59%	0.38%
$r = 0.03$	0.42%	27.74%	8.74%	0.32%	0.31%
Time discount factor					
$\boldsymbol{\beta} = \mathbf{0.74}$	**2.16%**	**27.93%**	**9.69%**	**1.84%**	**1.32%**
$\beta = 0.8$	1.33%	34.15%	9.82%	0.94%	0.46%
$\beta = 0.9$	0.40%	47.95%	11.77%	0.20%	0.12%
Endowment loss					
$\boldsymbol{\lambda}_s$= 0.012	**2.16%**	**27.93%**	**9.69%**	**1.84%**	**1.32%**
$\lambda_s = 0$	0.70%	26.30%	9.13%	0.66%	0.71%
$\boldsymbol{\lambda}_d$= 0.02	**2.16%**	**27.93%**	**9.69%**	**1.84%**	**1.32%**
$\lambda_d = 0.03$	1.22%	23.54%	14.24%	1.08%	0.58%
$\lambda_d = 0.04$	0.69%	21.49%	18.96%	0.63%	0.51%

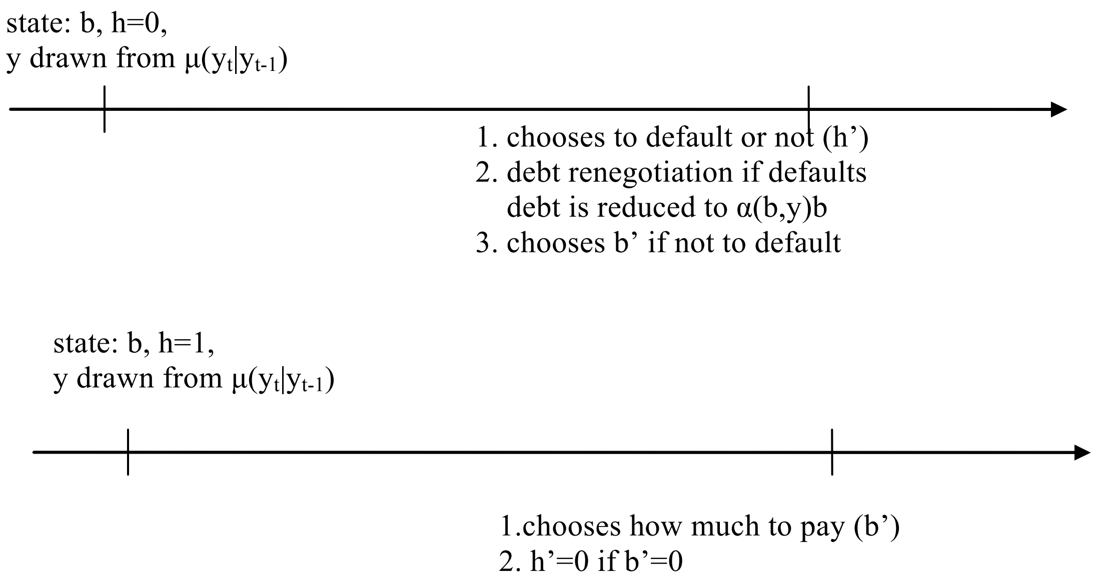

Figure 1: Timeline of the Model

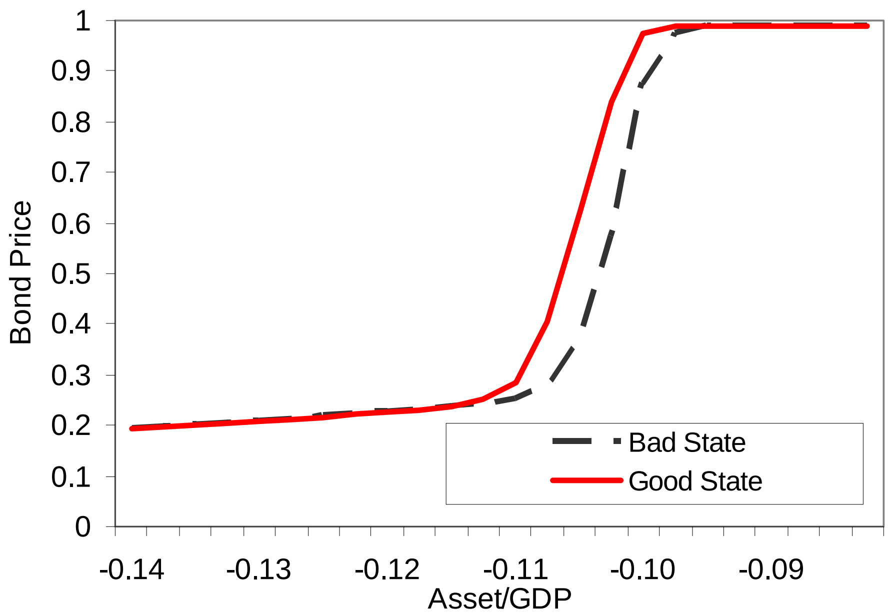

Figure 2: Bond Price in Benchmark Model

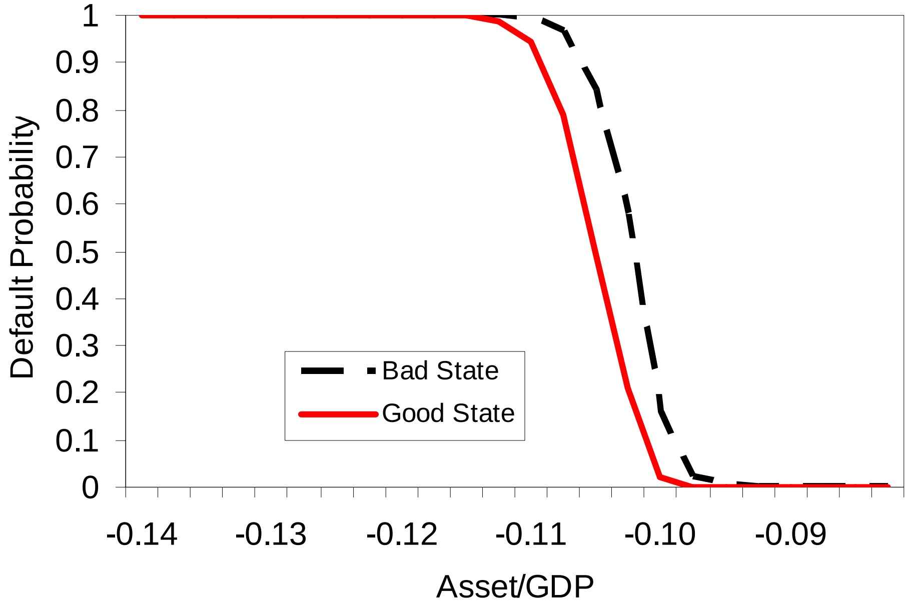

Figure 3: Default Probability in Benchmark Model

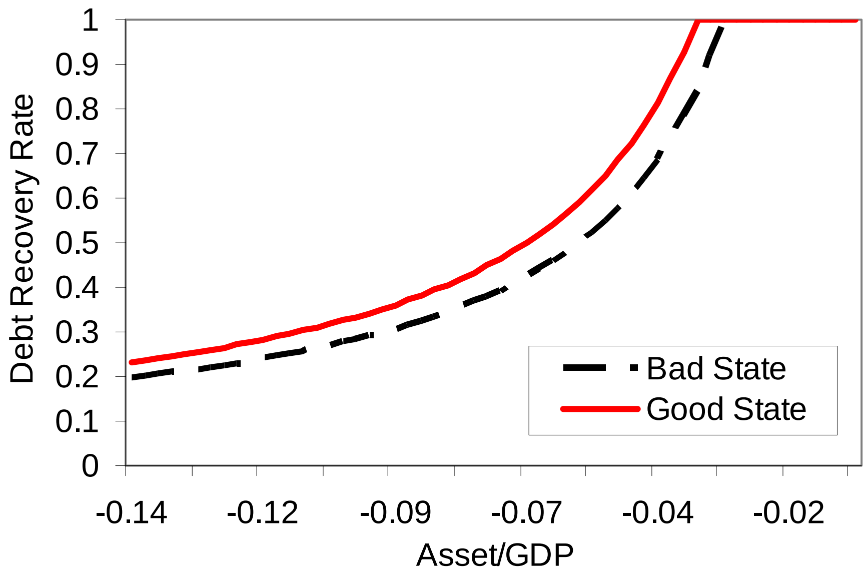

Figure 4: Recovery Rate in Benchmark Model

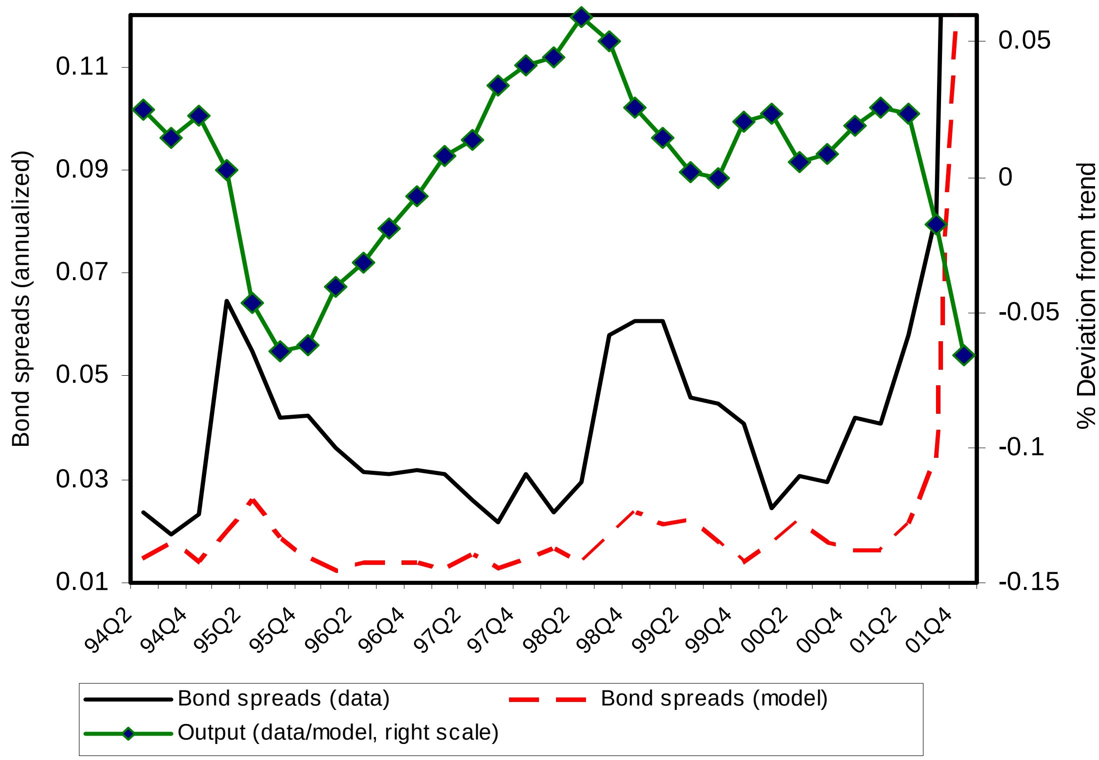

Figure 5: Output and Bond Spreads in the Data and in the Model (1994.2-2001.4)

Appendix

Proof of Theorem 1. The proof consists of three steps.

Step 1. Given any debt recovery schedule $\alpha(b,y) \in A$, we define a price correspondence $\varphi(q)$ that takes points in Q.

$$\varphi(q)(b',y;\alpha) = \begin{cases} \begin{array}{c} (1-p(q)(b',y;\alpha))/(1+r) \\ +p(q)(b',y;\alpha)\cdot\gamma(q)(b',y;\alpha)/(1+r) \end{array} & \text{if } b' \geq 0 \\ \\ 1/(1+r) & \text{if } b' \leq 0 \end{cases} \quad (1)$$

where $p(q)(b',y;\alpha)$ and $\gamma(q)(b',y;\alpha)$ satisfy (12) and (13). Thus, $\varphi(q)(b',y;\alpha)$ is the set of prices for a debt contract of type (b',y) that are consistent with zero profits given the price function q. We can show that $\varphi(q)(b',y;\alpha)$ is a closed interval in R and the correspondence $\varphi(q)(b',y;\alpha)$ has a closed graph (see Lemma App 5 and Lemma 8 in Chatterjee et al. (2002) for similar proofs). Therefore $\varphi(q)(b',y;\alpha)$ is an upper hemicontinuous correspondence. For any $q \in Q$, let $\varphi(q) \subset Q$ be the product correspondence $\Pi_{b',y\in B\times Y}\varphi(q)(b',y;\alpha)$. Since $\varphi(q)(b',y;\alpha)$ is convex-valued for each b',y, $\varphi(q)$ is convex-valued as well. Furthermore, since $\varphi(q)(b',y;\alpha)$ is upper hemicontinuous with compact values for each b',y, the product correspondence $\varphi(q)$ is also upper hemicontinuous with compact values. (see Aliprantis and Border (1999), Thm 16.28). Therefore, $\varphi(q;\alpha)$ is a closed convex-valued correspondence that takes elements of the compact, convex set Q and returns sets in Q. By Kakutani-Fan-Glicksberg FPT (see Aliprantis and Border (1999), Thm 16.51) there is $q^* \in Q$ such that $q^* \in \varphi(q^*)$. Hence, there is an equilibrium bond price function $q^*(b',y)(\alpha)$ given the debt recovery schedule α.

Step 2. Given any bond price function $q(b,y) \in Q$, we define a debt recovery schedule correspondence $\psi(\alpha)$ that takes point in A.

$$\begin{aligned} \psi(\alpha)(b,y;q) \quad = \quad & \arg\max_{a\in[0,1]} \left[\left(\Delta^B(a;b,y,q,\alpha)\right)^{\theta}\left(\Delta^L(a;b,y,q,\alpha)\right)^{1-\theta}\right] \\ & \text{s.t. } \Delta^B(a;b,y,q,\alpha) \geq 0 \\ & \quad \Delta^L(a;b,y,q,\alpha) \geq 0 \end{aligned} \quad (2)$$

$\psi(\alpha)(b,y;q)$ is the set of deb recovery rates for debt contract of type (b,y) that are consistent with Nash bargaining game.

Given q, for each b',y, $\psi(\alpha)(b',y;q)$ is an upper hemicontinuous correspondence with nonempty compact values from Berge's Maximum Theorem (see Aliprantis and Border (1999) Thm 16.31 and the technical appendix for details). For any $\alpha \in A$, let $\psi(\alpha;a) \subset A$ be the product correspondence $\Pi_{b',y\in L\times Y}\psi(\alpha)(b',y;q)$. Since $\psi(q)(b',y;\alpha)$ is upper hemicontinuous with compact values for each b',y, the product correspondence $\psi(q;\alpha)$ is also upper hemicontinuous with compact values. (see Aliprantis and Border (1999), Thm 16.28). For bargaining power $\theta \in \Theta$, $\psi(\alpha)(b',y;q)$ is single-valued, so is the product correspondence $\psi(q;\alpha)$. Therefore, $\psi(q;\alpha)$ is a closed convex-valued correspondence that takes elements of the compact, convex set A and returns sets in A. By Kakutani-Fan-Glicksberg FPT (see Aliprantis and Border (1999), Thm 16.51) there is $\alpha^* \in A$ such that $\alpha^* \in \omega(\alpha^*;q)$. Hence, there exists an equilibrium debt recovery schedule $\alpha^*(b',y)(\alpha)$ given the bond price function q.

Step 3. We construct a functional mapping operator $T: Q\times A \to Q\times A$ such that

$$T(q,\alpha)(b,y) = \begin{bmatrix} \varphi(q)(b,y;q,\alpha) \\ \psi(\alpha)(b,y;q,\alpha) \end{bmatrix}$$

Because $\varphi(q)(b',y;q,\alpha)$ and $\psi(\alpha)(b,y;q,\alpha)$ are upper hemicontinuous, $T(q,\alpha)$ is upper hemicontinous. (see Aliprantis and Border (1999) Thm 16.23). Therefore, the correspondence $T(q,\alpha)$ has a closed graph. We can also show that $T(q,\alpha)$ is convex valued. Suppose $(q_1,\alpha_1) \in T(q,\alpha)$ and $(q_2,\alpha_2) \in T(q,\alpha)$. Because $\varphi(q)(b',y;q,\alpha)$ is convex valued, $\gamma q_1 + (1-\gamma)q_2 \in \varphi(q;\alpha)$. Because $\psi(\alpha)(b,y;q,\alpha)$ is single valued, $\alpha_1 = \alpha_2 = \gamma\alpha_1 + (1-\gamma)\alpha_2 \in \psi(\alpha; q)$. Therefore, $(\gamma q_1 + (1-\gamma) q_2, \gamma\alpha_1 + (1-\gamma)\alpha_2)$. Hence, we can apply Kakutani's fixed point theorem and show the existence of a fixed point.

$$T(q^*, \alpha)(b, y) = (q^*, \alpha^*)$$

A recursive equilibrium exists. ■

Proof of Theorem 2. Because $\Delta^B(a; b, y)$ and $\Delta^L(a; b, y)$ are both function of al, define $\Delta^B(a; b, y) = \widetilde{\Delta}^B(ab; y)$, and $\Delta^L(a; b, y) = \widetilde{\Delta}^L(ab; y)$. The bargaining problem is equivalent to the following

$$\begin{aligned} &\max_{al} \left[\left(\widetilde{\Delta}^B(ab; y) \right)^{\theta} \left(\widetilde{\Delta}^L(ab; y) \right)^{1-\theta} \right] \\ &\text{s.t. } \widetilde{\Delta}^B(al; y) \geq 0 \\ &\quad \widetilde{\Delta}^L(al; y) \geq 0 \end{aligned}$$

where the functional form of $\widetilde{\Delta}^B(al; y)$ and $\widetilde{\Delta}^L(al; y)$ are transformations of $\Delta^B(a; b, y)$ and $\Delta^L(a; b, y)$. For bargaining power $\theta \in \Theta$, given (b, y), the renegotiation surplus has a unique optimum. In the transformed problem, the optimal solution is solely a function of endowment y and we denote it as $b_y \leq 0$. The bargaining over debt reduction has constraint $a \in [0, 1]$. When $b \leq b_y$, the constraint $a \in [0, 1]$ is not binding, so $a = \frac{b_y}{b}$. If $b \geq b_y$, the constraint $a \in [0, 1]$ is binding, so $a = 1$.

Therefore,

$$\psi(\alpha; q)(b, y) = \begin{cases} \frac{b_y}{b} \text{ if } b \leq b_y \\ 1 \text{ if } b \geq b_y \end{cases}$$

Because an equilibrium debt recovery rate function is a fixed point of the correspondence $\psi(\alpha; q)(b, y)$, the debt recovery rate also satisfies

$$\alpha(b, y) = \begin{cases} \frac{b_y}{b} \text{ if } b \leq b_y \\ 1 \text{ if } b \geq b_y \end{cases}$$

■

Proof of Theorem 3. It is easy to show that $v(b, 0, y)$ is increasing in b (see technical appendix for details). Since the equilibrium debt recovery schedule satisfies Theorem 2, given endowment y, the debt arrear after defaulting is independent of b. Thus, the utility from defaulting is independent of b. Therefore, if $v(b^1, 0, y) = u((1-\lambda_d) y) + \beta v(b_y, 1, y)$, then it must be the case that $v(b^0, 0, y) = u((1-\lambda_d) y) + \beta v(b_y, 1, y)$. Hence, any y that belongs in $\overline{D}(b^1)$ must also belong in $\overline{D}(b^0)$. ■

Proof of Theorem 4. Let $d^*(b, 0, y')$ be the equilibrium default functions. Equilibrium default probability is then given by

$$p(b', y) = \int_G d^*(b', 0, y') \, d\mu(y'|y)$$

From Theorem 3, if $d^*(b^1, 0, y') = 1$, then $d^*(b^0, 0, y') = 1$. Therefore,

$$p(b^0, y) \geq p(b^1, y)$$

■

Proof of Theorem 5. Let $p^*(b, y)$ be the equilibrium default probability function and $\alpha^*(b, y)$ be the equilibrium debt recovery schedule. The expected debt recovery rate is then given by

$$\gamma(b', y) = \frac{\int_Y d(b', 0, y') \, \alpha(b', y') \, d\mu(y'|y)}{\int_Y d(b', 0, y') \, d\mu(y'|y)}$$

From Theorem 2, given y, for $b^0 < b^1 \leq b_y \leq 0$, $\alpha^*(b^0, y) < \alpha^*(b^1, y) \leq 1$. Therefore, the equilibrium expected debt recovery rate $\gamma^*(b^0, y) < \gamma^*(b^1, y) \leq 1$. And from Theorem 4, $p^*(b^0, y) \geq p^*(b^1, y)$. For the country's indebtedness, the equilibrium bond price is given by

$$\begin{aligned} q(b',y) &= \frac{1-p(b',y)}{1+r} + \frac{p(b',y)\cdot\gamma(b',y)}{1+r} \\ &= \frac{1-p(b',y)(1-\gamma(b',y))}{1+r} \end{aligned}$$

Hence, we obtain that

$$q\left(b^0,y\right) \le q\left(b^1,y\right)$$

■

Proof of Theorem 6. Because $u(.)$ is concave function, given b, for all $b' \le 0$,

$$\frac{d}{dl'}\left[u((1-\lambda_d)y+b) - u((1-\lambda_d)y+b-b'/(1+r))\right] \ge 0$$

If $b \in L_{--}$ and $b \ge b$, for all $b' < 0$,

$$\begin{aligned} & u((1-\lambda_d)y+b) - u((1-\lambda_d)y+b-b'/(1+r)) \\ \ge\ & u((1-\lambda_d)y+b) + u((1-\lambda_d)y+b-b'/(1+r)) \\ \ge\ & \beta y^{1-\sigma}\int_Y v(0,0,y')\,d\mu(y'|y) + \beta\int_Y v(b',1,y')\,d\mu(y'|y) \end{aligned}$$

Thus,

$$\begin{aligned} & u((1-\lambda_d)y+b) + \beta\int_Y v(0,0,y')\,d\mu(y'|y) \\ \ge\ & \sup_{b'<0} u((1-\lambda_d)y+b-b'/(1+r)) + \beta\int_Y v(b',1,y')\,d\mu(y'|y) \end{aligned}$$

which implies

$$v(b,1,y) = u((1-\lambda_d)y+b) + \beta\int_Y v(0,0,y')\,d\mu(y'|y)$$

If $b \in L_{--}$ and $b \ge b$, for all $b' < 0$, suppose

$$\begin{aligned} & u((1-\lambda_d)y+b) + \beta\int_Y v(0,0,y')\,d\mu(y'|y) \\ >\ & \sup_{b'<0} u((1-\lambda_d)y+b-b'/(1+r)) + \beta\int_Y v(b',1,y')\,d\mu(y'|y) \end{aligned}$$

then, according to the above analysis,

$$\begin{aligned} & u((1-\lambda_d)y+b) + \beta\int_Y v(0,0,y')\,d\mu(y'|y) \\ >\ & \sup_{b'<0} u((1-\lambda_d)y+b-b'/(1+r)) + \beta\int_Y v(b',1,y')\,d\mu(y'|y) \end{aligned}$$

contradiction. ■